A NEW CALCULATION OF THE GEOGRAPHIC CENTER OF PENNSYLVANIA

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Introduction

Prospective freshmen at Penn State University are sometimes told that the flagpole on Old Main marks the exact geographic center of Pennsylvania. The Centre County courthouse in Bellefonte has also been accorded this distinction. Each site functions as a symbolic center, but neither happens to be the true geographic center. Both institutions were already in place before it was even practical for anyone to attempt to determine this point precisely. Any such calculation requires both accurate maps and a means of accurately measuring areas from maps; the latter was not possible until the invention of the planimeter in the late 1850s. Accurate tallies of the areas of the states were not even available until the Bureau of the Census undertook the task in 1881, suggesting that calculations of geographic centers would have been unlikely before this time (Gannett 1881, 1906).

Centre County, formed in 1800, takes its name from Centre Furnace, an iron furnace built in 1791-92. Centre Furnace took its name, in turn, from its unambiguous location at the center of the state. No doubt one of the founders of the furnace noticed that the diagonals of the nearly-rectangular state crossed on their property. (The state was more rectangular than it is today, since the northwestern Erie Triangle was not purchased from the United States until 1792). While this approach was once popularly believed to accurately mark the center, it is really only a rough approximation (Hayford 1902).

In some states, the precise location of the center has been a point of contention and the subject of governmental intervention. In Kentucky, for instance, there are three official claims in three different counties (*Kentucky* 2001). The city of Pittsville was deemed the geographic center of Wisconsin by gubernatorial decree in 1952, though off by more than 20 kilometers (Barmore 1993). In Pennsylvania, no real controversy seems to have ever existed. Despite the informal claims of University Park and Bellefonte boosters, the only genuine candidate location of the center of Pennsylvania appears to be that calculated by E.M. Douglas of the United States Geological Survey in 1923, which placed the center 2.5 miles southwest of Bellefonte. This value is clearly imprecise and of indeterminable accuracy, since no details about the calculation are provided. The 1923 publication also includes estimates of the centers of all of the other states, a list that has been kept in print, without revision, to the present (Douglas 1930, Van Zandt 1966, Van Zandt 1976, USGS 1991, USGS 2000).

The USGS never claimed that their published list of geographic centers was anything more than "sufficiently accurate for ordinary purposes". In the 1966 and subsequent editions, another disclaimer was added that no single, true center can ever be determined because of the multiplicity of methods for calculating a center. On the contrary, the geographic center is uniquely defined as the point at which a portion of the globe would balance if it was peeled off in a layer of negligible and uniform thickness (Barmore 1992, 1993). The center of a hemisphere is easy to imagine: the geographic center of the northern hemisphere is the North Pole, and the geographic center of the western hemisphere is where 90° W meets the equator, near the Galapagos Islands. If the area of interest is an irregular shape, then the calculation is more complex and a correct solution is constrained by the limits of map precision and surveying error. Still, this not the same as identifying multiple centers. Recent technological advances have made the calculation of a geographic center easier, but anyone armed with a complete set of quadrangles, slide rule and planimeter could have done better than Douglas any time after 1923.

Calculation

The technique used in this research to recalculate the center of Pennsylvania takes advantage of the five minute by five minute grid representing the World Geodetic System (WGS84) datum included in DeLorme's 1:150,000 *Pennsylvania Atlas and Gazetteer* (DeLorme 1996). Pennsylvania is described by a total of 1,902 grid cells. 1,824 of these have regular geometry, including 137 cells along the regular borders of the state. There are also 78 grid cells with irregular geometry along the irregular borders of the state. The geographic center of the state is the weighted average of the center points of each of the grid cells, weighted by their area.

The areas and centers of the regular grid cells can be determined with perfect accuracy if the earth is assumed a sphere. Of course, the earth is more truly described as an ellipsoid, but this distinction has a negligible impact on the calculation of the center and will be taken up later in the paper. The areas and centers of the irregular grid cells must be estimated. These estimates introduce error into the calculation, but the net effect of the error is small since only about 2% of the land area of the state falls in this category.

For the regular grid cells, areas and centers were calculated mathematically. Areas were calculated using the formula:

$$\mathrm{Sin}\,(\mathrm{lat}_2) - \mathrm{Sin}(\mathrm{lat}_1) * (\mathrm{lon}_2 - \mathrm{lon}_1) * 4\mathrm{L} / \pi^2 \qquad (1)$$

Where lat_1, lat_2, lon_1 and lon_2 describe the latitude and longitude coordinates of the boundary of the grid cell in radians, and L is the distance along the surface of the earth from the equator to the pole, taken to be exactly 10 million meters. Note that $\mathrm{lon}_2 - \mathrm{lon}_1$ equals five minutes.

The centers of the grid cells are described by a latitude and a longitude coordinate. The longitude coordinate is simply the average of the longitudes of the eastern and western boundaries of the cell. The latitude coordinate is slightly south of the average of the northern and southern boundaries of the grid cell, because the southern boundary is wider than the northern boundary, owing to the spherical shape of the earth. The latitude coordinate can be calculated as:

$$\frac{\int \theta \;\cos\theta \;d\theta}{\int \cos\theta \;\; d\theta} \qquad (2)$$

which has as its solution:

$$\frac{lat_2 \sin(lat_2) + \cos(lat_2) - lat_1 \sin(lat_1) - \cos(lat_1)}{\sin(lat_2) - \sin(lat_1)} \quad (3)$$

For the irregular grid cells, areas were estimated using planimetry. The planimeter used was capable of estimating areas accurately to within about 0.5%, but to be conservative, this was rounded to 1%. The centers of the irregular grid cells were estimated based on the shape of the land area in these cells. These were taken to be accurate to within 30 seconds, again conservatively. More precise measurements than these could have been obtained using digital boundary files and GIS technology, but since the approach used in this paper already yields a center that is accurate to within 35 meters, additional precision did not seem called for.

Assessing Measurement Error

All of the state boundaries, as well as the non-spherical shape of the earth, contribute to measurement error. In the case of the irregular boundaries, the accuracy and precision of areas and centers is limited by the quality of the printed maps and the measurement accuracy of the planimeter. For the regular boundaries, error results from historical surveying errors. The regular boundaries of Pennsylvania (the northern, southern and western boundaries, as well as the Pennsylvania-New York boundary in the Erie Triangle) are only nominally straight lines. These lines were surveyed between 1764 and 1790 and passed through areas that were primarily wilderness. The practical limits of astronomical surveying techniques coupled with the difficult terrain meant that the surveying parties were prone to drift away from the true line by as much as 300 meters. Clear nighttime skies and a prominent vantage point permitted corrections to be made, yielding an almost-imperceptible zigzag back and forth across the true line (Russ 1966).

The governments of Pennsylvania and the adjoining states authorized resurveys in the late nineteenth century to correct these earlier errors, and to reset the boundaries where the original markers had been lost (Commonwealth 1887, 1909). Correcting the errors was quickly found to be impractical, as it would involve revising long agreed-upon property lines. A portion of the village of

Lawrenceville in Tioga County, Pennsylvania, for example, would have had to have been reassigned to New York, with some individual houses straddling two states. Such adjustments were understandably unpopular, though there were property owners in favor of them, including a lumber mill operator who was hoping to have his mill reassigned to Pennsylvania, where the cost of business was lower (Commonwealth 1887, 562). In general, the original historic boundaries were preserved, except in some locations that still remained unsettled or were public property.

The deviations in these "straight" boundaries are small and largely random and have virtually no impact on the calculation of the geographic center. The only error of a systematic nature appears to be the Pennsylvania-Ohio line, which drifts westward about thirty meters as one travels from the Ohio River to Lake Erie, and shifts the geographic center about four meters to the west and two meters north. The reports of the resurveys describe each the boundaries in precise detail. At well-defined points along the boundaries, such as at road and railroad crossings, the latitude and longitude coordinates are given to the nearest tenth of an second or finer, well within ten meter accuracy. These well-defined points were used to assign the latitude and/or longitude values of the corners of the 137 regular grid cells along the boundaries of the state.

The earth is slightly flatter than a true sphere. If this flattening is to be taken into account, then the equation below must be used in place of equation 2, where the earth is taken to be an ellipsoid with semimajor axis a and semiminor axis b:

$$\frac{\int \theta \ (b/a) \sqrt{a^2 - b^2 \sin^2\theta} \ d\theta}{\int (b/a) \sqrt{a^2 - b^2 \sin^2\theta} \ d\theta} \qquad (4)$$

The problem this equation is that it is unsolvable. The square root term is known as the "elliptic integral" and cannot be integrated (Thomas and Finney 1984, 530). It can, however, be approximated using the trapezoidal rule, which involves breaking up the area under the curve into trapezoids and summing their areas. As the number of trapezoids increases, the exact solution is approached.

Table 1 compares the results of calculating the centroid of an area bounded by latitude 39.72° and 42° and symmetric about some arbitrary longitude (that is, a regularly shaped area of the same geographic extent as Pennsylvania), assuming the earth is both a sphere and an ellipsoid. The more accurate ellipsoid-based calculation places the centroid 0.00007° further north, about eight meters. (The WGS84 reference ellipsoid was used for this calculation, although the results are not affected by the choice of ellipsoid.) Using the exact shape of Pennsylvania here would make for a highly complicated calculation, but the result would be extremely close to eight meters.

TABLE 1

Geographic Center of a Regular Area between 39.7217 and 42 Degrees Latitude

	Latitude
Spherical earth	40.85432°
Ellipsoidal earth, estimated using the trapezoidal rule and 30' sections	42.85419°
15' sections	42.85434°
7.5' sections	42.85438°
5' sections	42.85439°
2.5' sections	42.85439°

The various sources of error were incorporated into the final calculation. For each irregular grid cell, up to 1% was randomly added or subtracted from the estimated area, and up to 30 seconds were randomly added or subtracted from the estimated center location in both dimensions. Random noise was also added to each of the grid cells along the regular boundaries to capture the surveying error. The calculation was repeated many times so as to determine a mean and standard deviation. Since each error component was measured conservatively, the standard deviation is an overestimate.

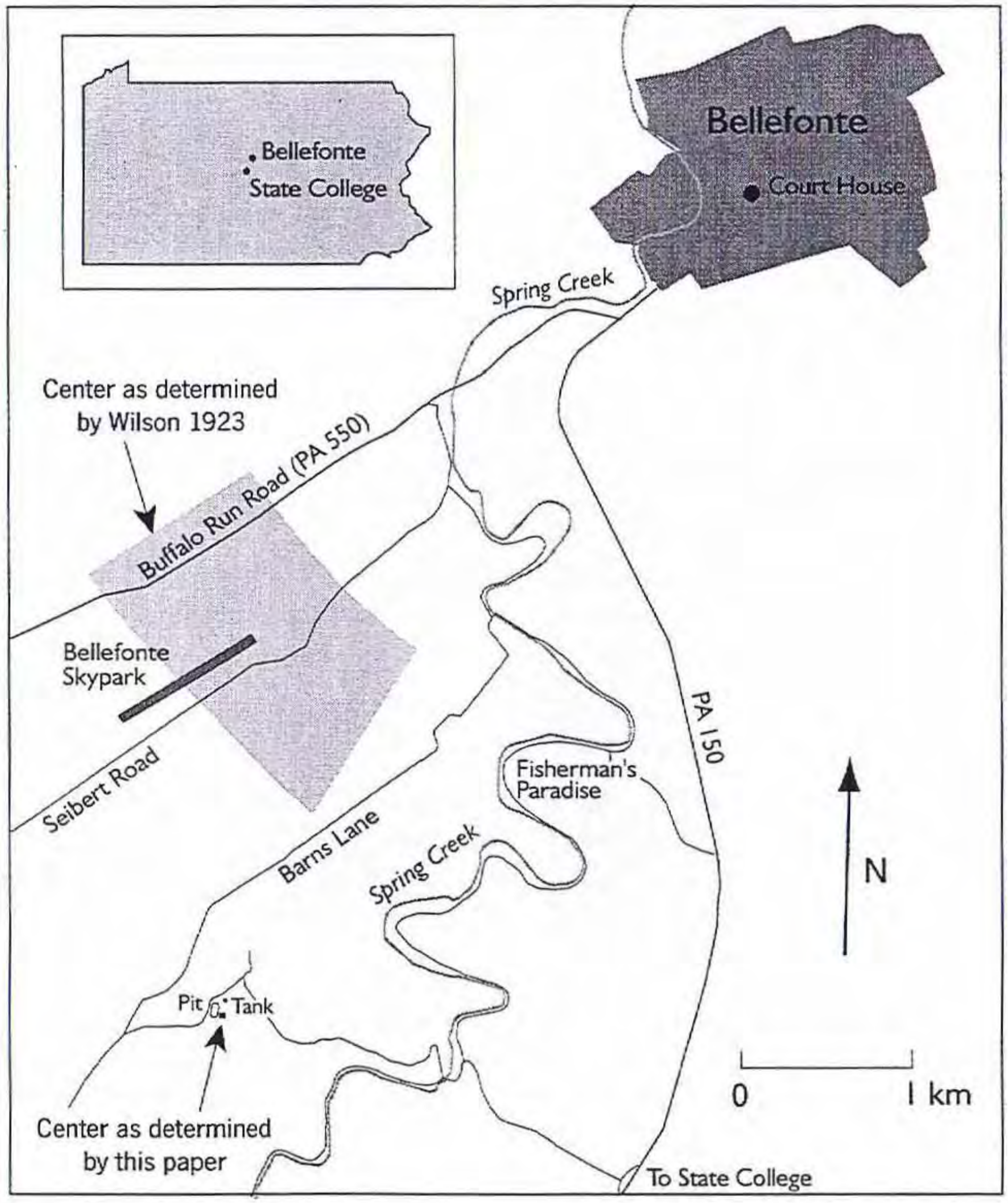

Figure 1. Map of the Geographic Center of Pennsylvania.

Conclusion

The latitude of the center of Pennsylvania is found to be 40.86833° ± 0.00007° (about 15 meter accuracy). The longitude is 77.8124° ± 0.0002° (about 35 meter accuracy). The longitude is less accurate than the latitude because the greatest source of uncertainty is the irregular eastern border with New Jersey and New York. When the ellipsoidal shape of the earth is accounted for, the latitude shifts northward to 40.86840° ± 0.00008° (about 20 meter accuracy). The location of this point is in Benner Township, Centre County, roughly midway between the boroughs of Bellefonte and State College, near a small quarry along an unnamed lane on the property of Rockview State Penitentiary. Spring Creek flows just to the east (Figure 1). Also shown on this figure are the possible locations of the center based on the 1923 determination "2.5 miles southwest of Bellefonte".

Geographic centers are perhaps little more than geographic curiosities and their measurement an occasion to dust off calculus textbooks. In the case of Pennsylvania, though, the center conveys greater significance, linked to early settlement and industry and eventually a county name. Some sort of commemorative marker calling attention to this point seems called for. The measured point is not highly accessible, but approximately two kilometers to the southwest of this point is the University Park airport. No doubt many passengers waiting at the terminal would be intrigued to know that they are very truly in the center of Pennsylvania.

Literature Cited

Barmore, F.E. 1992. The earth isn't flat. And it isn't round either! Some significant and little known effects of the earth's elliptical shape. *Wisconsin Geographer* 8: 1-9. Reprinted in *Solstice: An Electronic Journal of Geography and Mathematics* 4(1): 26-38.

Barmore, F.E. 1993. Center Here, Center There; Center, Center Everywhere. *Wisconsin Geographer* 9: 8-21. Reprinted in *Solstice: An Electronic Journal of Geography and Mathematics* 5(1): 12-25.

Commonwealth of Pennsylvania. Secretary of Internal Affairs. 1887. Report of the Secretary of Internal Affairs of the Commonwealth of Pennsylvania Containing Reports of the Surveys and Resurveys of the Boundary Lines, Accompanied with Maps of the Same. Harrisburg: Edwin K. Meyers.

Commonwealth of Pennsylvania. Secretary of Internal Affairs. 1909. *Report on the Resurvey of the Maryland-Pennsylvania Boundary, Part of the Mason and Dixon Line*. Harrisburg: Harrisburg Publishing Company.

DeLorme. 1996. *Pennsylvania Atlas and Gazetteer. Fourth Edition.* Freeport, ME: DeLorme.

Douglas, E.M. 1923. *Boundaries, Areas, Geographic Centers and Altitudes of the United States and the Several States, etc. (Geological Survey Bulletin 689).* Washington, DC: United States Geological Survey.

Douglas, E.M. 1930. *Boundaries, Areas, Geographic Centers and Altitudes of the United States and the Several States, etc. 2nd ed. (Geological Survey Bulletin 689).* Washington, DC: United States Geological Survey.

Gannett, H. 1881. *The Areas of the United States, the Several States and Territories, and their Counties.* Washington, DC: GPO.

Gannett, H. 1906. *The Areas of the United States, the States, and the Territories (Geological Survey Bulletin 302).* Washington, DC: United States Geological Survey.

Hayford, J.F. 1902. What is the Center of an Area, or the Center of a Population? Journal of the American Statistical Association, New Series 58(8): 47-58.

Kentucky Atlas and Gazetteer. On-line: www.uky.edu/Kentucky Atlas/ ky-geographic_center.html. Accessed May 13, 2001.

Russ, W.A. 1966. *How Pennsylvania Acquired its Present Boundaries*. University Park: Pennsylvania Historical Association.

Thomas, G.B. and R.L. Finney. 1986. *Calculus and Analytic Geometry, 6th ed.* Reading, MA: Addison-Wesley.

United States Geological Survey. 1991. *Elevations and Distances in the United States*. Washington, DC: United States Geological Survey.

United States Geological Survey. 2000. *Elevations and Distances in the United States. Online edition.* On line: http://mac.usgs.gov/mac/isb/pubs/booklets/elvadist/elvadist.html. Accessed May 5, 2001.

Van Zandt, F.K. 1966. *Boundaries of the United States and the Several States (Geological Survey Bulletin 1212).* Washington, DC: United States Geological Survey.

Van Zandt, F.K. 1976. *Boundaries of the United States and the Several States (Geological Survey Professional Paper 909).* Washington, DC: United States Geological Survey.