Predicting Sovereign Fiscal Crises: High-Debt Developed Countries

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Abstract

How large can debt get before triggering a crisis? Since debt is the expected present value of future primary surpluses, the answer depends on a country’s technical and political ability to raise future primary surpluses. We estimate surplus feedback rules on debt for ten high-debt developed countries and find substantial heterogeneity in quantitative responsiveness. However, for all countries, an increase in debt creates a sustained increase in the primary surplus, with the surplus reaching a peak in the future. Therefore, following an increase in debt, countries do not raise the surplus to its peak and maintain the peak forever, the assumption implicit in the standard practice of setting peak debt at the present value of the peak surplus. Our implied debt limit is much lower than the standard measure. We estimate debt limits following the global financial crisis in 2008 and find substantial heterogeneity. We separate countries into risk categories based on fiscal space. Greece and Portugal eroded their fiscal space several years, prior to their fiscal crises, placing them in the highest risk category and predicting the crises that followed. Canada and Belgium maintained large enough fiscal space to achieve safe status. Other countries reduced fiscal space, with three eroding fiscal space in 2014, warning of future crises.

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1 Introduction

How large can government debt get before triggering a crisis? Greece lost access to credit markets when debt exceeded 130 percent of GDP. In contrast, Belgium successfully retained market access with similar debt relative to GDP. What determines the value of debt beyond which a country loses access to financial markets? Why do countries exhibit heterogeneity in debt tolerance? We focus on ten high-debt, developed countries, two of which lost access to markets after the 2008 global financial crisis. The high-debt countries include Belgium, Canada, France, Greece, Italy, Japan, Portugal, Spain, the US, and the UK.

We assume that fiscal crises are due to insolvency created by fiscal limits (Bi 2012, Davig, Leeper and Walker 2011). Every country faces a limit on the value of the primary surplus relative to GDP that it can raise to repay debt. If debt repayments require larger current and expected future values for the primary surplus, then the sovereign is insolvent. Creditors refuse to lend into a position of insolvency, creating a fiscal crisis. We develop a model, using historical data on primary surpluses and debt, both relative to GDP, to explain why countries with similar debt levels have different experiences with crises. Additionally, we use the model to place high-debt countries into three risk categories, "safe", "risky", "highest risk", in each year following the 2008 financial crisis.

The standard method of estimating a debt limit is to first identify a surplus limit, and then set the debt limit at the present value of the surplus limit (Tanner 2013, Collard et al. 2015, Bi 2012, Davig, Leeper and Walker 2011). The problem with this approach is that it ignores dynamics. Countries do not seem to respond to an increase in debt with an immediate move towards very large surpluses which they maintain indefinitely.\footnote{Our empirical evidence confirms this.} For example, in recent negotiations with the Troika, Greece raised its surplus to the agreed value only after several years and explicitly stated that it could not maintain the required surplus of 3.5% of GDP indefinitely.

Our approach explicitly includes dynamics. We follow others in using the maximum historical value as one estimate for the surplus limit. However, we depart from the literature in its assumption that the surplus limit can be maintained forever. Instead, we
add fiscal policy dynamics to the literature on fiscal limits by estimating fiscal feedback rules, relating the current primary surplus to its own lag and to lagged debt.\(^2\) We use the results to estimate country-specific debt limits, fiscal space, and fiscal risk.

First, we estimate country-specific debt limits for each of our ten countries, using estimates of fiscal feedback rules for the primary surplus. The estimated parameters of the fiscal feedback rule predict adjustment paths for future primary surpluses and debt, conditional on initial debt-surplus pairs. When debt is above its long-run equilibrium level and the surplus is too low to bring it down, both debt and the surplus rise along the adjustment path. Debt reaches a peak while the surplus is still rising. Eventually the surplus peaks, and debt and the surplus fall together to their long-run equilibrium values. Peak values are not long-run equilibrium values, implying that peak values are not maintained indefinitely, consistent with casual observation. Additionally, the estimated adjustment paths imply values for the debt limit. The path with the peak value of the primary surplus at the surplus limit contains the largest possible future primary surpluses. This path is a boundary separating solvent positions below the path from insolvent ones above. Therefore, the debt limit is the peak value of debt along this boundary path.

Our measure of the debt limit is lower than the standard measure, given by the present value of a maximum surplus, since the estimated fiscal dynamics imply that a country will not maintain its maximum surplus forever. We find considerable heterogeneity in debt limits across countries based on heterogeneity in peak surpluses and in surplus responsiveness to debt. Japan, Belgium, and Canada have the highest debt limits and Portugal has the lowest. These debt limits are important determinants of debt tolerance, partly explaining why countries with similar levels of debt have different crisis experiences.

Second, we obtain measures of fiscal space for each country. Given our estimate of the debt limit, the standard procedure measures fiscal space as the difference between the debt limit and actual debt. However, this measure ignores dynamics. Our estimates imply that a country takes time to adjust its primary surplus to a shock or a change in debt. Therefore, a country with a larger current primary surplus has a larger expected present value of future primary surpluses, implying that it can sustain larger debt. Our measure of fiscal space depends on current values of both the primary surplus and debt. We define a boundary path as the expected adjustment path for the debt and primary surplus with peaks at debt and surplus limits. Current values for debt-surplus pairs above

\(^2\)Celasun, Debrun, and Ostry (2007) also use a fiscal feedback rule in assessing fiscal risk. They interact their estimate of the rule with the value of expected future shocks based on an estimated VAR, to predict probability bounds on future values of debt. Debt is viewed as non-sustainable if these probability bounds indicate rising debt.
the boundary path are not consistent with equilibrium because debt and/or the primary surplus would be expected to exceed their maximum values over time. We redefine fiscal space as the difference between the current value of debt and its value along the boundary path for a particular primary surplus. A larger primary surplus generally implies a higher position of debt along the boundary path and therefore larger fiscal space.\(^3\)

Third, fiscal space, together with the probability distribution of shocks, allows us to categorize a country’s risk of a one-period-ahead fiscal crisis. Countries with low debt relative to the boundary path, have large fiscal space and low risk. We compute measures of fiscal space for each of our ten high-debt developed countries for each year following the 2008 global financial crisis, and use this measure to place countries into three risk categories. We label a country as "safe" if fiscal space is so large that the probability of receiving shocks large enough to send debt above the boundary path next period is virtually zero. Alternatively, the country is at "highest risk" if it has exhausted its fiscal space. Intermediate values of fiscal space place countries in the "risky" category, with risk increasing at an increasing rate as fiscal space shrinks (Daniel and Shiamptanis 2012).

Greece and Portugal begin the period in the "highest risk" category, having exhausted fiscal space. Portugal succumbed to crisis three years later, even though it had lower debt than all the other countries, and Greece succumbed two years later with debt lower than that in Belgium, Canada, Italy and Japan. For both Greece and Portugal, several years of zero fiscal space put them in the "highest risk" category, predicting the crises they experienced. In contrast, Canada and Belgium sustain large enough fiscal space to maintain "safe" status throughout, even though Belgium is the highest debt country after Japan. The UK and Italy experience falling fiscal space and therefore rising risk throughout the period, but never exhaust fiscal space. Other countries, including the US, do exhaust their fiscal space for some years. France, Spain, and Japan have no fiscal space in 2014, placing them in the "highest risk" category and warning of possible future fiscal crises.

Our work builds on that of others who have used debt limits to assess insolvency risk. Tanner (2013) introduced the traditional measure, in which he identifies an upper bound on the surplus and infers a debt limit from the government’s intertemporal budget constraint as the present value of the maximum surplus. Collard et al. (2015) modify the Tanner analysis using an upper bound on the surplus, adjusted for the probability that a country defaults on debt. They calculate maximum sustainable debt as the value of

\(^3\)This result is consistent with empirical evidence (Chakrabarti and Zeaiter 2014). Once the surplus gets high enough, an increase no longer increases fiscal space as explained in Section 2.4.
debt beyond which debt becomes explosive due to its default premium. Bi (2012) and Davig, Leeper and Walker (2011) assume that there is a fiscal limit on the value of the surplus determined by the value of tax revenue at the top of the Laffer curve, which in turn implies a fiscal limit on debt. Countries further down their Laffer curves are less risky because they are further from their fiscal limit. Ghosh et al. (2013) offer a measure of a debt limit based on the assumption that future surpluses are determined by a nonlinear function of powers of debt which is homogeneous across countries. In their panel model, the cubic power of debt has a negative coefficient, enabling them to use country-specific interest rates to identify country-specific values of debt beyond which debt explodes. Since no creditor would lend to a borrower whose debt is explosive, their debt limit is the value of debt at which the system becomes explosive.4 Celasun, Debrun, and Ostry (2007) estimate a fiscal feedback rule and interact estimated coefficients with the value of expected future shocks based on an estimated VAR, to predict probability bounds on future values of debt. Debt is viewed as non-sustainable if these probability bounds indicate rising debt.

This paper is organized as follows. The next section derives the debt limit and fiscal space. Section 3 is empirical, with estimates of fiscal feedback rules. Section 4 uses our estimates to measure fiscal space and assess the risk of a fiscal crisis following the global financial crisis for our ten high-debt developed countries. Section 5 provides conclusions.

2 Fiscal Limits and Fiscal Space

2.1 Surplus Limit

Our debt limit measure is derived from the surplus limit and the government’s fiscal feedback rule. Consider the surplus limit first. Every sovereign faces a limit on its ability to raise government surpluses with tax increases and spending reductions, and therefore a limit on its ability to repay and service debt. Davig et al. (2011), and Bi (2012) motivate the surplus limit by the top of the Laffer curve for distortionary taxes. However, the concept can be more general. A limit on the surplus can be due to the inability to reduce government spending, perhaps due to the dependence of economic activity on the provision of public goods, and to the inability to raise tax rates for other reasons,

4This measure requires the assumption that responsiveness to debt continues to fall as debt rises to its limit, a value well outside of values of debt in the sample. Debt limits cannot be identified if the coefficient on the cubic power of debt is not negative, or if a country’s interest rate implies instability over all values of debt.
including tax evasion (Daniel 2014). Bi et al. (2013) argue that the surplus limit could also be political, whereby the democratic process cannot raise the surplus sufficiently to service the debt. We denote the absolute maximum value of the primary surplus relative to GDP as $\tilde{s}$, and express the surplus limit by

$$s_t \leq \tilde{s} \quad \forall t. \quad (1)$$

### 2.2 Dynamic Behavior of the Surplus

Our central premise is that a government does not instantaneously move its primary surplus to $\tilde{s}$, following an adverse fiscal shock, and keep it there indefinitely. To determine how surpluses actually do behave, we follow Bohn (1998, 2007) and assume that the government follows a fiscal feedback rule in which the primary surplus systematically reacts to economic variables. Consider a benchmark specification, in which the primary surplus ($s_t$) responds to both to its own lag and to lagged debt ($d_{t-1}$) according to

$$s_t = c + \beta s_{t-1} + \gamma d_{t-1} + \epsilon_t, \quad s_t \leq \tilde{s}, \quad (2)$$

where debt and the primary surplus are both expressed as a fraction of GDP. The parameter $c$ is a constant governing the long-run value of the primary surplus, and $\epsilon_t$ represents zero-mean stochastic shocks, due both to the policy process and to business cycles. We allow the primary surplus to exhibit persistence by including its lag and assume that $0 \leq \beta < 1$, which is consistent with our empirical evidence. The parameter $\beta$ captures the inertia in the legislative process, and the parameter $\gamma$ captures the government’s responsiveness to debt. We assume that the government follows this rule as long as the implied primary surplus is consistent with equilibrium as defined below. \(^5\)

### 2.3 Dynamic Behavior of Debt

Debt evolves according to the government’s flow budget constraint, given by

$$d_t = \delta_t \left( \frac{1 + i_{t-1}}{1 + \rho_t} \right) d_{t-1} - s_t, \quad (3)$$

\(^5\)When this fiscal feedback rule is inconsistent with equilibrium, the fiscal feedback rule determines the largest surpluses the government can raise, allowing a reduction in the surplus, but not an increase, to satisfy equilibrium requirements. See section 2.4.1: Additional Requirements.
\( \delta_t \) denotes the fraction of debt that is repaid, \( i_{t-1} \) is the domestic interest rate, \( \rho_t \) is the growth rate of domestic output and \( \frac{1 + i_{t-1}}{1 + \rho_t} \) has the interpretation as the domestic growth-adjusted interest rate.\(^6\)

We assume that the government has access to an international lender who is willing to lend any amount to the domestic government as long as he expects to receive the risk-free world interest rate \( (i) \), which we assume is constant. Given this assumption, interest rate parity determines the domestic interest rate according to

\[
1 + i = (1 + i_{t-1}) E_{t-1} \delta_t, \tag{4}
\]

where \( E_{t-1} \delta_t \) reflects expectations of default. Expectations of default \( (E_{t-1} \delta_t < 1) \) require the domestic interest rate \( (i_{t-1}) \) to rise to offer a risk-neutral investor an expected rate of return equal to that in the market.

Assume that the inverse of the gross domestic growth rate \( \left( \frac{1}{1 + \rho_t} \right) \) is distributed iid about a mean of \( \frac{1}{1 + \rho} \) such that

\[
\frac{1}{1 + \rho_t} = \frac{1}{1 + \rho} \zeta_t, \tag{5}
\]

where \( \zeta_t \) captures stochastic growth shocks and \( E_{t-1} \zeta_t = 1 \). Using this assumption, define interest rates adjusted by the mean domestic growth rate as

\[
(1 + r) = \frac{1 + i}{1 + \rho}, \quad (1 + r_{t-1}) = \frac{1 + i_{t-1}}{1 + \rho}. \tag{6}
\]

Using equations (5) and (6), the equation for the evolution of debt (3) becomes

\[
d_t = \zeta_t \delta_t (1 + r_{t-1}) d_{t-1} - s_t. \tag{7}
\]

Substituting equation (6) into interest rate parity, equation (4), and dividing both sides by \( 1 + \rho \) yields

\[
1 + r = (1 + r_{t-1}) E_{t-1} \delta_t. \tag{8}
\]

Define \( \alpha_t \) as

\[
\alpha_t = (1 - \delta_t) (1 + r_{t-1}) d_{t-1}, \tag{9}
\]

where \( \alpha_t \) has the interpretation as the capital loss due to default. Using equation (9) and substituting from equation (8), unexpected capital loss due to default can be expressed

\(^6\)We can view \( i_t \) and \( \rho_t \) either as nominal or real with no effect on the derivation.
as
\[ \alpha_t - E_{t-1}\alpha_t = [-\delta_t (1 + r_{t-1}) d_{t-1} + (1 + r) d_{t-1}] . \]  
(10)

Substituting \( \delta_t (1 + r_{t-1}) d_{t-1} \) from equation (10) into the equation (7) yields

\[ d_t = (1 + r) d_{t-1} - s_t - \eta_t + \psi_t. \]  
(11)

where
\[ \psi_t = [\zeta_t - 1] (1 + r) d_{t-1}, \]  
(12)
\[ \eta_t = [\alpha_t - E_{t-1}\alpha_t] \zeta_t. \]  
(13)

Equation (12) captures the impact of the growth shock on debt, and equation (13) captures the impact of unexpected capital loss on debt.

Equation (11) is an equation for the evolution of debt which is linear in lagged debt, the primary surplus, and mean-zero stochastic terms. An adverse growth shock \( (\zeta_t > 1) \) raises the growth-adjusted interest rate, increasing \( \psi_t \), thereby raising debt. The expectation of capital loss due to default \( (E_{t-1}\alpha_t > 0) \) raises the domestic interest rate to include a default premium \( (\eta_t < 0) \), thereby raising debt. Actual capital loss due to default \( (\alpha_t > 0) \) reduces debt \( (\eta_t > 0) \). When the expectation of default equals actual default, there is no debt reduction \( (\eta_t = 0) \) because the interest rate fully adjusts to offset the future default.

Solving equation (11) forward for debt, imposing \( \lim_{J \to \infty} \frac{d_{t+J}}{(1+r)^J} = 0 \) and the surplus limit, yields an expression for the government’s intertemporal budget constraint,\(^7\)

\[ d_t = \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^{j+1} \left[ s_{t+1+j} + \eta_{t+1+j} + \psi_{t+1+j} \right] s_{t+1+j} < \bar{s} \forall j. \]  
(14)

Satisfaction of the government’s intertemporal budget constraint, equation (14), does not require that the government never default, or equivalently that \( \alpha_{t+1+j} = 0 \ \forall j \). Default can occur, but it provides revenue only if it is larger than its expected value yielding \( \eta_{t+1+j} > 0 \). Therefore, systematic and expected default cannot provide revenue. However, actual default can be necessary to restore intertemporal budget balance.

Taking the expectation of equation (14) yields the expected intertemporal budget

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\(^7\)The government’s no Ponzi game constraint rules out a positive value. The no Ponzi game constraint for the household (or the aggregate of the remaining agents in the market) rules out a negative value.
constraint as\(^8\)

\[
d_t = \sum_{j=0}^{\infty} \left( \frac{1}{1 + r} \right)^{j+1} E_t(s_{t+1+j}|s_t, d_t) \quad E_t s_{t+1+j} < \bar{s} \forall j,
\]

validating that default cannot generate expected (and systematic) revenue. Satisfaction of the expected intertemporal budget constraint requires that the government be expected to generate future surpluses whose present value equals the value of debt, subject to the surplus limit.

2.4 Equilibrium for Government Bonds

**Definition 1.** Equilibrium in the market for government bonds: Given the world interest rate, \(i\), stochastic processes for \(\epsilon_t\) and \(\zeta_t\), the surplus limit, \(\bar{s}\), and the dynamic equation for the evolution of the primary surplus, equation (2), an equilibrium is values for \(\{s_t, d_t, i_t, \delta_t\}\), such that expectations are rational, international creditors expect to receive \(i\) on domestic government debt (equation 4), the primary surplus and debt do not exceed their limits, and the government’s flow and intertemporal budget constraints, equations (3) and (14), are satisfied.

In this paper we focus on equilibrium values for \(s_t, d_t, and \delta_t\), given an equilibrium value for \(i_t\). Daniel and Shiamptanis (2012) shows how to compute the equilibrium value for \(i_t\).

2.4.1 Satisfaction of Fiscal Limits in Expectation

We first derive restrictions to assure that the limits are satisfied in expectations, and then derive restrictions to assure that they are actually satisfied. Both are required for equilibrium.

Taking equations (11) and (2) \(j\) periods forward and taking the time \(t\) expectation yields

\[
E_t d_{t+j} = (1 + r) E_t d_{t+j-1} - E_t s_{t+j},
\]

\[
E_t s_{t+j} = \gamma E_t d_{t+j-1} + \beta E_t s_{t+j-1} + c,
\]

where we set \(E_t \eta_{t+j} = E_t \psi_{t+j} = E_t \epsilon_{t+j} = 0.9\)

\[
E_t \eta_{t+j} = E_t (\zeta_{t+1+j} (\alpha_{t+1+j} - E_{t+j} \alpha_{t+1+j})) = E_t (\zeta_{t+1+j} \alpha_{t+1+j}) - E_t (\zeta_{t+1+j} E_{t+j} \alpha_{t+1+j}) = E_t (\zeta_{t+1+j} \alpha_{t+1+j}) - E_t (\zeta_{t+1+j} \alpha_{t+1+j}) = 0
\]

\(9\)These expectations do not have to be zero, particularly when there is a positive probability of default. However, restrictions we derive below when they are zero, are necessary for equilibrium.
**Dynamic Stability** Satisfaction of fiscal limits in expectation requires that the dynamic system in expectations of debt and the primary surplus, equations (16) and (17), not explode, equivalently that the system be dynamically stable. This requires both eigenvalues of the matrix

\[
\begin{bmatrix}
1 + r & -1 \\
\gamma & \beta
\end{bmatrix}
\]

be inside the unit circle. Since both the determinant and trace of the matrix are positive, both eigenvalues are positive. Therefore, letting \( \theta_1 \) and \( \theta_2 \) denote the two eigenvalues of the system, the requirement for stability is

\[
(1 - \theta_1) (1 - \theta_2) > 0 \implies \theta_1 \theta_2 - (\theta_1 + \theta_2) + 1 = \text{determinant} - \text{trace} + 1 > 0
\]

Substituting for values of the determinant and trace yields

\[
\beta (1 + r) + \gamma - (1 + r + \beta) + 1 > 0,
\]

requiring

\[
\gamma > r(1 - \beta). \quad (18)
\]

If the system in the debt and primary surplus were not dynamically stable, then the expectations for the debt and the primary surplus could become unbounded over time, eventually exceeding any fiscal limit. Unboundedness in the negative direction does not violate fiscal limits, but unboundedness in the positive direction does. And since shocks affect initial positions, and therefore the direction of the explosion, we must rule out all explosion in order to rule out positive explosion. Therefore, the first requirement for equilibrium is a restriction on the parameters of the government’s fiscal feedback rule for the primary surplus, equation (18), to yield stability.

When the system is stable, values for the debt and primary surplus are expected to reach their long-run equilibrium values. We derive these by dropping time subscripts in equations (16) and (17) and solving for debt and the primary surplus as

\[
d^* = \frac{-c}{\gamma - r (1 - \beta)}, \quad s^* = \frac{-rc}{\gamma - r (1 - \beta)}.
\]

Given the stability requirement, equation (18), a \( c < 0 \) yields positive long-run values of the debt \( (d^*) \) and the primary surplus \( (s^*) \).
Additional Requirements: Boundary Path  Global stability is necessary, but not sufficient, to assure that the debt and the primary surplus, following equations (16) and (17), are not expected to exceed their fiscal limits. To understand this, we use a phase diagram in the expected future values of the primary surplus and debt over time for given initial values. To derive the equations for the phase diagram, subtract the lagged values of the left-hand side variable from both sides of equations (16) and (17) and set expected changes in each variable to zero to yield

\[ E_t d_{t+j-1} | (\Delta E_t d_{t+j} = 0) = \frac{c + \beta E_t s_{t+j-1}}{r - \gamma}, \]  

(20)

\[ E_t d_{t+j-1} | (\Delta E_t s_{t+j} = 0) = \frac{-c + (1 - \beta) E_t s_{t+j-1}}{\gamma}. \]  

(21)

Figure 1 contains the phase diagram. The \( \Delta E_t d_{t+j} = 0 \) has a negative slope and the \( \Delta E_t s_{t+j} = 0 \) curve has a positive slope. Arrows of motion for both curves point toward each curve, confirming a globally stable model. Point E represents long-run equilibrium values for debt and the primary surplus, given by equations (19). Adjustment paths ACE and BWLE reflect paths for expected future values of the primary surplus and debt for initial values of the debt-surplus pair at A and B respectively.

The highest possible future surpluses must be consistent with the fiscal feedback rule and with a surplus limit (\( \bar{s} \)). Therefore, the expected future adjustment path, labeled BWLE, which peaks at \( \bar{s} \) (point L) contains the set of highest possible future surpluses. This implies that the debt limit (\( \bar{d} \)) is the peak for debt along this path at point W.

We use Figure 1 to determine the boundary path, which separates values of the debt-surplus pair which are feasible in equilibrium from values which are not. For an initial debt-surplus pair on or below BWL, expected future values of the debt and the primary surplus do not exceed fiscal limits, implying that these values satisfy the equilibrium criterion that expected future surpluses and debt satisfy fiscal limits. At the other extreme, a value of debt above the debt limit (\( \bar{d} \)) does not satisfy this criterion and is therefore not an equilibrium.\(^\text{10}\)

Consider the case where shocks have sent the debt-surplus pair above the upward-sloping segment of BW, but below the horizontal line \( \bar{d} \). For these positions, expected future values of debt violate the debt limit. To avoid violation, the country would need higher surpluses than those implied by the fiscal feedback rule, an impossibility. Therefore, these positions are not consistent with equilibrium and require default to restore equilib-

\(^{10}\)We have assumed that values of the surplus above \( \bar{s} \) are not attainable.
rium. Finally, consider the case where shocks have sent the debt-surplus pairs above WL, but below WZ. In these positions, the dynamics are reducing debt, implying that along the fiscal feedback rule, debt is not expected to violate its debt limit, but the surplus is. The country can avoid violating the surplus limit by reducing its primary surplus to reach WL. From a position along WL, it is expected to reach a long-run equilibrium without violating either fiscal limit. Therefore, positions above WL and below WZ do not require default to be consistent with equilibrium.\footnote{Another way to infer equilibrium for this type of debt-surplus pair is to realize that the country could throw away some of its primary surplus to move horizontally to the adjustment path BWLE. If it is expected to satisfy fiscal limits when it throws some surplus away, then it must be expected to satisfy fiscal limits without throwing away any surplus.}

Fiscal solvency requires satisfaction of the government’s actual and expected intertemporal budget constraints, equations (14) and (15) and is necessary for equilibrium. The government is fiscally solvent without default whenever the initial debt-surplus pair lies below the path BWL, implying that expected future surpluses and debt do not violate fiscal limits. Additionally, if the initial debt-surplus pair is above WL but below WZ, the government can restore solvency and equilibrium by reducing the surplus without default. However, if the initial debt-surplus pair is above WZ, default is necessary to restore solvency and equilibrium. Therefore, the boundary path separating solvent from insolvent positions requiring default is given by BWZ.

2.4.2 Satisfaction of Actual Fiscal Limits

The previous section demonstrates that values for the debt-surplus pair above BWZ are not equilibrium values. However, shocks could send the system into such positions, requiring default to restore equilibrium. We assume that default occurs in the magnitude necessary for the debt-surplus pair to reach the boundary path. Therefore, the equilibrium value of default \( \delta_t \) assures that debt falls by the amount necessary to reach the boundary path.

2.5 Fiscal Space

The traditional definition of fiscal space is the difference between the debt limit and the current value of debt. Figure 1 is drawn with a debt limit of \( \tilde{d} \), determined as the peak of the adjustment path whose primary surplus peak is the surplus limit, \( \tilde{s} \). Assume that the actual debt-surplus pair is at point A. The traditional measure of fiscal space is the debt limit, \( \tilde{d} \), minus the value of debt at point A. However, this traditional measure
ignores dynamics. If the country received a shock to debt equal to this measure of fiscal space, then the debt-surplus pair would be above the boundary path, a position requiring default. The largest shock the country could receive and avoid default is smaller, implying a smaller fiscal space.

The dynamic behavior of debt and the primary surplus requires an alternative measure of fiscal space. We define fiscal space as the largest one-period-ahead increase in debt for which the country is expected to remain solvent, equivalently remain below the boundary path BWZ. If the economy begins at debt-surplus pair A in Figure 1, then it is expected to transition along the adjustment path AE to point C in the absence of any shocks.\textsuperscript{12} Fiscal space is the maximum increase in debt from point C, subject to the constraint that debt not be above BWZ, following adverse changes in the exogenous shocks $($\textit{\psi}_{t+1}, \textit{\epsilon}_{t+1})$, or in the endogenous value of $\eta_{t+1}$ due to expected default $($\textit{\eta}_{t+1} = -\zeta_{t+1}E_t\alpha_{t+1} < 0).$\textsuperscript{13} Fiscal space must be greater than or equal to zero to justify lending.

Either an increase in expected default $(\textit{\eta}_{t+1} < 0)$ or an adverse debt shock due to growth $(\textit{\psi}_{t+1} > 0)$ would raise debt with an unchanged surplus. The vertical distance between point C and the boundary path at point D, the length of CD, is one measure of fiscal space. Alternatively, an adverse surplus shock $(\textit{\epsilon}_{t+1} < 0)$ would raise debt and reduce the primary surplus by equal amounts. The reduction in the primary surplus sends the economy from point C to point G, while the equal increase in debt moves it vertically from point G to point F. Therefore, another measure of fiscal space is the vertical length of GF (which is equal to the horizontal length GC). Fiscal space for an expected default or debt shock (CD) exceeds fiscal space for a surplus shock (GF) because the reduced primary surplus yields a lower expected present value of future surpluses implying a lower intertemporal-budget-balancing value for debt. Additionally, the shocks could occur in combination, yielding measures of fiscal space between these two distances.

Therefore, when the relevant portion of the boundary path is upward-sloping (BW), fiscal space is not one number but a range of values, captured by the vertical distances between the FD portion of the boundary path and GC. We therefore show that fiscal space depends on the value of the primary surplus as well as the value of debt. When the relevant portion of the boundary path is flat (WZ), the two measures of fiscal space are identical.

\textsuperscript{12}Recall $E_t\textit{\psi}_{t+1} = E_t\textit{\epsilon}_{t+1} = E_t\textit{\eta}_{t+1} = 0.$
\textsuperscript{13}Fiscal space is analogous to the distance variable in Daniel and Shiamptanis (2012).
2.6 Fiscal Risk

Fiscal risk is the probability that exogenous shocks \((\psi_{t+1}, \epsilon_{t+1})\), together with endogenous expectations of default \((\eta_{t+1})\), send the debt-surplus pair beyond the boundary path. We establish criteria for a country to be almost safe. The probability of receiving one shock greater than three standard deviations is only 0.135%, a probability comfortably close to zero. And the probability of receiving two independent shocks this large is \((0.00135)^2\), a very tiny number. Even if the shocks are perfectly correlated, the probability of receiving three standard deviation shocks or greater for both variables is only the probability of receiving one at 0.00135. Therefore, we use three standard deviations as the benchmark value for shocks separating countries which are almost perfectly safe from those which are not.

Consider an economy that is expected to reach point C from an initial position at point A in the absence of any shocks. Let the vertical distance from C to H be equal to an adverse debt shock of \(\psi_{t+1}\), created by a three standard deviation shock to \(\zeta_{t+1}\). And let the horizontal distance from H to K equal an adverse surplus shock of three standard deviations to \(\epsilon_{t+1}\). The adverse surplus shock sends the system from point H diagonally to point J. When the trapezoid labeled CHJK lies fully below the boundary path BWZ, virtually no exogenous pair of shocks \((\psi_{t+1}, \epsilon_{t+1})\) could cause default, and endogenous expectations of default are zero. A three standard-deviation shock to \(\zeta_{t+1}\) would send the system to point H. A three standard-deviation shock to \(\epsilon_{t+1}\) would send it to point K, and three standard-deviation shocks to both would send it to point J.

However, when the trapezoid is not fully below the boundary path BWZ, as in the case in Figure 1, some combinations of \(\psi_{t+1}\) and \(\epsilon_{t+1}\) could exhaust fiscal space and cause default. This creates expectations of default \((\eta_{t+1} = -\zeta_{t+1}E_t\alpha_{t+1} < 0)\), raising the expected value of debt above point C. Therefore, prior to any realization of default, when expectations of default are positive, debt is expected to travel to a point above C from the initial point A due to the default premium on the interest rate. The higher value of debt due to these expectations implies that the exogenous shocks \((\psi_{t+1}, \epsilon_{t+1})\) necessary to send the system above the boundary path and create default are smaller than the available fiscal space.

We use measures of fiscal space to place countries into one of three categories for fiscal risk. We classify the country as "safe", when fiscal space is so large that the trapezoid CHJK lies fully below the boundary path BWZ. For this case, expectations of default are virtually zero. For intermediate measures of fiscal space, those for which the trapezoid intersects the boundary path BWZ, as in the case in Figure 1, the country is
"risky", and expectations of default are positive. The higher interest rate due to default expectations raises debt reducing the magnitude of the minimum shock creating default. Due to default expectations, default risk is increasing at an increasing rate as fiscal space shrinks. Finally, when fiscal space is zero with a debt-surplus pair along the boundary path BWZ, the country is in the "highest-risk" category, and expectations of default are high.

3 Estimates of Fiscal Feedback Rules

To construct the boundary path BWZ, we need estimates for fiscal feedback rules and interest rates. We estimate fiscal feedback rules for ten high-debt countries, Belgium, Canada, France, Greece, Italy, Japan, Portugal, Spain, UK and US using annual data during the period 1970-2007. We cut our sample in 2007 for two reasons. Prior to 2007, countries experienced neither fiscal crises nor high-debt high-surplus regions, both of which could have caused them to violate their fiscal feedback rules. Second, we want to use data prior to the global financial crisis (in sample data) to assess solvency risks after the global financial crisis between 2008 and 2014 (out of sample).

Prior to estimation we make two empirically-motivated changes to the fiscal feedback rule in equation (2). First, we allow the primary surplus to depend upon the value of the output gap ($\tilde{y}_t$) and temporary government expenditures ($\tilde{g}_t$) as in Bohn (1998, 2007) and Mendoza and Ostry (2008). This requires that we replace $\epsilon_t$ with $\delta_1\tilde{y}_t + \delta_2\tilde{g}_t + \tilde{\epsilon}_t$, thereby specifying three different types of shocks to the primary surplus. The first type of shock is an output-gap shock ($\tilde{y}_t$), which tends to raise the primary surplus by more than output, thereby increasing $s_t$. The second is a shock to temporary government spending ($\tilde{g}_t$), and the third ($\tilde{\epsilon}_t$) aggregates all other shocks, particularly those due to the political process.

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14 Using a bounded distribution of shocks, Daniel and Shiamptanis (2012) show that risk of default is increasing at an increasing rate as fiscal space shrinks toward zero, becoming unity along the boundary path. For a debt-surplus pair along the boundary path, the endogenous value of expected future default is high enough that only the best shock could avert default. With the bounded shocks in Daniel and Shiamptanis (2012), the probability of the best shock is zero, implying that the probability of a crisis is unity.

15 Daniel and Shiamptanis (2012) show that the probability of default in this case is unity. However, we do not claim such a large default probability because our estimate of the boundary path could be too low.

16 Details on data are contained in the Data Appendix.

17 See Section 2.4.

18 The gap variables are determined by subtracting the trend component, estimated using the Hodrick-Prescott filter, from each observation.
The second change to the benchmark fiscal feedback rule is that we account for the possible effect of different values for the growth-adjusted interest rate on the parameters of the fiscal feedback rule. Consider carefully the motivation for this modification.

3.1 Interest Rate and the Fiscal Feedback Rule

Equations (19) imply that for countries with positive values for long-run primary surplus and debt ($c < 0$), both values are increasing in the growth-adjusted interest rate. If countries want to keep long-run debt below a certain target value after the interest rate rises, then they could raise the surplus-responsiveness to government debt ($\gamma$), or raise the constant ($c$) to reduce the structural deficit. Therefore, we allow the interest rate to affect the parameters of the fiscal feedback rule.

Growth-adjusted interest rates for all countries have similar movements over the sample 1970-2007. They are negative early, beginning with the 1970’s and continuing into the 1980’s. The negativity comes from both high inflation and real growth, compared to the nominal interest rate. In the mid-1980’s, growth-adjusted interest rates rise with a sharp increase in nominal interest rates, and they remain positive until the late-1990’s for some countries and until the end of the sample for others. For all countries except Japan, the interest rates are higher in the middle of the sample than they are early or late. Japan’s interest rates rise throughout the sample.

We view these periods in which interest rates took on very different values as different interest rate regimes and test whether the constant ($c$) and responsiveness ($\gamma$) take on different values in different interest rate regimes. To identify different interest rate regimes for each country, we test for multiple break points using the sequential procedure of Bai and Perron (1998, 2003). We allow for up to 5 breaks and serial correlation in the errors. At the 5 percent significance level, we find three separate interest rate regimes for all countries except Italy, which has only two. The dates for each regime over the sample 1970-2007, together with the mean value of the growth-adjusted interest rates (percents) in each regime, $r_h$, are given in Table 1, where $h \in (1, 2, 3)$ denotes the interest rate regime.

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$^{19}$We obtain almost identical results when we use the Bayesian Information Criterion and the modified Schwarz criterion of Liu et al. (1997).
3.2 Empirical Results

The estimating equation, modified to include explicit surplus shocks and different interest rate regimes, is given by

\[ s_t = c_h + \beta s_{t-1} + \gamma_h d_{t-1} + \delta_1 \bar{y}_t + \delta_2 \bar{g}_t + \bar{\epsilon}_t, \]

(22)

where \( \gamma_h \) and \( c_h \) denote interest-rate-regime specific values for responsiveness and the constant. We use the break dates from Table 1 to construct dummy variables for each interest rate regime, and use them to estimate equation (22).

Results are contained in Table 2. The results imply that surplus responsiveness increases in interest rates, as necessary to mitigate the impact of higher interest rates on long-run values. We find that in the first sub-period, labeled regime 1, when interest rates are negative for all countries, responsiveness \( (\gamma_1) \) is never both positive and significantly different from zero. Therefore, in regime 1 with negative interest rates, countries were either responding negatively to debt or were not responding at all. In the middle period, regime 2, when interest rates for all countries rise and become positive, responsiveness \( (\gamma_2) \) rises above zero for all countries and is statistically significant at the 1% percent level for most countries. In regime 3 when interest rates fall for all countries except Japan, responsiveness falls \( (\gamma_3) \) for all countries except Japan. For Japan, the interest rate rises and responsiveness rises. Therefore, for all countries, responsiveness increases systematically with the interest rate.\(^{20}\) There is no systematic change in the constant across interest rate regimes.

We test whether the interest rate regimes are distinct with F-tests for the equality of the regimes. Table 3 reports the p-values of the F-tests. For almost all the countries, the early low-interest rate regime, regime 1, is statistically different from the very highest one. The highest interest rate regime is regime 2 for all countries except Japan and is regime 3 for Japan. Additionally, for almost all the countries, regime 2, which has maximum interest rates for all countries except Japan, is statistically different from regime 3.\(^{21}\)

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\(^{20}\) As a check on the need to use a fiscal rule with different parameters for different interest rate regimes, we estimated fiscal rules imposing identical behavior across interest rate regimes. We often failed to find significance for the responsiveness of debt to the surplus, reinforcing our result that the positive and significant responsiveness appears when interest rates are relatively high.

\(^{21}\) There are two anomalies in Table 3. First, for the UK, the high interest rate regime, regime 2, is not statistically different from the later lower interest rate regime, regime 3. This might be because the UK experienced the smallest fall in the interest rate between the two regimes. Second, for the US, regime 1 is not statistically different from regime 2 or from regime 3, yet, regime 2 is statistically different from regime 3. The problem seems to be that estimates in regime 1 are imprecise, implying that regime 1 is not very different from either regime 2 or regime 3.
Next we consider stability in the three interest rate regimes. Stability requires comparison of the responsiveness in the particular interest rate regime \( \gamma_h \) with an interest rate term in that regime \( r_h (1 - \beta) \) from equation (18). The interest rates are negative in the first regime for all countries, and the fiscal feedback rules satisfy the stability criterion for all countries except France, Greece, Portugal, and the UK.\(^{22}\) In regime 2, the responsiveness is high enough to satisfy the stability requirement for all countries. In regime 3, despite the fall in the responsiveness, it remains higher than the interest rate term, satisfying the stability requirement for all countries.

We compute the value of long-run debt, equation (19), for each country in each interest rate regime which satisfies the stability requirement from equation (18). We use the parameter values from Table 2 with the interest rates from Table 1. Long-run values are given in Table 4. For most of the countries, long-run debt is highest in the regime for which the interest rate is highest, implying that countries do not fully adjust the parameters of the fiscal feedback rule to counter the increase in the long-run value of debt implied by a higher interest rate.

4 Fiscal Space and Risk Categories

We compute out-of-sample estimates of fiscal space over the period between 2008 and 2014 for each country. Our estimates of fiscal space predict the two fiscal crises which occurred and allow us to separate other countries into risk categories. To construct adjustment paths for expected future values of debt and primary surpluses, we use our estimated fiscal feedback rules, conditional on a particular interest rate regime. To construct the boundary paths, we also need estimates for \( s \). Therefore, to measure fiscal space, we must first determine both the interest rate regime, beginning in 2008, and the value of \( s \).

4.1 Interest Rate Regime

We need the interest rate regime beginning in 2008. To determine whether each country was in a low or high interest rate regime between 2008 and 2014, we repeat the break test now using the full sample (1970-2014). For seven countries, namely Belgium, Canada, France, Italy, Japan, UK and the US, we find that there is no change in the interest rate regime from regime 3 after 2007. This justifies use of the estimated coefficients for the

\(^{22}\)Failure to satisfy stability criteria when the interest rate is negative should not imply that agents do not lend because governments do not face a binding intertemporal budget constraint. The more they borrow, the more revenue they receive due to the negative interest rate.
regime 3 fiscal feedback rule, from Table 2, together with the regime 3 interest rates, from Table 1, to estimate expected adjustment paths.\footnote{For Canada and the UK, the regime 3 interest rate is slightly negative, implying that these countries do not actually face an intertemporal government budget constraint. Our measure of the growth-adjusted interest rate is an ex post realized rate when a government would be using an ex ante expected rate. We do not think it is reasonable that Canada and the UK expected that their budget constraints did not bind over this period. Therefore, for these two countries, we assume that the expected growth-adjusted interest rate was higher than the ex post realized rate and substitute the small positive value for the US growth-adjusted interest rate in regime 3.}

For the remaining three countries, namely Greece, Portugal, and Spain, we find evidence of a high-interest rate regime beginning in 2008. We assume a return to the high interest rate regime 2, for which we have estimates from Tables 1 and 2.\footnote{Growth-adjusted interest rates could have risen due to the introduction of risk premia or to the reduction in growth. The measure of the growth-adjusted interest rate we need has no risk premium. Therefore, if this were the only possible reason for a break, then we would ignore it. However, the sharp reduction in growth could be responsible for the break. For this reason, we accept the evidence for a break. In the absence of information on regime 4, we assume a return to regime 2.}

4.2 Estimates of $\bar{s}$ and $\bar{d}$

To estimate the surplus limit, $\bar{s}$, we use historical data, together with our estimated dynamics.\footnote{We can also justify using historical information to determine ability to pay for the same reasons that private credit markets use a household’s history of borrowing and lending to set credit limits.} We entertain two estimates of the surplus limit and take the maximum. For the first, we follow Tanner (2013) and Collard et al (2015) and choose $\bar{s}$ to equal to the maximum historical surplus in our sample. For the second, we use adjustment paths associated with every historical debt-surplus pair to estimate the maximum surplus and debt along each adjustment path. Since agents were lending at historical debt-surplus pairs, these adjustment paths must be consistent with solvency without default. The maximum surplus implied by this technique is the peak surplus on the highest adjustment path. We make calculations using both approaches, and select the one which provides the overall largest value for $\bar{s}$.

Estimates for surplus limits for each country for 2008 are given in Table 5. For Belgium, Canada, France, Greece, Italy, Portugal, Spain, and the UK, the historical maximum surplus is larger. For Japan and the US, the historical debt-surplus pairs in 1994 and 2005, respectively, provide a larger value for the maximum surplus. We set point L in Figure 1 equal to the maximum surplus, and then construct the adjustment path (BWLE).

Next, we use the adjustment path, with peak at $\bar{s}$, for each country, to estimate the debt limit, $\bar{d}$, as the peak debt (point W) along the path. Estimates for the debt limits for each country for 2008 are given in Table 5. Countries exhibit large variations in their...
debt limits, yielding differences in debt tolerance.

Additionally, calculations for $d$ along the boundary path differ considerably from calculations based on the standard measure, which assumes that a country moves to $\bar{s}$ immediately and retains the value forever. As an example, consider Greece, whose historical maximum surplus in 2008 is 3.06 percent of GDP. Using the regime 2 estimated coefficients and interest rate, our estimate for the debt limit in 2008 is 108.02 percent of GDP, which is the peak of the debt along the boundary path with peak surplus equal to 3.06 percent of GDP. In contrast, if we compute the debt limit as the present value of the historical maximum of the surplus, we obtain the much larger value of 197.42 percent of GDP. Given the recent financial history of Greece, the second value is unreasonable. This measure of the debt limit did not predict the Greek crisis.

Our measures for the surplus and debt limits can be time-varying when countries have not yet experienced their maximum surplus. Our 2008 measures are based on historical behavior of the debt and the primary surplus through 2007. However, the true maximum surplus could be higher implying that the actual boundary path exceeds our estimated boundary path. Going forward from 2008, if shocks send the debt-surplus pair to a point implying a larger peak surplus, and if the country retains access to financial markets, we update our estimates for $\bar{s}$ and $\bar{d}$ to lie along the adjustment path for the realized debt-surplus pair, shifting the boundary path upward.

Finally, a country could lose access to the private markets and receive official loans. Since the value of debt in this case is not market determined, the official loan does not have to obey debt limits, implying that the expected present value of the largest future surpluses need not be large enough to equal debt. When a country loses access to the markets, we do not update our measure of the surplus limit.

4.3 Ten High-Debt Developed Countries

We compute measures of fiscal space for each country for each year and place each country into a risk category based on fiscal space. We use each country’s risk category to predict the two fiscal crises which occurred and to separate remaining countries into risk categories. Table 6 contains the bounds on fiscal space and the value of debt for each country, either for the crisis year or for the end of our sample, 2014. The high-debt countries we study belong to all three risk categories. The two countries with the highest fiscal space are "safe", the three with zero fiscal space are at "highest risk", while the others are in the intermediate category with varying risk.

If a country can achieve a larger value for $\bar{s}$ than historically indicated, then our esti-
mate of \( \overline{s} \) would be too low, overstating the risk of crises. Therefore, our prediction of a crisis is conditional on a country not being able to achieve higher surpluses than historically indicated. The ability to raise surpluses higher than those experienced historically puts the country in uncharted territory, where we do not have actual evidence to evaluate actual ability to succeed.

In our model, creditors refuse to lend when the economy is above the boundary locus, and default restores solvency after a crisis. However, when comparing with real-world experience, politics can interfere with market outcomes, separating the timing for loss of market access from that of default. Therefore, we date a fiscal solvency crisis as the date of loss of market access, where private loans are replaced by official loans, even if the actual default comes later (or not yet).\(^{26}\)

### 4.3.1 Belgium and Canada

Belgium and Canada are two high-debt countries which both belong to the "safe" category over the entire period following the 2008 global financial crisis. Figure (2) contains the boundary paths for Belgium and Canada, their debt-surplus pairs between 2008 and 2014, and the trapezoids for the position with the highest adjustment path. The trapezoids lie fully below the boundary paths, implying that three standard deviations shocks to both debt and the surplus would not breach the boundary path. Therefore, our model successfully predicts that neither country has been in danger of crisis over this period. These two countries provide a test of the model. The model predicts that these countries are "safe", and both countries have been free of debt crises.

### 4.3.2 Greece and Portugal

Consider the opposite case, Greece and Portugal, which both experienced fiscal crises. Figure (3) contains the boundary paths for both countries in the years leading up to their fiscal crises, together with their debt-surplus pairs. For both countries, shocks, together with default expectations, did send the 2008 debt-surplus pairs above the 2008 boundary paths. Since both countries continued to participate in financial markets in 2008 without a fiscal crisis, we update the 2009 boundary paths to be the expected adjustment path from the 2008 debt-surplus pairs. This in turn, places the debt-surplus pairs on their respective boundary paths, implying zero fiscal space and placing both countries in the "highest risk" category.

\(^{26}\)Concessionary loans would change the budget constraint. Our model assumes that there are no concessionary loans.
From this position of high risk, adverse shocks in 2009 again pushed both countries above their 2009 boundary paths, implying continuing high risk for both. Since neither country lost access to financial markets, we update the boundary paths again. Eventually, Greece lost access to financial markets early in 2010, and Portugal in 2011. Two to three years in the "highest risk" category, predicted both fiscal crises. Therefore, our method successfully predicts both crises, providing another test of the model.

Note that debt tolerance for the two countries was very different. Based on Greece’s 2010 boundary path, Greece would have been categorized as "risky", not as "highest risk", if it had had Portugal’s lower pre-crisis debt (Table 6). Additionally, note the large increases in debt after the crisis for both countries. These values are consistent with our concern that debt might no longer satisfy expected intertemporal budget constraints after official loans supplement private loans.

4.3.3 US

Figure (4) presents the initial boundary path for the US, together with historical values for the debt-surplus pairs between 2008 and 2014. The US begins in 2008 with large fiscal space, placing it in the "safe" category. However, the fiscal space shrinks over time. Starting in 2010, the trapezoid does not lie fully below the boundary path, implying that fiscal space is not large enough to keep the US out of the "risky" category. In 2012, shocks send the debt-surplus pair outside its boundary path. Since the US did not face a fiscal crisis in 2012, we update the boundary path to be the expected adjustment path from the 2012 debt-surplus pair, placing the debt-surplus pair on the boundary path. This puts the US in the "highest risk" category. In 2013, the US experiences favorable fiscal shocks, creating positive fiscal space, moving the US from "highest risk" to "risky". The US is the only country in our sample which entered and exited the "highest risk" category.

4.3.4 Japan, France, and Spain

Japan’s experience is contained in Figure (5). We present the initial boundary path for the Japan, together with values for the debt-surplus pairs between 2008 and 2014. In 2008, Japan has positive fiscal space, but the trapezoid does not lie fully below the boundary paths, placing the country in the "risky" category. Averse shocks in 2011 send the country outside its historical boundary path. Since Japan did not experience a fiscal crisis in 2011, we update the next year’s boundary path to be the expected adjustment path from the 2011 debt-surplus pair, placing debt-surplus pair on the boundary paths
implying the "highest risk" category. In 2012, adverse shocks push the debt-surplus pair above the 2012 boundary path such that Japan retains the "highest risk" status. This pattern of actual debt-surplus pairs falling outside the historical boundary paths continues throughout the period. Therefore, Japan remains at "highest risk" until the end of our sample and is an example of the highest debt country also being in the "highest risk" category.

Behavior for France and Spain is similar to Japan, except that these countries exhausted fiscal space later in the sample. Figure (5) presents their initial boundary paths together with values for the debt-surplus pairs between 2008 and 2014. These countries begin the period with large fiscal space which shrinks over time. Spain breached its boundary path in 2013, and France in 2014. In 2013, Spain, facing high interest rates for borrowing (the default risk premium), received an official loan without actually losing access to markets. Spain did access private credit markets in 2014, implying that its intertemporal budget constraint was expected to be satisfied. Even so, fiscal space was zero, placing Spain in the "highest risk" category. France has not accessed official loans, but exhausted fiscal space in 2014, placing it in the "highest risk" category. Table 6 shows that both countries had similar debt in 2014. Even so, the US had more debt, but its historical experience with surpluses implied that it faced less risk.

4.3.5 UK and Italy

Finally, consider the UK and Italy, whose boundary paths are drawn in Figure (6). Over the period, both countries experience falling fiscal space, thereby moving from the "safe" category to the "risky" category. Although they have not breached their respective historical boundary paths, fiscal space is not large enough to eliminate the possibility that future shocks could send the debt-surplus pairs beyond the historical boundary paths, as shown by the trapezoids, which do not lie fully below the boundary paths. Although Italy has the second highest debt in the sample (Table 6), fiscal space is positive, placing it in the "risky" category instead of the "highest risk" category.

4.3.6 Prediction Successes

Our measure of fiscal space correctly predicts the crises that Greece and Portugal experienced and correctly predicts the absence of crises in Canada and Belgium. For the remaining six countries, our measures indicate crisis risk, with Japan, France, and Spain facing "highest risk." For the highest risk countries to avoid crisis, they must be able to raise future surpluses higher than they have done historically. We must await future
experience to evaluate whether these highest risk countries succumb to crisis, as Greece and Portugal did, or whether they can attain higher surpluses than they have experienced historically. For the US, Italy, and the UK, these countries face risk, but not the "highest risk," implying that future outcomes with or without fiscal crises are consistent with our predictions. Therefore, we have only four countries with definite predictions and outcomes against which to judge our model, and all four are consistent with the model.

5 Conclusion

How do we determine whether or not a country has taken on so much debt that it risks a fiscal solvency crisis? Every country faces limits on the magnitude of the primary surpluses it can raise over time and therefore on the value of the debt it can sustain. And these fiscal limits differ across countries, as evidenced by the Greek crisis at a value of debt/GDP which Belgium successfully managed. Fiscal solvency requires that expected future paths for debt and the primary surplus remain below the debt limits while satisfying the government’s expected intertemporal budget constraint. We propose and implement a new and simple data-based method for estimating debt limits and fiscal space, and for separating countries into risk categories.

Our methodology relies on historical data in two ways. First, we need estimates of fiscal limits. For the surplus limit, we begin with the largest historical value of the primary surplus. Second, since a government does not move its primary surplus immediately to its limit and retain this value indefinitely, we need a way to predict expected future values for the surplus and debt. We estimate a fiscal feedback rule where the surplus responds to its own lag and to lagged debt, but is subject to stochastic shocks. Given a debt-surplus pair, the fiscal feedback rule provides predictions for future surpluses and debt. We use the estimated fiscal rules to construct adjustment paths which provide an alternative estimate for the surplus limit, as the peak of adjustment paths beginning at historical debt-surplus pairs. The surplus limit we use is the maximum of the historical value and this estimated value. The peak of the debt along this path becomes the debt limit. Surplus and debt limits differ substantially across countries, yielding different tolerance for debt.

To measure fiscal space and assess risk, we use the adjustment path, with peak at the surplus limit, to define a boundary path above which the country is insolvent without default. For a debt-surplus pair above the boundary path, expected future values of the debt and/or surplus are expected to violate fiscal limits, an impossibility. We define fiscal space as the magnitude of the increase in debt following an adverse shock which would
send the debt-surplus pair to the boundary path. Since countries face different fiscal limits, fiscal space is not necessarily lowest for the highest debt countries. We use our measures of fiscal space to separate countries into three risk categories. The "highest risk" category contains countries which have exhausted their fiscal space. This is uncharted territory – to retain solvency without default, they must be expected to raise surpluses above values deemed feasible by historical experience.

We use the model to predict fiscal crises following the 2008 global financial crisis. Both Greece and Portugal exhausted their fiscal space, placing them in the "highest risk" category. Several years in the "highest risk" category correctly predicted the fiscal crises they experienced. In contrast, Belgium and Canada remained in the "safe" category throughout, correctly predicting the absence of crisis. Therefore, for the two extreme cases, those which did default and those which showed no risk, the model successfully predicts outcomes.

All other countries experienced shrinking fiscal space after 2008, with adverse consequences for risk. The US exhausted fiscal space in 2012, but moved back to positive territory a year later. Japan exhausted fiscal space in 2011, never regaining positive fiscal space. France and Spain experienced falling fiscal space, with Spain receiving an official loan in 2013 and France exhausting fiscal space in 2014. Italy and the UK have experienced falling fiscal space and therefore rising risk, but have not exhausted their fiscal space. Our sample ends with France, Spain, and Japan in the "highest risk" category, with Japan having maintained that status since 2011. Continued solvency for these countries at highest risk rests on confidence from lenders that they can attain surpluses higher than historically observed, warning of potential future fiscal crises.

References


26
6 Data Appendix

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
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<tbody>
<tr>
<td>( s_t )</td>
<td>primary surplus as a percentage of GDP</td>
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<td>( d_t )</td>
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<td>OECD (Economic Outlook No. 96) and AMECO database</td>
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<td>( \tilde{g}_t )</td>
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The sample is from 1970-2014. We use 1970-2007 to estimate the fiscal feedback rules (in sample estimation), and we use 2008-2014 for the assessment of risk (out of sample).

Summary Statistics (1970-2007)

<table>
<thead>
<tr>
<th></th>
<th>Debt as % of GDP</th>
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<td></td>
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### Table 1 Interest Rate Regimes

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<td>-0.68</td>
</tr>
<tr>
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<td>-1.01</td>
<td>4.18</td>
<td>0.29</td>
</tr>
<tr>
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<td>-6.65</td>
<td>1.55</td>
<td>-3.93</td>
</tr>
<tr>
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<td>1.93</td>
<td></td>
</tr>
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<td>Japan</td>
<td>-7.97</td>
<td>0.23</td>
<td>1.69</td>
</tr>
<tr>
<td>Portugal</td>
<td>-7.27</td>
<td>2.65</td>
<td>-1.02</td>
</tr>
<tr>
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<td>2.10</td>
<td>-2.92</td>
</tr>
<tr>
<td>UK</td>
<td>-3.62</td>
<td>2.10</td>
<td>-0.11</td>
</tr>
<tr>
<td>US</td>
<td>-2.21</td>
<td>2.64</td>
<td>0.02</td>
</tr>
</tbody>
</table>

### Table 2 Fiscal Rules with Interest Rate Dependency

<table>
<thead>
<tr>
<th>Country</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$\beta$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$R^2$</th>
<th>$\sigma_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>-2.534</td>
<td>-10.907</td>
<td>-4.961</td>
<td>0.353</td>
<td>0.011</td>
<td>0.101</td>
<td>0.070</td>
<td>0.336</td>
<td>-0.223</td>
<td>0.907</td>
<td>1.264</td>
</tr>
<tr>
<td>Canada</td>
<td>-6.346</td>
<td>-6.164</td>
<td>-3.885</td>
<td>0.588</td>
<td>0.113</td>
<td>0.077</td>
<td>0.051</td>
<td>0.346</td>
<td>-0.396</td>
<td>0.960</td>
<td>0.711</td>
</tr>
<tr>
<td>France</td>
<td>0.499</td>
<td>0.517</td>
<td>0.646</td>
<td>0.290</td>
<td>-0.037</td>
<td>0.075</td>
<td>0.055</td>
<td>0.334</td>
<td>-0.559</td>
<td>0.683</td>
<td>0.620</td>
</tr>
<tr>
<td>Greece</td>
<td>1.672</td>
<td>-3.744</td>
<td>-6.416</td>
<td>0.368</td>
<td>-0.110</td>
<td>0.067</td>
<td>0.055</td>
<td>0.334</td>
<td>-0.132</td>
<td>0.761</td>
<td>0.944</td>
</tr>
<tr>
<td>Italy</td>
<td>-5.241</td>
<td>-7.711</td>
<td>-12.038</td>
<td>0.449</td>
<td>0.055</td>
<td>0.067</td>
<td>0.070</td>
<td>0.164</td>
<td>-0.153</td>
<td>0.760</td>
<td>1.132</td>
</tr>
<tr>
<td>Japan</td>
<td>-1.807</td>
<td>-2.275</td>
<td>-12.038</td>
<td>0.619</td>
<td>0.041</td>
<td>0.067</td>
<td>0.070</td>
<td>0.002</td>
<td>-0.241</td>
<td>0.760</td>
<td>1.495</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.637</td>
<td>-4.434</td>
<td>-12.038</td>
<td>0.253</td>
<td>-0.118</td>
<td>0.029</td>
<td>0.025</td>
<td>0.288</td>
<td>-0.138</td>
<td>0.760</td>
<td>1.495</td>
</tr>
<tr>
<td>Spain</td>
<td>-4.982</td>
<td>-4.781</td>
<td>-8.224</td>
<td>0.442</td>
<td>0.231</td>
<td>0.077</td>
<td>0.025</td>
<td>0.137</td>
<td>-0.657</td>
<td>0.712</td>
<td>1.318</td>
</tr>
<tr>
<td>UK</td>
<td>-4.623</td>
<td>-4.331</td>
<td>-8.224</td>
<td>0.604</td>
<td>0.184</td>
<td>0.077</td>
<td>0.035</td>
<td>0.305</td>
<td>-0.919</td>
<td>0.712</td>
<td>0.714</td>
</tr>
<tr>
<td>US</td>
<td>-11.022</td>
<td>-8.224</td>
<td>-5.248</td>
<td>0.222</td>
<td>0.042</td>
<td>0.160</td>
<td>0.037</td>
<td>0.288</td>
<td>-0.919</td>
<td>0.712</td>
<td>0.714</td>
</tr>
</tbody>
</table>

Note: The *, † and ‡ indicate statistical significance at the 10, 5 and 1 percent level, respectively.
Table 3 F-tests on Regime Equality

<table>
<thead>
<tr>
<th></th>
<th>Belgium</th>
<th>Canada</th>
<th>France</th>
<th>Greece</th>
<th>Italy</th>
<th>Japan</th>
<th>Portugal</th>
<th>Spain</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(\text{regime}_1 = \text{regime}_2) )</td>
<td>0.089</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.030</td>
<td>0.899</td>
<td>0.054</td>
<td>0.099</td>
<td>0.006</td>
<td>0.592</td>
</tr>
<tr>
<td>( p(\text{regime}_2 = \text{regime}_3) )</td>
<td>0.007</td>
<td>0.104</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
<td>0.004</td>
<td>0.089</td>
<td>0.013</td>
<td>0.668</td>
<td>0.000</td>
</tr>
<tr>
<td>( p(\text{regime}_1 = \text{regime}_3) )</td>
<td>0.508</td>
<td>0.027</td>
<td>0.386</td>
<td>0.000</td>
<td>-</td>
<td>0.021</td>
<td>0.175</td>
<td>0.008</td>
<td>0.079</td>
<td>0.385</td>
</tr>
</tbody>
</table>

Table 4 Long Run Debt (% of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Belgium</th>
<th>Canada</th>
<th>France</th>
<th>Greece</th>
<th>Italy</th>
<th>Japan</th>
<th>Portugal</th>
<th>Spain</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{regime}_1 )</td>
<td>86.49</td>
<td>50.61</td>
<td>(unstable)</td>
<td>(unstable)</td>
<td>57.57</td>
<td>25.38</td>
<td>(unstable)</td>
<td>18.53</td>
<td>(unstable)</td>
<td>50.23</td>
</tr>
<tr>
<td>( \text{regime}_2 )</td>
<td>141.53</td>
<td>98.40</td>
<td>83.36</td>
<td>89.19</td>
<td>105.35</td>
<td>79.75</td>
<td>67.27</td>
<td>73.15</td>
<td>48.59</td>
<td>54.95</td>
</tr>
<tr>
<td>( \text{regime}_3 )</td>
<td>71.99</td>
<td>72.74</td>
<td>81.50</td>
<td>78.72</td>
<td>-</td>
<td>188.75</td>
<td>84.80</td>
<td>21.03</td>
<td>40.58</td>
<td>69.07</td>
</tr>
</tbody>
</table>

Table 5 Fiscal limits (% of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Belgium</th>
<th>Canada</th>
<th>France</th>
<th>Greece</th>
<th>Italy</th>
<th>Japan</th>
<th>Portugal</th>
<th>Spain</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{s}_{2008} )</td>
<td>6.10</td>
<td>5.95</td>
<td>1.10</td>
<td>3.06</td>
<td>5.65</td>
<td>5.73</td>
<td>2.23</td>
<td>3.48</td>
<td>7.83</td>
<td>5.51</td>
</tr>
<tr>
<td>( \tilde{d}_{2008} )</td>
<td>137.97</td>
<td>142.04</td>
<td>94.06</td>
<td>108.02</td>
<td>135.42</td>
<td>208.18</td>
<td>71.74</td>
<td>90.60</td>
<td>99.16</td>
<td>110.16</td>
</tr>
</tbody>
</table>

Table 6 Fiscal Space (% of GDP)

<table>
<thead>
<tr>
<th>Fiscal space end of the sample</th>
<th>Belgium</th>
<th>Canada</th>
<th>France</th>
<th>Italy</th>
<th>Japan</th>
<th>Spain</th>
<th>UK</th>
<th>US</th>
<th>Greece</th>
<th>Portugal</th>
</tr>
</thead>
<tbody>
<tr>
<td>largest</td>
<td>33.79</td>
<td>48.02</td>
<td>0</td>
<td>4.82</td>
<td>0</td>
<td>0</td>
<td>11.71</td>
<td>3.10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>smallest</td>
<td>28.46</td>
<td>27.68</td>
<td>0</td>
<td>4.82</td>
<td>0</td>
<td>0</td>
<td>10.70</td>
<td>3.10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Debt level</td>
<td>106.47</td>
<td>93.86</td>
<td>95.02</td>
<td>132.11</td>
<td>229.99</td>
<td>97.67</td>
<td>89.36</td>
<td>109.71</td>
<td>130.7</td>
<td>97.6</td>
</tr>
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</table>

Note: Greece (Portugal) received its first bailout in May 2010 (May 2011) after losing access to private credit markets. We list debt at the end of the quarter prior to each crisis 2009 (2010).