MAT 540A Test No. 3 Fall 2014

Please turn in the solutions on Monday, October 13, in class. Each problem here is worth 2 points.

Question 1 (Qual Exam from August 2010)**.** The unreduced cone, cX, on a space X is given by

$$cX = X \times I / X \times 1,$$

where $I = [0,1]$ is the unit interval. Show that cS^1 is homeomorphic to D^2, where S^1 is the unit circle $\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ and D^2 is the unit disk $\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$.

Question 2 (Qual Exam from August 2010)**.** Let (X, x_0) be a space with basepoint x_0. The reduced cone, $CX = C(X, x_0)$, is given by

$$CX = X \times I / (X \times 1 \cup x_0 \times I).$$

Let x_0 be any point on S^1. Show that CS^1 is homeomorphic to D^2.

Question 3 (Qual Exam from January 2010)**.** Let $\sim$ denote the equivalence relation on the cylinder $S^1 \times [-1,1]$ defined by $(v,-1) \sim (v',-1)$ for all v, $v' \in S^1$, and $(v,1) \sim (v',1)$ for all v, $v' \in S^1$. Prove that the quotient space $S^1 \times [-1,1]/\sim$ is homeomorphic to the unit sphere $S^2 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$.

Question 4 (Qual Exam from August 2010)**.** Let $X = [0,1]/(0,1)$ and let

$$\pi \colon [0,1] \longrightarrow X$$

be the quotient map.

(a) Is X Hausdorff?

Consider the complement of $\pi(0)$ in X, with the subspace topology.

(b) Is this complement of $\pi(0)$ Hausdorff?

Consider the complement of $\pi(1/2)$ in X, with the subspace topology.

(c) Is this complement of $\pi(1/2)$ Hausdorff?

Question 5 (Qual Exam from January 2008)**.** Let $f \colon X \to Y$ be a continuous function, and let $G \subset X \times Y$ be its graph, that is, the subset $G = \{(x,y) \mid y = f(x)\}$. If Y is Hausdorff, prove that G is closed in $X \times Y$.