Discretionary Policy and Time Inconsistency

Walsh Chapter 7

1 Time Inconsistency Yields Inflation Bias

- Policy Objective

  - loss function

$$V = \frac{1}{2}\lambda\left(y - y_n - k\right)^2 + \frac{1}{2}\pi^2$$

    * monetary authority wants output (y) above the natural rate (y_n) by a fixed amount k, perhaps due to distortions caused by monopolistic competition or to politics

    * desired inflation is zero, and loss is increasing in inflation at an increasing rate

* expanding the loss function

$$V = -\lambda k \left(y - y_n\right) + \frac{1}{2}\pi^2 + \frac{1}{2}\lambda \left(y - y_n\right)^2 + \frac{1}{2}\lambda k^2$$

– alternative representation of loss function – drop quadratic terms in output

$$V = -\lambda k \left(y - y_n\right) + \frac{1}{2}\pi^2$$

* more output is preferred to less

* write as utility function for central banker, dropping k

$$U = \lambda \left(y - y_n\right) - \frac{1}{2}\pi^2$$

- Barro and Gordon (1983)
  - Model
    - Lucas aggregate supply curve due to nominal wage rigidity (wages set in advance based on π^e)

$$y = y_n + a\left(\pi - \pi^e\right) + e$$

    - money market equilibrium

$$\pi = \Delta m + v$$

- Central Bank behavior
  * π^e is set before bank chooses Δm
  * central bank observes e, but not v
  * e and v are uncorrelated

- Sequence of events
  * private sector sets the nominal wage based on π^e
  * the supply shock (e) is realized after the nominal wage is set
  * central bank chooses Δm, possibly responding to e
  * after Δm is set, the velocity shock (v) is realized
  * Central bank has an informational advantage (know e), giving it a stabilizing role

– Maximization with linear utility

$$E(U) = E\lambda\left[a\Delta m + av - a\pi^e + e\right] - \frac{1}{2}E(\Delta m + v)^2$$

* derivative with respect to Δm, taking π^e and e as given

$$\lambda a - \Delta m = 0$$

* Δm has marginal benefits as it raises output by creating a surprise

* marginal costs as it raises inflation, but at $\Delta m = 0$, these marginal costs are zero

* therefore, incentive to raise Δm above zero until marginal costs rise to marginal benefits

* actual inflation

$$\pi = \Delta m + v = \lambda a + v$$

- Rational Expectations

$$\pi^e = E(\Delta m) = \lambda a$$

* average inflation is λa

* no effect on output since fully anticipated

* inflation bias is increasing in the effect of a price surprise on output (a) and the weight on output in the objective function (λ)

* inflate until the marginal benefits of increasing output due to the inflation surprise (λa) equal the marginal costs due to higher inflation (π)

- Compare expected utility under discretion and commitment
  * Expected utility under discretion

$$E\left(U^d\right) = E\left[\lambda\left(av+e\right) - \frac{1}{2}\left(a\lambda+v\right)^2\right] = -\frac{1}{2}\left(a^2\lambda^2+\sigma_v^2\right)$$

  * Expected utility under commitment to a fixed money growth rule

$$\Delta m = 0$$

$$E\left(U^c\right) = E\left[\lambda\left(av+e\right) - \frac{1}{2}v^2\right] = -\frac{1}{2}\left(\sigma_v^2\right)$$

  * Discretion entails a loss in utility

– Alternative loss function to give money a stabilizing role in response to e

$$V = \frac{1}{2}\lambda\left[a\left(\Delta m + v - \pi^e\right) + e - k\right]^2 + \frac{1}{2}\left(\Delta m + v\right)^2$$

∗ Choose Δm after observing e, but before observing v to maximize $E\left(V\right)$

$$\lambda E\left[a\left(\Delta m + v - \pi^e\right) + e - k\right]a + \Delta m = 0$$

∗ Solving for Δm yields

$$\Delta m = \frac{a^2\lambda\pi^e + a\lambda\left(k - e\right)}{1 + a^2\lambda}$$

optimal policy depends on both π^e and e

* Rational expectations

$$\pi^e = E(\Delta m) = \frac{a^2\lambda\pi^e + a\lambda k}{1 + a^2\lambda}$$

- Solving for π^e yields

$$\pi^e = a\lambda k$$

- Substitute into Δm

$$\Delta m = \frac{\left(a^2\lambda\right)(a\lambda k) + a\lambda(k - e)}{1 + a^2\lambda} = a\lambda k - \frac{a\lambda}{1 + a^2\lambda}e$$

- Inflation under discretion becomes

$$\pi^d = \Delta m + v = a\lambda k - \frac{a\lambda}{1 + a^2\lambda} e + v$$

- positive average and expected inflation

$$\pi^e = a\lambda k$$

- no effect on output because anticipated

- optimal to offset a positive supply shock with a reduction in money growth and inflation

– Compare expected utility under discretion and commitment

* Expected loss under discretion

$$EV = \frac{1}{2}E\left\{\lambda\left[a\left(\Delta m + v - \pi^e\right) + e - k\right]^2 + \left(\Delta m + v\right)^2\right\}$$

$$\begin{aligned}
EV &= \frac{1}{2}E\left\{\lambda\left[-\frac{a^2\lambda}{1+a^2\lambda}e + e + av - k\right]^2 + \left(a\lambda k - \frac{a\lambda}{1+a^2\lambda}e + v\right)^2\right. \\
&= \frac{1}{2}\lambda\left[\frac{1+a^2\lambda}{\left(1+a^2\lambda\right)^2}\sigma_e^2 + \left(a^2 + \frac{1}{\lambda}\right)\sigma_v^2 + k^2\left(1+\lambda a^2\right)\right] \\
&= \frac{1}{2}\left[\frac{\lambda}{\left(1+a^2\lambda\right)}\sigma_e^2 + \left(1+a^2\lambda\right)\left(\sigma_v^2 + \lambda k^2\right)\right]
\end{aligned}$$

* Optimal policy under commitment when money is allowed to respond to e

$$\Delta m^c = b_0 + b_1 e$$

$$\pi^e = b_0$$

$$V^c = \frac{1}{2}\lambda\left[ab_1 e + e + av - k\right]^2 + \frac{1}{2}\left(b_0 + b_1 e + v\right)^2$$

· Values for b_0 and b_1 are chosen to minimize the unconditional expectation of V^c

· Derivative with respect to b_0

$$E\left[b_0 + b_1 e + v\right] = 0,$$

implying average and expected inflation is zero

$$b_0 = 0$$

- derivative with respect to b_1

$$E\left[\lambda\left(ab_1e + e + av - k\right)ae + \left(b_0 + b_1e + v\right)e\right] = 0$$

$$\left(\left(1 + \lambda a^2\right)b_1 + \lambda a\right)\sigma_e^2$$

$$b_1 = \frac{-\lambda a}{1 + a^2\lambda}$$

$$\Delta m^c = \frac{-\lambda a}{1 + a^2\lambda}e$$

implying the same response to a supply shock under commitment as under discretion

- Unconditional expectation of loss

$$
\begin{aligned}
EV^c &= E\left\{\frac{1}{2}\lambda\left[ab_1e+e+av-k\right]^2+\frac{1}{2}\left(b_0+b_1e+v\right)^2\right\} \\
&= \frac{1}{2}\left\{\lambda k^2+\left[\left(\frac{\lambda}{\left(1+a^2\lambda\right)^2}\right)+\left(\frac{-\lambda a}{1+a^2\lambda}\right)^2\right]\sigma_e^2\right\} \\
&\quad +\frac{1}{2}\left(1+\lambda a^2\right)\sigma_v^2 \\
&= \frac{1}{2}\left\{\lambda k^2+\left(\frac{\lambda\left(1+\lambda a^2\right)}{\left(1+a^2\lambda\right)^2}\right)\sigma_e^2+\left(1+\lambda a^2\right)\sigma_v^2\right\} \\
&= \frac{1}{2}\left\{\lambda k^2+\left(\frac{\lambda}{\left(1+a^2\lambda\right)}\right)\sigma_e^2+\left(1+\lambda a^2\right)\sigma_v^2\right\}
\end{aligned}
$$

- Loss under discretion is larger by $\frac{(a\lambda k)^2}{2}$

– Compare loss under discretion with loss under a rigid rule with $\Delta m = 0$

* loss with $\Delta m = 0$

$$EV = E\left\{\frac{1}{2}\lambda\left[a\left(\Delta m + v - \pi^{e}\right) + e - k\right]^{2} + \frac{1}{2}\left(\Delta m + v\right)^{2}\right\}$$

$$\begin{aligned} EV^{r} &= E\left\{\frac{1}{2}\lambda\left[av + e - k\right]^{2} + \frac{1}{2}\left(v\right)^{2}\right\} \\ &= \frac{1}{2}\left\{\lambda k^{2} + \lambda\sigma_{e}^{2} + \left(1 + \lambda a^{2}\right)\sigma_{v}^{2}\right\} \end{aligned}$$

* loss under discretion

$$\frac{1}{2}\left[\frac{\lambda}{\left(1 + a^{2}\lambda\right)}\sigma_{e}^{2} + \left(1 + a^{2}\lambda\right)\left(\sigma_{v}^{2} + \lambda k^{2}\right)\right]$$

* rigid rule preferred to discretion if

$$\lambda k^2+\lambda\sigma_e^2+\left(1+\lambda a^2\right)\sigma_v^2<\frac{\lambda}{\left(1+a^2\lambda\right)}\sigma_e^2+\left(1+a^2\lambda\right)\left(\sigma_v^2+\lambda k^2\right)$$

$$\frac{a^2\lambda}{\left(1+a^2\lambda\right)}\sigma_e^2<(a\lambda k)^2$$

  - if loss from not responding to supply shocks is less than loss from having money grow too fast

2 Inflation Bias

- Can reputation in a repeated game eliminate inflation bias?
  - Central Bank's objective function

$$U = \lambda (y - y_n) - \frac{1}{2}\pi^2$$

  - Public behavior: trigger strategy
    * if $\pi_{t-1} = \bar{\pi}$, then public expects $\pi_t^e = \bar{\pi} < a\lambda$
    * If $\pi_{t-1} > \bar{\pi}$, then public expects $\pi_t^e = a\lambda$, that is inflation under pure discretion
    * one-period punishment: if can set $\pi_{t+1} = \bar{\pi}$, then get $\pi_{t+2}^e = \bar{\pi} < a\lambda$ again

- Central Bank choses inflation to maximize

$$\sum_{i=0}^{\infty} E_t \beta^i \left(U_{t+i}\right)$$

  * Assume that $\pi_s = \bar{\pi}$ for $s < t$, then $\pi_t^e = \bar{\pi}$
  * Assume $e = 0$ and that central bank controls π_t directly (with $\pi = \Delta m + v$, v must be zero)
  * Substituting for $(y - y_n)$, the marginal benefit of higher inflation is higher output

$$U = \lambda a \left(\pi - \pi^e\right) - \frac{1}{2}\pi^2$$

* First order condtion on inflation yields discretionary policy

$$\frac{\partial U}{\partial \pi} = \lambda a - \pi = 0$$

* Utility with inflation at the discretionary level λa and expectations at $\bar{\pi}$ is

$$U(\lambda a, \bar{\pi}) = \lambda a (\lambda a - \bar{\pi}) - \frac{1}{2}(\lambda a)^2$$

* Utility with inflation at commitment level of $\bar{\pi}$ and expectations also at $\bar{\pi}$

$$U(\bar{\pi}, \bar{\pi}) = -\frac{1}{2}(\bar{\pi})^2$$

* Temptation to cheat is utility under discretion less utility under commitment

$$G(\bar{\pi}) = \lambda a (\lambda a - \bar{\pi}) - \frac{1}{2}\left[(\lambda a)^2 - (\bar{\pi})^2\right] = \frac{1}{2}(\lambda a - \bar{\pi})^2$$

· decreasing in $\bar{\pi}$ for small $\bar{\pi}$, at a decreasing rate (eventually increasing)

∗ Costs to cheating (enforcement): the public sets $\pi^e_{t+1} = a\lambda$ in the next period. Bank will set $\pi_{t+1} = \lambda a$.

$$\begin{aligned} C(\bar{\pi}) &= \beta\left[U(\bar{\pi}, \bar{\pi}) - U(\lambda a, \lambda a)\right] \\ &= \beta\left[-\frac{1}{2}(\bar{\pi})^2 + \frac{1}{2}(\lambda a)^2\right] = \frac{\beta}{2}\left((\lambda a)^2 - (\bar{\pi})^2\right) \end{aligned}$$

· costs to cheating is decreasing in $\bar{\pi}$

– Any $\bar{\pi}$ such that $C(\bar{\pi}) > G(\bar{\pi})$ can be supported as an equilibrium

∗ with the costs to cheating greater than the gain

∗ no incentive to deviate

* at social optimum $(\bar{\pi} = 0)$, gains to cheating exceed the costs, so not an equilibrium

$$C(0) - G(0) = (\lambda a)^2 \left[\frac{\beta}{2} - \frac{1}{2}\right] < 0$$

* set costs equal to gains and solve for $\bar{\pi}$

$$\frac{\lambda a (1-\beta)}{1+\beta} \leq \bar{\pi} \leq \lambda a$$

  · minimum sustainable inflation rate exceeds zero and is decreasing in β – the greater the weight on the future (larger β), the smaller the minimum sustainable inflation rate is because raises the value of discounted costs

- Problems
  - How does an atomistic public coordinate on the trigger strategy? possibly with a single union
  - Is it in the interest of the public to punish the central bank when it deviates, or does punishment hurt the public so it will not occur?
  - How can the public know the central bank has cheated when actual inflation is a stochastic outcome of policy?
    - Suppose the central bank reacts to private information on the velocity shock so that Δm differs from the expected value, but has not cheated on setting money growth to achieve desired inflation
    - Let central bank fix the exchange rate. If an unexpected change in money is required to peg the exchange rate, then it has not cheated. However, if the exchange rate changes, then it has cheated.

- Explaining bouts of inflation as times when inflation bias dominates
  - Backus and Driffill (1985)
    - Two types of central bankers
      - Optimizers who maximize expected discounted value of utility (unable to commit)
      - Inflation fighters who always pursue zero inflation (commit)
      - Public does not know type, but has prior beliefs about type
      - Both have incentive to announce that they are inflation fighters to get expectations of inflation low, so an announcement is not credible

- If central bank ever behaves as an optimizer (because it is an optimizer), its identity is revealed and public expects equilibrium under optimization. To avoid this outcome, optimizer has the incentive to behave as an inflation-fighter.

* Sequential equilibrium with a final period T

π^d is desired inflation rate for inflation-fighter (dry)

π^w is desired inflation rate for optimizer (wet)

- In final period T

$$\pi_T^d = 0 \qquad \pi_T^w = \lambda a$$

- In periods prior to T, wet central banker randomizes, and if inflates in period t, then $\pi_t = \lambda a$ until final period. If not, then keep $\pi_t = 0$

– Ball (1995)

  * Two types of central bankers

    · D sets inflation to zero (commits)

    · W minimizes loss

$$L^w = \sum_{i=0}^{\infty} \beta^i \left[\lambda \left(y_{t+i} - y_n - k \right)^2 + \pi_{t+i}^2 \right]$$

    · Central bank type is Markov

$$\begin{aligned} P\left(D_{t+1}|D_t\right) &= d \qquad P\left(W_{t+1}|D_t\right) = 1 - d \\ P\left(D_{t+1}|W_t\right) &= 1 - w \qquad P\left(W_{t+1}|W_t\right) = w \end{aligned}$$

if shifts in policy are unlikely, then d and w are relatively large

- Supply

$$\begin{aligned} y_t &= y_n + a\left(\pi_t - \pi_t^e\right) + e \\ e &\in (0, \bar{e}) \qquad P\left(e = 0\right) = 1 - q \qquad P\left(e = \bar{e} < 0\right) = q \end{aligned}$$

if supply shocks are unlikely, then q is small

* Timing

- Public forms expectations π_t^e

- Supply shock (e_t) is realized and observed

- Central bank type is realized and not observable

- Central bank sets π_t

* Nash equilibrium: actions depend only on variables that affect current payoffs (ruling out a trigger strategy which depends on π_{t-1}, when current payoffs do not depend on π_{t-1})

  - W sets $\pi = 0$ as long as $e = 0$

  - W sets π at discretionary level once $e = \bar{e}$. Inflation remains at the discretionary level until type D takes over.

  - When D takes over, sets inflation at 0. Inflation remains at zero until economy gets W and $\bar{e}$ together.

  - Explains periodic and persistent bouts of inflation in response to negative supply shocks

- Cukierman and Liviatan (1991)
  * Two types of central bankers
    - D commits to announced policy, but not necessarily to 0
    - W cannot commit
    - Share utility

$$U = \lambda\left(y - y_n\right) - \frac{1}{2}\pi^2$$

$$y - y_n = a\left(\pi - \pi^e\right)$$

* Two-period model
  - W sets discretionary rate of inflation λa
  - D's choice for inflation depends on the equilibrium type
* **Separating equilibrium**: behavior of central bank in first period reveals its type
  - In period 2, public knows central bank type
  - D can commit, so optimal to announce $\pi_2 = 0$; public knows D commits, so $\pi_2^e = 0$
  - In period 1, probability type is D is q

- Expected inflation: D announces π^a and W announces λa

$$\pi_1^e = q\pi^a + (1-q)\,\lambda a$$

- Type D chooses π^a to maximize utility given by

$$\begin{aligned} U_{sep}^D &= \lambda\,(y_1 - y_n) - \frac{1}{2}\pi_1^2 + \beta\left[\lambda\,(y_2 - y_n) - \frac{1}{2}\pi_2^2\right] \\ &= \lambda a\,(\pi_1 - \pi_1^e) - \frac{1}{2}\pi_1^2 \end{aligned}$$

since $y_2 = y_n$ and $\pi_2 = 0$

- Substituting for expected inflation, taking derivative and setting it to zero to solve for inflation

$$U_{sep}^D = \lambda a\,(\pi^a - q\pi^a - (1-q)\,\lambda a) - \frac{1}{2}(\pi^a)^2$$

$$\pi^a = \lambda a\,(1-q)$$

Inflation chosen by D is greater than zero, since expectations of inflation are greater than zero. Chooses positive inflation since faces imperfect credibililty. Since inflation is less than that of W, policy reveals his type

- Outcome: if D, then inflation in first period is positive, but less than expectations, output is below natural rate, and in second period output is at natural rate and inflation is zero. If W then inflation in both periods is λa. In first period, this exceeds expectations so output is above natural rate, and returns to natural rate in second.

- Utility for W in separating equilibrium

$$
\begin{aligned}
U^W_{sep} &= \lambda a \left(\lambda a - q\lambda a \left(1-q\right) - \left(1-q\right)\lambda a\right) - \frac{1}{2}\left(\lambda a\right)^2 - \beta\left[\frac{1}{2}\left(\lambda a\right)^2\right] \\
&= \left(\lambda a\right)^2\left(1-\left(1-q\right)\left(1+q\right)\right) - \frac{1}{2}\left(\lambda a\right)^2\left(1+\beta\right) \\
&= \left(\lambda a\right)^2\left[q^2 - \frac{1}{2}\left(1+\beta\right)\right]
\end{aligned}
$$

* **Pooling equlibrium**: both types behave identically in first period so identity is not revealed
  - W must make same announcement in period 1 and choose same inflation as D
  - D faces expectations in period 2

$$\pi_2^e = q\pi_2^a + (1-q)\,\lambda a$$

  - So chooses $\pi_2^a = \lambda a\,(1-q)$ as before
  - In period 1, D knows that W will mimic its actions so it chooses and delivers $\pi_1 = 0$
  - Outcome: inflation in period 1 is zero and output is at the natural rate. In period 2, inflation is either λa or $\lambda a\,(1-q)$, and output is above the natural rate if W and below if D

- Utility for W in pooling equilibrium, in which W mimics and inflates at 0 during period 1 and at λa during period 2

$$
\begin{aligned}
U^W_{pool} &= \beta\left[\lambda\left(y_2 - y_n\right) - \frac{1}{2}\pi_2^2\right] \\
&= \beta\left[\lambda a\left[\lambda a - q\left(\lambda a\left(1-q\right)\right) - \left(1-q\right)\lambda a\right] - \frac{1}{2}\left(\lambda a\right)^2\right] \\
&= \beta\left(\lambda a\right)^2\left[1 - \left(1-q\right)\left(1+q\right)\right] \\
&= \beta\left(\lambda a\right)^2\left[q^2 - \frac{1}{2}\right]
\end{aligned}
$$

* Assume have a separating equilibrium in which D inflates as $\lambda a(1-q)$ in period 1, and W inflates at λa. Will it be in $W's$ interest to deviate and mimic D by also inflating at $(1-q)\lambda a$ in period 1?

$$\begin{aligned}
U_m^W &= \lambda a\left[(1-q)\lambda a - q\lambda a(1-q) - (1-q)\lambda a\right] - \frac{1}{2}\left[(1-q)\lambda a\right]^2 \\
&\quad +\beta\lambda a(\lambda a - 0) - \beta\frac{1}{2}(\lambda a)^2 \\
&= (\lambda a)^2\left[(1-q) - (1-q)(1+q)\right] - \frac{1}{2}\left[(1-q)\lambda a\right]^2 + \frac{1}{2}\beta(\lambda a)^2 \\
&= (\lambda a)^2\left[(1-q) - \left(1-q^2\right)\right] - \frac{1}{2}\left[(1-q)\lambda a\right]^2 + \frac{1}{2}\beta(\lambda a)^2 \\
&= \frac{1}{2}(\lambda a)^2\left(q^2 - 1 + \beta\right)
\end{aligned}$$

Separating equiliubrium holds if W does not have an incentive to deviate, which requires

$$U_{sep}^W - U_m^W \geq 0$$

$$\frac{q^2}{2} - \beta \geq 0$$

requires small β, impatient agent

* Pooling equilbrium holds if W does not have the incentive to deviate (separate)

$$U_{dev}^{W} = \lambda a \left[\lambda a - 0\right] - \frac{1}{2}(\lambda a)^2 + \beta\left[\frac{1}{2}(\lambda a)^2\right] = \frac{1}{2}(\lambda a)^2 (1 - \beta)$$

$$U_{pool}^{W} - U_{dev}^{W} = (\lambda a)^2 \left[\beta q^2 - \frac{1}{2}\right] \geq 0$$

* requires large β, patient agent

– If neither inequality holds, intermediate-size β, then equilibrium is neither pooling nor separating, but mixed strategy

- Solve inflation bias by appointing a **conservative central banker** (Rogoff)
  - If the central bank is independent of elected fiscal authorities, then its preferences can differ
  - Define a conservative central banker as an individual who places a larger weight on inflation $(1+\delta)$ than the population (1)

$$\lambda\left(y_t - y_n - k\right)^2 + (1+\delta)\,\pi_t^2$$

    * Optimal inflation under discretion

$$\pi^d(\delta) = \Delta m + v = \frac{\lambda a k}{1+\delta} - \left(\frac{\lambda a}{1+\delta+a^2\lambda}\right)e + v$$

      · smaller response to a supply shock, so less inflation stabilization

      · smaller inflation bias

* Choose optimal degree of conservatism for the central banker by chosing δ to minimize expected loss function

$$
\begin{aligned}
E(V) &= \frac{1}{2}E\left\{\lambda\left[a\left(\pi^{d}(\delta)-\pi^{e}\right)+e-k\right]^{2}+\left[\pi^{d}(\delta)\right]^{2}\right\} \\
&= \frac{1}{2}E\lambda\left[-\left(\frac{\lambda a^{2}}{1+\delta+a^{2}\lambda}\right)e+av+e-k\right]^{2} \\
&\quad +\frac{1}{2}E\left[\frac{\lambda ak}{1+\delta}-\left(\frac{\lambda a}{1+\delta+a^{2}\lambda}\right)e+v\right]^{2} \\
&= \frac{1}{2}\lambda\left[\frac{(1+\delta)^{2}}{\left(1+\delta+a^{2}\lambda\right)^{2}}\sigma_{e}^{2}+a^{2}\sigma_{v}^{2}+k^{2}\right] \\
&\quad +\frac{1}{2}\left[\frac{(\lambda ak)^{2}}{(1+\delta)^{2}}+\frac{(\lambda a)^{2}}{\left(1+\delta+a^{2}\lambda\right)^{2}}\sigma_{e}^{2}+\sigma_{v}^{2}\right]
\end{aligned}
$$

- take derivative with respect to δ and solve for δ

$$\delta = \frac{k^2}{\sigma_e^2}\left(\frac{\left(1+\delta+a^2\lambda\right)}{1+\delta}\right)^3 \equiv g(\delta)$$

- Since $g(\delta) > 0$ for $\delta = 0$, and $\lim_{\delta\to\infty} g(\delta) = \frac{k^2}{\sigma_e^2}$, intersection of $g(\delta)$ with 45 degree line occurs for positive value of δ, implying that always want to appoint a central banker more conservative than the public

– Benefit of conservative central banker is lower inflation bias, but cost is less response to supply shock and therefore greater output variance,increasing in δ

$$\frac{(1+\delta)^2}{\left(1+\delta+a^2\lambda\right)^2}\sigma_e^2 + a^2\sigma_v^2$$

- Modification Lohman (1992): government appoints a weight-conservative central banker and overrides whenever there is a large supply shock

- Empirically, greater central bank independence is associated with lower average inflation, but not with greater output variablility

– Problems

* How does a government identify δ?

* How does a government commit to δ? What keeps a government from appointing a conservative central banker, letting the public set expectations, and then firing the central banker and appointing one with the government's preferences?

* What prevents further delagation to an even more conservative central banker?

* To get optimal policy, should we focus on preferences of person in charge or on incentives?

- Solve inflation bias by commiting to a **contract** with a central banker (Walsh 1995)
  - View the problem with discretion as central bankers respond optimally to incentives they face, but the incentives are wrong
  - Model
    * Agggregate supply

$$y = y_n + a\left(\pi - \pi^e\right) + e$$

    * money growth and inflation

$$\pi = \Delta m + v$$

    * social loss

$$V = \frac{1}{2}\lambda\left(y - y_n - k\right)^2 + \frac{1}{2}\pi^2$$

* central bank chooses Δm
  - takes π^e as given
  - observes supply shock (e) before setting Δm, giving policy a stabilization role
  - cannot observe v
  - v and e are uncorrelated

* central bank receives transfers from government and utility depends on these transfers and on social loss

$$U = t - V$$

- government is principal who designs transfer function (t) to induce the central bank to choose socially optimal commitment

$$\Delta m = \Delta m^c(e)$$

$$\Delta m^c(e) = \frac{-\lambda a e}{1 + \lambda a^2}$$

- government cannot observe e, so cannot make transfers conditional on e. Instead, make them conditional on π.

$$\pi^c(e) = \Delta m^c(e) + v$$

- transfer policy implements optimal policy if π^c maximizes

$$E^{cb}\left[t\left(\pi\left(e\right)\right)-V\right]$$

where E^{cb} denotes central bank's expectation conditional on e

- central bank chooses $\Delta m^{cb}\left(e\right)$ to maximize utility, yielding

$$\Delta m^{cb}\left(e\right)=\frac{\lambda ak+\left(\lambda a^2\right)\pi^e-\lambda ae+E^{cb}\left(t'\left(\pi\right)\right)}{1+\lambda a^2}$$

where $t'=\frac{\partial t(\pi)}{\partial\pi}$, and where want a policy which leaves intact optimal social response to e, while eliminating inflation bias

- Letting E denote the public's expectations, taking expectations of policy, setting $E\left(\Delta m^{cb}\left(e\right)\right)=\pi^e$, and solving for policy yields

$$E\left[\Delta m^{cb}\left(e\right)\right]=\lambda ak+E\left(t'\left(\pi\right)\right)$$

- Substituting for expectations in the policy decision

$$
\begin{aligned}
\Delta m^{cb}(e) &= \frac{\lambda ak + \left(\lambda a^2\right)(\lambda ak + E(t'(\pi))) - \lambda ae + E^{cb}(t'(\pi))}{1+\lambda a^2} \\
&= \lambda ak + \frac{\lambda a^2 E(t'(\pi)) + E^{cb}(t'(\pi))}{1+\lambda a^2} - \frac{\lambda ae}{1+\lambda a^2}
\end{aligned}
$$

- Setting $\Delta m^{cb}(e) = \Delta m^{c}(e) = \frac{-\lambda ae}{1+\lambda a^2}$ requires

$$\lambda ak + \frac{\lambda a^2 E(t'(\pi)) + E^{cb}(t'(\pi))}{1+\lambda a^2} = 0$$

$$\lambda ak + \left(t'(\pi)\right) = 0$$

$$t'(\pi) = -\lambda ak$$

$$t = t_0 - \lambda ak\pi$$

- t_0 is chosen high enough to assure participation

- eliminates inflation bias while allowing optimal response to supply shock: key insight is that inflation bias is constant and independent of e. Policy works by raising the marginal cost of inflation from perspective of central bank

- **Institutional reform**, which raises cost of inflation, can serve as a commitment device
  - Political business cycles
    * Party A wants π^A, and party B wants π^B with $\pi^A > \pi^B$
    * Probability that party A wins is q, implying expectations of inflation given by

$$\pi^e = q\pi^A + (1 - q)\,\pi^B$$

    * If party A wins, get expansion, whereas if party B wins, get contraction
    * Inflation surprises often occur after an election
  - Design institutions to try and eliminate political business cycles

- Appointment of members of monetary policy board
  - Appoint members with long terms overlapping those of elected officials
  - Let one party appoint and other confirm
- Inflationary bias as young tries to transfer wealth from old generation
  - Solve by including young and old on policy board
- Punish central bankers who deviate from announced agreement on inflation (New Zealand)
- European Central Bank
  - member countries with different shocks and therefore different preferences for inflation delegate monetary policy to a single monetary

authority

∗ countries with severe supply shocks have incentive to abandon Union if monetary authority does not deviate from optimal commitment policy

- **Targeting Rules**
  - Flexible targeting rules impose penalty on central bank if it deviates from a specific target to reduce flexibility, but allow tradeoffs with other objectives
  - Strict targeting rules require a central bank to achieve a specific objective

- Flexible targeting rules
  - Let π^* be socially optimal rate of inflation and π^T be target rate of inflation, where need not be equal.

- Objective function

$$V^{cb} = \frac{1}{2} E_t \left\{ \lambda \left(y_t - y_n - k \right)^2 + \left(\pi_t - \pi^* \right)^2 + h \left(\pi_t - \pi^T \right)^2 \right\}$$

- Remaining equations of model

$$y = y_n + a \left(\pi - \pi^e \right) + e$$

$$\pi = \Delta m + v$$

- Socially optimal commitment policy is

$$\Delta m^s = \pi^* - \frac{\lambda a e}{1 + \lambda a^2}$$

* Policy under discretion: choose Δm to minimize loss function

$$
\begin{aligned}
V^{cb} \;&=\; \frac{1}{2}E\lambda\left(a\left(\Delta m + v - \pi_t^e\right) + e - k\right)^2 \\
&\quad + \frac{1}{2}E\left\{\left(\Delta m + v - \pi^*\right)^2 + h\left(\Delta m + v - \pi^T\right)^2\right\}
\end{aligned}
$$

* First order condition, conditional on values for π^e and e, but on an unknown value for v

$$\lambda\left[a\left(\Delta m - \pi_t^e\right) + e - k\right]a + \Delta m - \pi^* + h\left(\Delta m - \pi^T\right) = 0$$

$$\Delta m = \frac{\lambda a^2 \pi^e - \lambda a e + \lambda a k + \pi^* + h\pi^T}{1 + h + \lambda a^2}$$

* Letting expected inflation equal expected money growth

$$\pi^e = E\Delta m = \frac{\lambda a^2 \pi^e + \lambda a k + \pi^* + h\pi^T}{1 + h + \lambda a^2}$$

$$\pi^e = \frac{\lambda a k + \pi^* + h\pi^T}{1+h}$$

* Substituting

$$\Delta m^T = \left(\frac{\lambda a^2}{1+h+\lambda a^2}\right)\left(\frac{\lambda a k + \pi^* + h\pi^T}{1+h}\right) + \frac{\lambda a k - \lambda a e + \pi^* + h\pi^T}{1+h+\lambda a^2}$$

$$\Delta m^T = \left(\frac{\lambda a k + \pi^* + h\pi^T}{1+h}\right) - \frac{\lambda a e}{1+h+\lambda a^2}$$

* If target rate is socially optimal rate, then

$$\Delta m^T = \pi^* + \left(\frac{\lambda a k}{1+h}\right) - \frac{\lambda a e}{1+h+\lambda a^2}$$

where $h = 0$ yields time consistent rate without targeting

- Results
  - targeting reduces the inflation bias by raising the marginal cost of inflation due to the additional penalty
  - also reduces the responsiveness to a supply shock
- h parameter plays same weight as **Rogoff's conservative central banker** when $\pi^T = \pi^*$, such that a positive value for h dominates purely discretionary policy
- Targeting rule is replicated by **Walsh's optimal linear contract** when $\pi^T < \pi^*$ and replace π^* with π^T in social loss

$$
\begin{aligned}
V^{cb} &= \frac{1}{2}\left\{E\lambda\left(y-y_n-k\right)^2+E\left(\pi-\pi^T\right)^2+hE\left(\pi-\pi^T\right)^2\right\} \\
&= \frac{1}{2}\left\{E\lambda\left(y-y_n-k\right)^2+HE\left(\pi-\pi^T\right)^2\right\} \\
&= \frac{1}{2}\left\{E\lambda\left(y-y_n-k\right)^2+HE\left(\pi-\pi^*+\pi^*-\pi^T\right)^2\right\} \\
&= \frac{1}{2}E\lambda\left(y-y_n-k\right)^2 \\
&\quad +\frac{1}{2}HE\left(\left(\pi-\pi^*\right)^2+\left(\pi^*-\pi^T\right)^2+2\left(\pi-\pi^*\right)\left(\pi^*-\pi^T\right)\right)
\end{aligned}
$$

- – * Compare with utility of central banker under optimal contract with $t=t_0-\lambda ak$

$$
U=t-V=\frac{1}{2}\left\{\lambda\left(y-y_n-k\right)^2+\pi^2\right\}+\lambda ak\pi-t_0
$$

- do not want a conservative central banker

$$H = 1 \text{ if } h = 0$$

- need socially optimal inflation zero

$$(\pi - \pi^*)^2 = \pi^2 \text{ if } \pi^* = 0$$

- need inflation target below socially optimal target to offset inflation bias

$$(\pi - \pi^*)\left(\pi^* - \pi^T\right) = \pi\left(\pi^* - \pi^T\right) = \lambda ak\pi \text{ if } \pi^T = \pi^* - \lambda ak$$

- Strict targeting rules
  - Central bank is told to achieve a target value for certrain variables with no tradeoffs allowed
  - Money growth target

$$\Delta m = \Delta m^T = \pi^*$$

    * Social loss

$$V = \frac{1}{2} E_t \left\{ \lambda \left(y - y_n - k \right)^2 + \left(\pi - \pi^* \right)^2 \right\}$$

$$
\begin{aligned}
V\left(\Delta m^T\right) &= \frac{1}{2}E\lambda\left(a\left(\Delta m^T + v - \pi_t^e\right) + e - k\right)^2 \\
&\quad + E\left(\Delta m^T + v - \pi^*\right)^2 \\
&= \frac{1}{2}E\left\{\lambda\left(av + e - k\right)^2 + (v)^2\right\} \\
&= \frac{1}{2}\left[\left(\lambda a^2 + 1\right)\sigma_v^2 + \lambda\sigma_e^2 + \lambda k^2\right]
\end{aligned}
$$

* Loss under pure discretion is

$$
V^d = \frac{1}{2}\left[\left(\lambda a^2 + 1\right)\sigma_v^2 + \frac{\lambda}{\left(1 + \lambda a^2\right)}\sigma_e^2 + \lambda\left(1 + \lambda a^2\right)k^2\right]
$$

Difference

$$
V\left(\Delta m^T\right) - V^d = \frac{1}{2}\left[\frac{\lambda a^2}{\left(1 + \lambda a^2\right)}\sigma_e^2 - (\lambda a k)^2\right]
$$

Prefer strict rule to discretion if difference in losses is negative, that is if inflation bias is large relative to need to stabilize economy

– Nominal income target – weighs fluctuations in inflation output equally

- Is inflation bias important?
  - High inflation is not necessarily evidence of inflation bias due to role of inflation in generating seigniorage
  - Openness of the economy affects the tradeoff between inflation and output
    - Monetary expansion in an open economy depreciates exchange rate
    - With prices in both currencies sticky, price of foreign goods rises, producing more inflation the more open the economy
    - For a given price surprise for domestic goods, and therefore a given output stimulus, get more inflation
    - Therefore, more open economies could have a smaller inflation bias

- * Romer tested in a sample of 114 countries post 1973 and found generally supportive evidence, but not for OECD countries

- Different inflation experiences over time could be due to a failure to understand that the long-run Phillips Curve is verticle (1970's)