MAT 540A Comments on Test No. 2 Fall 2014

Question 1. The main point is this: we learned how to construct the unique coarsest topology that contains any collection of subsets of X that we would like to call open. We just need to check that each point of X belongs to at least one of our subsets, so our collection forms a subbasis. Starting with a subbasis, arbitrary finite unions give a basis for a topology. Finally, arbitrary unions generate the topology. This is by design the smallest topology that contains the given collection od subsets.

For the purposes of this problem, we need to include all complements of single points in X in order to make each single point closed. This is a covering of X as long as X has more than one point. That's our subbasis. In the case when X is a single point (did you miss this case?), the answer is clear anyway. It is the discrete topology just as for any finite set.

Question 2. I am a fan of the following example showing that the two subspace topologies on Y might be the same while the topologies on X are different (one strictly finer than the other). It was given in two papers. Just take Y to be a single point $\{y\}$ in any set X. X has to be bigger in order to accomodate the two different topologies. No matter what those topologies are, there is only one topology on $\{y\}$.