Sovereign Default in a Rich Country

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Abstract

Rising debt following the Great Recession climaxed in the Greek debt crisis. Yet, this crisis did not evolve as a typical strategic default crisis. Debt/GDP reached levels much higher than could be supported by standard costs to default. Debt was countercyclical in the decade prior to the crisis. As the crisis approached, debt spiked, instead of falling. We argue that a rich-country sovereign faces different incentives for default, which alter debt dynamics. We modify the strategic default model to have a reputational equilibrium by introducing multiple contracts with spillovers in value. Default occurs only due to inability to repay and is excusable if the sovereign repays what she is able. We calibrate to the 2010 Greek crisis, and demonstrate that the rich-country default model allows much larger values for debt/GDP than strategic default and matches Greek debt dynamics prior to the crisis. We use the calibrated model to explain crisis severity, including magnitudes for endogenous risk premia, haircuts, and default duration.

Keywords: Strategic Default, Fiscal Limits, Excusable Default, Sovereign Default, Fiscal Risk, Ability to Pay, Haircuts

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1 Introduction

Prior to the Great Recession of 2008, sovereign default was confined to emerging-market and poor countries. With the recession, government debt/GDP in most industrial countries increased substantially, in many cases to unprecedented levels. These large debt ratios, accompanied by financing difficulties in some European countries, raised the possibility of sovereign default by rich countries. The Greek crisis, which began in the first quarter of 2010 and was followed by default, confirmed this possibility. However, the Greek crisis did not evolve as a typical strategic default crisis (Arellano 2008). Debt/GDP reached levels much higher than could be supported by standard costs to default. Debt was countercyclical in the decade prior to the crisis. As the crisis approached, debt spiked, instead of falling. Over the same period, other European countries also experienced sharply rising debt, but did not default.

The objective of this paper is to develop a stochastic general equilibrium model which can match these characteristics of the Greek crisis. In the canonical strategic default model, the fall in debt as a crisis approaches is an analytical result, not a calibrated result.[1] Therefore, recalibration alone cannot change debt dynamics in the strategic default model to match those of Greece. We modify the canonical model to incorporate reasons that the optimizing problem faced by a rich-country sovereign could differ from that faced by an emerging-country sovereign. The end product is a stochastic general equilibrium model of sovereign default for a rich country. The rich-country default model replicates the behavior of debt leading up to the Greek crisis and is consistent with other European countries avoiding default, even though they have debt/GDP substantially larger than standard default costs.

The first modification is based on Cole and Kehoe's (1997, 1998) set-up of a reputational equilibrium and allows accumulation of larger debt/GDP than in the canonical strategic default model. They consider an economy, in which the sovereign has multiple global contracts, all of which require trust to have value. There is asymmetric information about the sovereign's type. She could be an honest type, who upholds all contracts, or an optimizing type who upholds each contract only when expedient. Violation of one contract reveals her type as optimizing, thereby destroying trust and the value of all contracts. When a sovereign with an honest reputation considers a deviation from honoring one contract, she takes account of the reputational cost whereby violation of one contract

[1] The appendix in Arellano (2008), repeated in Schmitt-Grohe and Uribe's text (2017, p.515), presents a proof when income shocks are iid, and problem 13.10 (p.588) in the text extends this to the case with a serially-correlated income process.

would destroy all contracts. Therefore, beginning with a reputational equilibrium in which the sovereign has multiple contracts with spillovers in value, the cost of strategic default exceeds the cost of violating the debt contract in isolation. If the cost of default, inclusive of the lost value of reputation, exceeds the value of debt repudiation in all states, then the reputational equilibrium can be maintained and there is no default. Alternatively, if the value of global contract is too small, then there are states in which the sovereign would repudiate debt, revealing her type. Thereafter, the strategic default model applies.

The value of the set of global contracts, essentially the value of reputation, differs across countries. Rich countries tend to have greater connectedness to other countries through more implicit and explicit contracts. International meetings on global issues occur between rich countries, as with the G-20, or smaller groups. Many of these contracts facilitate trade in goods and assets, improving economic efficiency allowing higher output. Therefore, rich countries have a higher value for reputation, implying that loss of reputation would be more costly. Our first modification of the canonical strategic default model is to raise the cost of strategic default for a rich country to include the value of reputation, which would be lost with strategic default. For rich countries with strong global connectedness, we assume that this additional cost is large enough to support a reputational equilibrium.

However, in a reputational equilibrium, there is no default. To allow default, we make two additional modifications. First, following Davig, Leeper and Walker (2010, 2011) and Bi (2012), we assume that each country has state-dependent fiscal limits on her ability to pay. The fiscal limits have economic and political determinants and are increasing in current output. Second, following Grossman and Van Huyck (1988), we introduce excusable default. Explicitly, default is excusable and elicits no penalty as long as the sovereign pays what she is able. If stochastic shocks reduce the fiscal limit below debt, the sovereign defaults but maintains reputation by repaying as much as she is able. Together these modifications imply that the optimizing sovereign will never choose strategic default, but will choose excusable default when unable to repay. This alternative cause of default implies different debt dynamics as a crisis approaches.

An overview of our rich-country default model is as follows. The optimizing sovereign faces stochastic endowment shocks and fiscal limits on ability to repay. She chooses debt and debt repayment to maximize utility of consumption. A large enough value to global contracts supports a reputational equilibrium in which the sovereign commits to repay debt up to her fiscal limits. Default occurs when stochastic shocks to the endowment reduce the state-dependent fiscal limit below outstanding debt. Default is due to inability to fully repay. As long as the sovereign repays what she is able, the default is excusable and does not destroy reputation. Since the sovereign values her reputation highly, she optimally chooses to maintain reputation in default by repaying debt up to her ability.

The rich country's decision to shun strategic default and to repay debt up to fiscal limits, in order to avoid loss of reputation, implies that the incentives for the sovereign to accumulate debt and to default are fundamentally different from those in strategic default. These different incentives alter debt dynamics leading up to default. First, the larger cost to strategic default and the associated choice to repay debt up to fiscal limits allows accumulation of larger values for debt/GDP. Second, in contrast with strategic default, the possibility of excusable default does not reverse consumption-smoothing incentives.[2] This is because repayment up to fiscal limits in default mitigates the interest rate response to an increase in the probability of default. Third, the small responsiveness of the interest rate to the probability of excusable default implies that, when debt is near the fiscal limit, the sovereign has the incentive to take on even more debt, creating a spike in debt.[3] Finally, in the rich-country model of sovereign default, the sovereign always repays debt up to fiscal limits, yielding an endogenous and optimal determination of the haircut – always above the optimal value of zero in strategic default.

We calibrate the model to match the Greek crisis and demonstrate quantitatively that the rich-country default model can match the characteristics of debt leading up to the Greek crisis. We use the results of the calibrated model to describe other features of debt crises, including the interest rate spike as a crisis approaches, and the magnitude and duration of haircuts.

Other European countries experienced rising debt/GDP with the Great Recession, but did not default. The fact that they have not defaulted, even as debt/GDP has risen, implies that fiscal limits are even higher than recent values of debt. For most countries, debt/GDP has remained lower than the value which created Greece's crisis. However, Italy's debt/GDP has continued to grow, surpassing the critical value for Greece, and Italy has continued servicing debt. The implication is that Italy's fiscal limits are higher than those of Greece. Its history of bringing debt down with large primary surpluses is consistent with this inference, but this history does not guarantee that she will avoid default should future recession reduce ability to pay.

Related Literature This paper brings together four literatures on sovereign risk

[2] Arellano (2008) demonstrated that the possibility of strategic default reverses consumption-smoothing incentives, allowing the relatively higher volatility of consumption compared to output, created by pro-cyclical debt. This outcome can be implemented with lump-sum taxes financed by pro-cyclical borrowing, consistent with pro-cyclical debt in many emerging economies.

[3] Stiglitz (1981) first demonstrated that the possibility of default cuts off the lower portion of the risk distribution, thereby incentivizing risk-taking.

and default, including those on strategic default, excusable default, fiscal limits, and reputational equilibrium, with the objective of explaining three characteristics of the Greek crisis: 1) higher debt ratios than can be supported by standard default costs; 2) countercyclical debt; and 3) the sharp upward spike in debt as the crisis approached. Eaton and Gersovitz (1982), Aguiar and Gopinath (2006), and Arellano (2008) are the seminal papers on strategic default. Aguiar and Amador (2013) and Aguiar, Chatterjee, Cole, and Strangeby (2016) provide extensions and survey extensions offered by others. Other authors have addressed raising debt ratios and generating countercyclical debt separately, but we are aware of no modifications of the strategic default model which address all three characteristics of the Greek crisis together, or which explain the upward spike in debt prior to the crisis.

Increasing equilibrium debt ratios to match those in emerging markets has been addressed using models, which raise the cost of default, due to the damage default inflicts on the financial system and therefore on output (Brutti 2011, Gennaioli, Martin, and Rossi 2014, Padilla 2014, Perez 2015), and models, which generate partial repayment, either exogenously (Uribe and Schmitt-Grohe 2017), or endogenously through an additional financial contract (Salomao 2017), or through debt renegotiation (Yue 2010, Mendoza and Yue 2012, Benjamin and Wright 2008). Chatterjee and Eyigungor (2012) use longer-maturity debt to achieve ratios as high as those in Argentina. Arellano, Mateos-Planos, and Rios-Rull (2013) generate partial repayment by assuming that there is an output loss proportional to the size of unpaid debt.

Paluszynski (2017), Bocola and Dovis (2016) and Bocola, Bornstein and Dovis (2018) introduce modifications of the strategic default model to obtain countercyclical debt. Paluszynski uses learning about a rare disaster to explain that debt could increase in the early phase of a disaster while agents do not expect default. However, debt must be falling as the crisis approaches. Bocola and Dovis (2016) and Bocola, Bornstein and Dovis (2018) introduce a minimum government consumption to incentivize the government to increase debt when output falls. However, as the crisis approaches the upward spike in interest rates sends debt downward.

The fiscal limits literature is based on the concept that every government faces a limit on the revenue net of spending that it can extract from its citizens. The fiscal limit is defined as the maximum debt the government can repay in a particular income state. Default occurs when the government cannot fully repay. Fiscal limits have been used as explanations of default (Bi, 2012, Bi and Leeper, 2012, Bi and Tram, 2014, Daniel and Shiamptanis 2013, 2018)), policy switching (Davig, Leeper, and Walker, 2010, 2011), and fiscal consolidation (Bi, Leeper and Leith 2013), among others. In default models with fiscal limits, debt/GDP ratios for rich countries look more like those in the data, but the behavior of the sovereign is determined either empirically or exogenously, not optimally, as in our model.

Excusable default is based on the fact that shocks can make full repayment of debt impossible (Grossman and Van Huyck 1988). When full repayment is impossible, default is excusable and does not trigger a punishment, as long as the agent pays what she is able. Bulow and Rogoff (1989a) present a model in which renegotiation in some states is part of the implicit contract, yielding similar outcomes to those of excusable default.

Bulow and Rogoff (1989b) demonstrate that a reputational equilibrium is not possible when the sovereign can borrow and save at the world risk-free interest rate and when reputation for upholding one contract cannot spill-over into reputation in other contracts. They suggest, however, that a reputational equilibrium might be possible when there are multiple contracts with spillovers in value. Cole and Kehoe (1997,1998) follow up on this suggestion and create a model with a reputational equilibrium.

The paper is organized as follows. Section 2 presents the theoretical model, including the debt contract and optimizing behavior by the sovereign. Section 3 provides the calibration to the Greek crisis. Section 4 provides a quantitative description of financial crises using the calibrated model. Section 5 concludes.

2 Model

2.1 Background

2.1.1 Strategic Default

Our baseline model is the canonical strategic default model (Arellano 2008). The domestic economy is small, open and subject to stochastic endowment shocks. Endowment income (y) on each date is drawn from a bounded distribution, indexed by $j \in [1, \bar{j}]$. The bounded distribution approximates a distribution in which income is determined by

$$\ln y' = (1-\rho)\ln \bar{y} + \rho \ln y + \epsilon' \qquad 0 < \rho < 1 \qquad \epsilon \sim N(0, \sigma_\epsilon^2),$$

where the prime denotes a one-period-ahead value. With this specification, high income today implies high expected future income.

The sovereign can trade one-period bonds with risk-neutral international lenders to maximize the expected present value of utility of consumption of the representative agent, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right), \tag{1}$$

where E_0 denotes the conditional expectation at time zero, $0 < \beta < 1$, c_t denotes consumption in period t, and $u\left(\cdot\right)$ is strictly increasing and concave. The sovereign is not committed to repay debt, but faces costs if she decides to repudiate debt that she could repay. Following a debt repudiation, the sovereign will be in bad standing for a random period of time. While in bad standing, she will be unable to access international markets for loans, and output will be lower by $L\left(y\right)$, with the loss in output increasing in the value of current output. The price of the government bond reflects the likelihood of default such that creditors are willing to lend any amount as long as they expect to receive the exogenous market rate of return on debt.

We follow Schmitt-Grohe and Uribe (2017) in laying out the equations for the canonical strategic default model. The value of repaying and continuing to participate in financial markets is given by

$$V^c\left(y, D\right) = \max_{D'} \left[u\left(c^c\right) + \beta E_y V^g\left(D', y'\right)\right], \tag{2}$$

where c^c is given by the budget constraint as

$$c^c = y + q(D', y)D' - D, \tag{3}$$

and where E_y is the expectation conditioned on y and $V^g\left(D', y'\right)$ is the value of being in good financial standing next period. The value of being in bad financial standing is given by

$$V^b\left(y\right) = u(c^b) + \beta\theta E_y V^g\left(0, y'\right) + \beta\left(1-\theta\right) E_y V^b\left(y'\right), \tag{4}$$

with c^b given by

$$c^b = y - L\left(y\right), \tag{5}$$

and where θ is the probability of returning to good financial standing next period. The value of being in good financial standing is given by

$$V^g\left(y, D\right) = \max\left[V^c\left(y, D\right), V^b\left(y\right)\right]. \tag{6}$$

Define $\hat{\jmath}$ as the lowest income state in which full repayment occurs. The price of debt necessary to satisfy risk-neutral lenders $\left(q^{sd}\right)$ is given by

$$q^{sd}\left(D',y\right)=\frac{1-F\left(\hat{\jmath}|h\right)}{\left(1+r^{*}\right)}, \tag{7}$$

where r^* is the fixed risk-free international interest rate, $F\left(\hat{\jmath}|h\right)$ is the cumulative distribution of output up to state $\hat{\jmath}$, and the superscript on q denotes the bond price with the possibility of strategic default. The numerator is the probability of repayment.

We define an equilibrium with the possibility of strategic default as:

Given the stochastic process for income, the lack of commitment to repay, costs of failure to repay, and a price function for debt $q^{sd}\left(D',y\right)$*, equilibrium is a set of policy functions for government debt holdings* $D'\left(D,y\right)$ *and consumption,* $c^{c}\left(D,y\right), c^{b}\left(y\right)$*, and a cutoff value for states determining repayment* $\hat{\jmath}\left(D\right)$*, for which the sovereign maximizes expected utility, budget constraints clear, and the price function for debt* $q^{sd}\left(D',y\right)$ *assures risk-neutral lenders an expected return equal to the exogenous risk-free rate of return.*

2.1.2 Reputation

We add asymmetric information, reputation, and spillovers in value across multiple global contracts to this canonical model. Following Cole and Kehoe (1997, 1998), we assume that there is asymmetric information about the sovereign's type. She could be honest, and always honor contracts, or optimizing, and honor contracts only when it is in her interest to do so. We assume that, if the sovereign has never violated a contract, she has a reputation for honesty. However, if the sovereign violates any contract, she reveals her type as optimizing, thereby destroying her reputation and the value of all contracts. The loss of value in one contract spills over to other contracts by revealing type and destroying trust. Since the violation reveals type, the loss of value is permanent.

Many of the sovereign's contracts facilitate trade in goods and assets, allowing a more efficient allocation of resources. Loss of this trade reduces allocational efficiency, implying lower output. In our endowment economy, we assume that the cost of lost reputation is a permanent proportional reduction in output in all states (y) reducing output in state y from y to ηy where $0 < \eta < 1$.[4] Reputation (O,H) adds a third state variable to the sovereign's value function since realized output in each output state, y, depends on reputation.

[4] Since we are using an endowment economy, we do not have the opportunity to microfound the efficiency loss due to lost gains to trade. We view the output loss as a reduced-form way of modeling the loss, much as Arellano's lost output due to strategic default is also reduced form.

This environment has two different types of equlibria, depending on the sovereign's reputation. Since the sovereign in strategic default is known to be the optimizing type, we can interpret the value functions for that model, given by equations (2), (4) and (6) as containing the third state variable O. With this interpretation and with the reinterpretation of output as the lower output achievable without international contracts, the strategic default model applies.

The sovereign with an honest reputation faces a different set of incentives when considering default because she has a reputation which default would destroy. Her value functions have H as the third state variable and are expressed as:

$$V^c(y, D, H) = \max_{D'} \left[u(c^c) + \beta E_y V^g(D', y', H') \right], \tag{8}$$

$$V^b(y, H) = u(c^b) + \beta\theta E_y V^g(0, y', O') + \beta(1 - \theta) E_y V^b(y', O'), \tag{9}$$

$$V^g(y, D, H) = \max \left[V^c(y, D, H), V^b(y, H) \right], \tag{10}$$

where c^c and c^b given by equations (3) and (5) and $V^b(y, O)$ and $V^g(0, y, O)$ are given by the value functions in equations (4) and (6) with income in state y given by ηy. A sovereign with an honest reputation is expected to repay in all output states, implying that $\hat{\jmath} = 1$, setting the bond price at the inverse of the gross risk-free international interest rate,

$$q^R = \frac{1}{1 + r^*},$$

where the superscript R denotes the value in a reputational equilibrium.

A strategic default would destroy reputation. The value of being in good financial standing with an honest reputation exceeds the value of good financial standing with an optimizing reputation, due to the higher output associated with good reputation. Although the sovereign could expect to regain good financial standing again after default, she could never regain an honest reputation. Therefore, having an honest reputation raises the cost of default.

This additional cost of default admits an additional type of equilibrium. If the cost of reputational loss is large enough, equivalently if η is small enough that $V^c(y, D, H) > V^b(y, H)$ for all values of output and admissible debt, defined as debt chosen in equilibrium,[5] then $V^g(y, D, H) = V^c(y, D, H)$. The sovereign never chooses strategic default. There is a cut-off value for η, below which $V^g(y, D, H) = V^c(y, D, H)$ for all values of

[5] This requires that the inequality hold for the smallest output state and the largest debt ever chosen in equilibrium.

output and admissible debt, allowing a reputational equilibrium, in which the sovereign's behavior is consistent with her reputation.

A reputational equilibrium is defined as follows:

Given the stochastic process for income, asymmetric information about type, a reputation for honesty, costs of failure to repay including a value for η below the cutoff, and a price for debt q^R, a reputational equilibrium is a set of policy functions for government debt holdings $D'(D,y)$ and consumption, $c^c(D,y), c^b(y)$, and a cutoff value for states determining repayment $\hat{\jmath}(D)$, such that the sovereign maximizes expected expected utility, her budget constraints clear, the price for debt q^R assures risk-neutral lenders an expected return equal to the exogenous risk-free rate of return, and the sovereign's actions are consistent with her honest reputation.

2.2 Model of Excusable Default Due to Fiscal Limits

Although a sovereign with a sufficiently high value of reputation would never default strategically, she could default due to inability to repay. We add this possibility to the model, by assuming that every country faces fiscal limits on her ability to repay debt. These limits arise from economic and political constraints on raising tax revenue and reducing government spending. Following Davig, Leeper and Walker (2010, 2011) and Bi (2012), we assume that fiscal limits on debt repayment are increasing in current output $(A(y))$. We also assume that fiscal limits exceed standard default costs, consistent with the assumption in the strategic default literature that default occurs due to benefits exceeding costs and not to inability to repay. We simplify by assuming that the value of state-dependent fiscal limits is known.

Finally, we introduce excusable default, whereby there is no explicit punishment to default as long as the sovereign repays what she is able (Grossman and Van Huyck 1988). We could include other consequences, not punishments since default is excusable, but the empirical evidence on the consequences and costs of actual defaults is controversial.[6] We simplify the theory by omitting them.[7]

In this section we add fiscal limits and excusable default to the model of reputation. These additions do not change the behavior of the sovereign with an optimizing reputation.

[6] The data are consistent with the assumption that default and recession occur together, but Yeyati and Panizza (2011) argue that the recession precedes and causes the default. Additionally, Benjamin and Wright (2008) provide evidence that many countries emerge from default with debt higher than that for which they entered, suggesting ability to borrow while in default. Arellano, Mateos-Planos, and Rios-Rull (2013) also allow a sovereign continue to borrow in default.

[7] We add a small deadweight cost of default in the calibration and discuss its implications there.

She continues to behave as in the strategic default model. Consider the incentives faced by the sovereign who has an honest reputation.

2.2.1 Optimization Problem for a Sovereign with an Honest Reputation

A sovereign can maintain her honest reputation in one of two ways. First, she can repay the current face value of debt (D). Second, if she is unable to repay the face value, she can repay as much as she is able, given by the state-dependent fiscal limit $(A(y))$. When the sovereign repays as much as she can, but less than the face value of debt, we define her action as excusable default. Since default is excusable, there are no restrictions on borrowing, even in a default period. The budget constraint for a sovereign, who faces a low enough value for η that she optimally chooses to maintain reputation, is given by

$$c^c = y + q(D', y)D' - \min\{A, D\}. \tag{11}$$

The set of value functions allowing excusable default is given equations (8), (9), (10), and (5) with equation (11) replacing equation (3).

2.2.2 Debt Price and Interest Rate

The price of debt is determined to assure that risk-neutral international creditors expect to receive the market rate of return on government bonds, given by r^*. A sovereign with an honest reputation is always expected to repay up to her ability, but is no longer expected to fully repay in all states. When unable to fully repay, repayment up to fiscal limits maintains reputation.

Redefine $\hat{\jmath}$ as the lowest income state in which full repayment occurs. Income states below $\hat{\jmath}$ are excusable default states, and income states above are full repayment states. The value for $\hat{\jmath}$ is increasing in D. For values of $D < A_1$, debt is safe and $\hat{\jmath} = 1$, its lower support. For higher values of debt, the cutoff state is implicitly defined by

$$D = A_{\hat{\jmath}} \text{ for } D \geq A_1, \tag{12}$$

since the repayments in the marginal state equal debt. As debt rises, the fiscal limit is equal to debt only if the income state rises, allowing the increase in $A_{\hat{\jmath}}$.

The value of debt sold today (qD') equals the expected present value of receipts tomorrow, yielding

$$qD' = \frac{D'\left[1 - F(\hat{\jmath}|h)\right] + \int_{j=1}^{\hat{\jmath}} A_j f(j|h)\, dj}{(1 + r^*)}, \tag{13}$$

where $f(j|h)$ is the density function for the distribution of income levels indexed j, conditional on beginning in income state indexed h, and

$$F(\hat{\jmath}|h) = \int_{j=1}^{\hat{\jmath}} f(j|h)\, dj$$

is the cumulative distribution, conditional on beginning in state h. There are two components to expected receipts tomorrow. The first is the face value of debt (D'), multiplied by the probability of full repayment $[1 - F(\hat{\jmath}|h)]$. The second is the value of repayments in each default state, $(A_j | j < \hat{\jmath})$, multiplied by their probabilities $(f(j|h))$.

2.2.3 Reputational Equilibrium with Excusable Default

Define the cutoff value for η as the value below which $V^c(y, D, H) > V^b(y, H)$ with c^c given by equation (11), for all values of output and admissible debt. With η below the cutoff, $V^g(y, D, H) = V^c(y, D, H)$. The sovereign never chooses strategic default, thereby choosing to maintain her honest reputation. A reputational equilibrium with excusable default due to fiscal limits is defined as:

Given the stochastic process for income, lack of commitment to repay, asymmetric information about type, a reputation for honesty, fiscal limits with the possibility of excusable default, costs of failure to repay when able including a value for η below the cutoff, and a price function for debt $q(D', y)$, a reputational equilibrium is defined as a set of policy functions for government debt holdings $D'(D, y)$ and consumption, $c^c(D, y), c^b(y)$, and a cutoff value for states determining full repayment $\hat{\jmath}(D)$, that the sovereign maximizes expected utility, her budget constraints clear, the price function for debt $q(D', y)$ assures risk-neutral lenders an expected return equal to the exogenous risk-free rate of return, and the sovereign's actions are consistent with her honest reputation.

Equilibrium Behavior of the Interest Rate Since the sovereign pays the minimum of debt and the fiscal limit in each state, payments in states with default $\left(\int_{j=1}^{\hat{\jmath}} A_j f(j|h)\, dj\right)$ are positive. Therefore, the price of debt is higher (equation 13), and the interest rate lower, compared with a model in which default entails zero payments, equation (7).

Using equation (13), the derivative of the price of debt with respect to its face value is given by

$$\frac{\partial q}{\partial D'} = -\left[\frac{D' - A_{\hat{\jmath}}}{D'}\right] \frac{f(\hat{\jmath}|h)}{1 + r^*} \frac{\partial \hat{\jmath}}{\partial D'} - \frac{\int_{j=1}^{\hat{\jmath}} A_j f(j|h)\, dj}{(1 + r^*)(D')^2}. \tag{14}$$

Recognizing that $A_{\hat{\jmath}} = D'$ from equation (12) and simplifying yields

$$\frac{\partial q}{\partial D'} = -\frac{\int_{j=1}^{\hat{\jmath}} A_j f(j|h)\, dj}{(1+r^*)(D')^2} \leq 0. \tag{15}$$

When the face value of debt is low enough that it is less than the fiscal limit in the worst state ($D' < A_1$), all debt is safe ($\hat{\jmath} = 1; F(1|h) = 0$), and $q = \frac{1}{1+r^*}$. Since the integral has unity as the upper and lower limit, the derivative is zero. However, once the face value of debt rises above A_1, $\hat{\jmath}$ rises, and the price of debt falls as debt rises.

When debt is risky, an increase in the face value of debt increases resources from borrowing (qD') by less than the price of debt. Taking the derivative of equation (13) with respect to D' yields

$$\frac{\partial (qD')}{\partial D'} = \frac{[1 - F(\hat{\jmath}|h)] + f(\hat{\jmath}|h)[A_{\hat{\jmath}} - D'] \frac{\partial \hat{\jmath}}{\partial D'}}{1+r^*}. \tag{16}$$

Noting that $A_{\hat{\jmath}} = D'$ from equation (12) and simplifying yields

$$\frac{\partial (qD')}{\partial D'} = \frac{[1 - F(\hat{\jmath}|h)]}{1+r^*} \geq 0. \tag{17}$$

When $D' < A_1$, all debt is safe and $F(\hat{\jmath}|h) = 0$. The effect of an increase in the face value of debt on the proceeds from borrowing is the inverse of the gross risk-free interest rate, equivalently the price of debt. However, once debt is large enough to be risky, implying that the probability of default is positive ($F(\hat{\jmath}|h) > 0$), an increase in D' requires a decrease in q such that the proceeds from borrowing rise by less than $\frac{1}{1+r^*}$.

Upper Bound on Debt The foregoing implies that there is an upper bound on borrowing (qD'). From equation (17), qD' is increasing in D' until D' reaches the the fiscal limit in the highest state possible next period, conditional on the current state. Define this state as $\tilde{\jmath}_h$. Using equation (13) with $\tilde{\jmath}$ replacing $\hat{\jmath}$, and $F\left(\tilde{\jmath}|h\right) = 1$, the upper bound on sovereign borrowing is determined by the expected present-value of repayments in default, conditional on income in the initial state, h.

$$qD' \leq (qD')_{ub} = \frac{\int_{j=1}^{\tilde{\jmath}} A_j f(j|h)\, dj}{(1+r^*)}, \tag{18}$$

where h is the initial state. When income is below the median, higher initial income implies a higher upper bound due to the autoregressive behavior of income.

The upper bound on qD' also implies an endogenous upper bound on D'. Once the face value of debt rises so much that $qD' = (qD')_{ub}$, the sovereign does not choose further increases in D'. Larger D' would be accompanied by a proportionate fall in q such the increase in future debt obligations would not be accompanied by an increase in borrowing proceeds and current consumption, a suboptimal move. Therefore, D' is subject to a state-dependent upper bound. We refer to this upper bound as a debt limit (which differs from the fiscal limit) and note that it is increasing in the state. This Laffer-curve-type behavior of the proceeds from borrowing is also present in strategic default models.

Default Risk Premium When the probability of default is positive, the domestic interest rate carries a default-risk premium, given by

$$r - r^* = \frac{1}{q} - (1 + r^*) = \frac{(1 + r^*) \int_{j=1}^{\hat{\jmath}} (D' - A_j) f(j|h)\, dj}{D' - \int_{j=1}^{\hat{\jmath}} (D' - A_j) f(j|h)\, dj}, \tag{19}$$

where the second equality uses equation (13). An increase in debt causes $\hat{\jmath}$ to rise, creating a an increase in the interest premium by increasing the measure of states with default.

Haircut The size of the "haircut" in default is endogenous and depends on the fiscal limit relative to outstanding debt. The sovereign with current ability to pay A and face value of debt D, optimally chooses payment equal to $\min\{A, D\}$. Therefore, the size of the "haircut" is given by

$$H = \frac{D - A}{D} \text{ for } D > A.$$

Role of Interest Rate in Counter-cyclical Debt Using equation (16), compare the effect of an increase in debt (D') on proceeds from borrowing (qD') in this model with that in the model of strategic default, in which either there are no debt payments in default ($A_j = 0$; Arellano 2008), or the payments are some fixed fraction of debt. In the strategic default model, default occurs only if the gains to default, based on the difference between what the sovereign owes and what she repays, exceed the value of the cost. Under strategic default, the value in equation (16) for $A_{\hat{\jmath}} - D'$, where we interpret $A_{\hat{\jmath}}$ as debt repayments, must be negative with D' sufficiently larger than $A_{\hat{\jmath}}$, for the net value to exceed the cost, justifying default. Therefore, the second term in equation (16) is large and negative, instead of zero, implying that an increase in borrowing creates a smaller increase in the proceeds from borrowing than in our model. This is because the price of debt falls relatively more as debt rises.

The corresponding large increase in the interest rate in the strategic default model is responsible for the result that the sovereign saves when there is a positive probability of default, even though consumption-smoothing would require borrowing. Since the interest rate rises less with the rich-country's repayment according to ability, the sovereign is more likely to choose to borrow to smooth consumption even in the neighborhood of default.

Spike in Debt Near a Crisis Once the probability of default becomes large enough, the sovereign chooses a large increase in debt, creating a spike in debt. Since the sovereign defaults in states $j < \hat{\jmath}$, and fully repays in others, we can rewrite the continuation value function for a sovereign with an honest reputation using equation (8) as

$$V^c(y, D, H) = \max_{D'} \left\{ u(c^c) + \beta \left[\int_{j=1}^{\hat{\jmath}} V^c(y', A(y'), H') f(j)\, dj + \int_{j=\hat{\jmath}}^{\bar{\jmath}} V^c(y', D', H') f(j)\, dj \right] \right\},$$

with c^c given by equation (11). The only distinction between full repayment states and excusable default states next period is initial debt, implying different arguments for the future value functions in full repayment versus default states, but not different future value functions.

The first order condition with respect to D' is given by[8]

$$\frac{\partial u(c)}{\partial c} \frac{\partial (qD')}{\partial D'} - \beta \int_{j=\hat{\jmath}}^{\bar{\jmath}} \left(\frac{\partial u(c')}{\partial c'} \right) f(j|h)\, dj = 0,$$

where c' should be understood as depending on j. Note that since the current choice of debt does not affect the value of repayments in a default state, that these terms are missing. Substituting from equation (17) yields

$$\frac{\partial u(c)}{\partial c} = \beta (1 + r^*) \left[\frac{\int_{j=\hat{\jmath}}^{\bar{\jmath}} \left(\frac{\partial u(c')}{\partial c'} \right) f(j|h)\, dj}{\int_{j=\hat{\jmath}}^{\bar{\jmath}} f(j|h)\, dj} \right] = \beta (1 + r^*) E \left\{ \left(\frac{\partial u(c')}{\partial c'} \right) | (j > \hat{\jmath}) \right\}. \tag{20}$$

The right hand side of equation (20) is the expected marginal utility of consumption next period, conditional on obtaining states in which full repayment occurs. Since payment in default states is not related to the amount borrowed, states below $\hat{\jmath}$ are not included in the integral for expected future marginal utility of consumption. At the optimum, the marginal utility of current consumption equals the expected marginal utility of future

[8] The term multiplying $\frac{\partial \hat{\jmath}}{\partial D'}$ vanishes since at $\hat{\jmath}$, $A(y') = D'$.

consumption, conditional on full repayment, multiplied by $\beta(1+r^*)$. Since consumption is higher in states in which full repayment occurs, the marginal utility of expected future consumption, conditional on full repayment, is lower than the unconditional marginal utility of expected future consumption. Therefore, when default is possible, the marginal utility of current consumption must be lower and current consumption higher.[9]

A positive probability of default next period ($\hat{\jmath} > 1$) decreases the right hand side of the Euler equation (20) because expected future marginal utility is included only for states with full repayment, and consumption is higher in those states than in default states. The lower expected marginal utility of future consumption requires that the marginal utility of current consumption also fall, thereby increasing current consumption. Therefore, consumption and the choice of debt next period are higher when the probability of default is positive.

The possibility of paying only what the sovereign is able and not actual debt cuts off the lower portion of the risk distribution encouraging the sovereign to increase consumption and debt, thereby taking on more risky behavior. This is Stiglitz's (1981) classic result that the availability of bankruptcy increases risk-taking behavior. The result is opposite that in the strategic default model, in which the sovereign saves in states for which the probability of default is positive.

It is important to emphasize that the spike in debt, when the probability of a crisis is high, is an analytical result and therefore does not rely on calibration. Similarly, the fall in debt in the strategic default model when crisis probability is high, is an analytical result.[10]

Probability of Default The probability of default is the probability of transiting from current income state h to an income state lower than $\hat{\jmath}$, where income state $\hat{\jmath}$ is the lowest income state in which full repayment occurs, given D'. Since income is autoregressive, the probability of transiting to a lower income state is higher the smaller is h, equivalently, the lower is current income. The probability of transiting to income state $\hat{\jmath}-1$ or lower, from a given state above, is also higher the larger is $\hat{\jmath}$. The value for $\hat{\jmath}$ is increasing in debt. Therefore, the probability of transiting from income state h to income state $\hat{\jmath}-1$ or lower is decreasing in income (the higher h, the lower the probability of transiting to a low enough value) and increasing in debt (which makes $\hat{\jmath}$ high). Therefore, the probability of

[9] With no repayments in default, $\frac{\partial(qD')}{\partial D'}$ is smaller, implying that its inverse is larger, raising $\frac{\partial u(c)}{\partial c}$, thereby reducing consumption through smaller D'. This is the effect of the large increase in the interest rate in the neighborhood of default, offsetting the effect of the Stiglitz risk-taking.

[10] See footnote 1.

default is increasing in debt and decreasing in income.

2.2.4 Alternative Debt Dynamics with Alternative Default Motive

It is important to understand that the reputational equilibrium, with limited ability to repay and excusable default when the sovereign repays up to her ability, is not simply a modification of the strategic default model to have larger default costs, i.e. a recalibration. The assumption of multiple contracts with spillovers is a plausible way to raise the cost of strategic default for a sovereign with an honest reputation. However, the model with asymmetric information and no fiscal limits has only two types of equilibria, one with strategic default if the value of reputation is low (η is high), and one with no default if the value of reputation is high (η is low). If we drop the assumption of asymmetric information and modify the strategic default model only with higher costs of default, then debt ratios can be higher, but debt dynamics in the neighborhood of default retain the characteristics of the strategic default model. Explicitly, debt is procyclical prior to the crisis and falls, instead of spiking upwards, just prior to the crisis.

Additionally, the model for rich-country default is not a modification of the strategic default model to allow fiscal constraints to alter incentives within the strategic default framework as in Arellano and Bai (2016), Bocola and Dovis (2016), and Bocola Bornstein and Dovis (2018).[11] In these papers, the default decision remains strategic, and the models cannot jointly account for the three features of debt dynamics in Greece prior to the crisis that we seek to explain.

Our model uses asymmetric information and a reputational equilibrium to rule out strategic default, providing the opening for an alternative motivation for default, that of inability to repay. The sovereign in a reputational equilibrium never chooses strategic default and chooses excusable default when unable to fully repay. This different incentive for default is responsible for different debt dynamics leading up to a crisis.

2.2.5 Application of Excusable Default to Rich Countries and to Greece

The value of global contracts differs across countries. Rich countries tend to have greater connectedness to other countries through more implicit and explicit contracts. International meetings on global issues occur between rich countries, as with the G-20, or

[11] Arellano and Bai (2016) introduce a fixed tax rate and utility of government spending to show that inability to raise the tax rate in recession can reduce government spending so much that the sovereign's incentive for default rises. Bocola and Dovis (2016) and Bocola, Bornstein and Dovis (2018) introduce a floor on government spending to provide an incentive for debt accumulation when output is low, reversing the procyclical debt in strategic default.

smaller groups. Many of these global links facilitate trade among countries, permitting higher output. Since rich countries are more likely to have global contracts than poor ones, they have more to lose from a loss of reputation. If poor or emerging-market countries do not have valuable global contracts, then they are less likely to be able to sustain a reputational equilibrium. Therefore, the canonical strategic default model explains default in these countries, those for which the model was designed. In contrast, global contracts have more value in rich countries, implying that to avoid a highly-costly reputational loss, rich countries eschew strategic default. This behavior switches the motivation for default to inability to repay due to fiscal limits. This is the case we choose to present as the explanation for the Greek crisis and as our model of default for a rich country.

We view Greece as a country with an honest reputation and a high value of global contracts. Although Greece had a history of default, prior to the default we seek to explain, she had not experienced one since 1974, when democracy replaced the military junta. Even though our model assumes that sovereign type is fixed, application to history requires some subtlety. We assume that after Greece made changes to enable her to join the European Economic Community in 1981, that she was perceived as an "honest" type, without modeling how that perception evolved. Greece further demonstrated the high value she placed on admission to the European Monetary Union in making additional policy changes necessary to join the Monetary Union in 2001, stabilizing debt/GDP and reducing inflation, and in her decision not to leave the Union in the wake of the crisis. The benefits of these contracts included unrestricted access to the entire European market and a lower nominal interest rate which nearly matched that of Germany.

As an illustration of the value of European Monetary Union contracts to Greece, Alexis Tsipras, the Greek prime minister, argued in a 2015 interview with *The Guardian*, when Greece was considering leaving the Union, that the costs of leaving the Union would be astronomical. He states:" A disorderly default would not only have led to a collapse of the banking system and a disappearance of all deposits, but it would force you to print a currency which would be drastically devalued because there is no reserve to support it. A pensioner who got 800 euros would get 800 drachmas and it would only last him three days and not a month." This statement illustrates that the sovereign felt that "a disorderly default", which we can interpret as a strategic default, would have had adverse consequences for all contracts with the EMU and the EU, thereby raising the cost of a strategic default beyond the standard cost in the canonical model.

We have no way to empirically measure the value of global contracts for Greece or for any other country. We assume that their value was high enough to sustain a reputational equilibrium and compare debt dynamics generated by this assumption with what they would have been if the value of contracts was lower such that default was strategic. The result that excusable default can match debt dynamics, while strategic default cannot, is our ultimate justification for the assumption of a large value of reputation.

3 Calibrated Model and Functional Forms

3.1 Functional Forms

3.1.1 Fiscal Limit on Debt Repayment

We model the state-dependant limits on ability-to-pay $(A(y))$ using the fiscal limits literature (Davig, Leeper and Walker (2010, 2011), and Bi (2012)), which assumes that every country has a limit on her ability to raise primary surpluses to repay debt. In this literature, the fiscal limit on debt is a wealth concept, given by the expected present value of future maximum primary surpluses. Maximum surpluses are determined by the tax rate at the top of the Laffer curve, together with exogenous, possibly state-dependent, paths for transfers and government spending. A wealth concept is a more appropriate representation of ability-to-pay than a liquidity concept for our model since the sovereign can borrow, even in a state with excusable default. Maximum primary surpluses, relative to income, are increasing in income because the tax rate at the peak of the Laffer curve is increasing in income and because it is more feasible politically to raise tax revenue when income is high.

Components of the primary surplus, which are not explicitly due to a Laffer curve, are fungible across time. A government could raise the same expected present-value primary surplus with a high primary surplus over a short period of time as with a small primary surplus over a long period. For example, under strong European pressure, Greece recently succeeded in meeting bailout targets for its primary surplus of 3.5 percent of GDP, even with very low output levels. However, Greek surpluses this large are not viewed as sustainable. In quantifying fiscal limits, we would not want to infer that Greece could continue to produce such large surpluses/GDP at low output or that it could generate even larger surpluses/GDP for higher output. Similarly, a country which sets a tax rate at the top of the Laffer curve under external political pressure is likely to abandon the severe austerity at some future date for political reasons. Austerity forever is not politically viable.[12] The maximum surplus that is politically sustainable over time differs from the

[12] Bi, Leeper, and Leith (2013) allow the actual fiscal limit on debt to be a fraction of expected future

maximum surplus a country could achieve in a particular period. Therefore, we do not determine the fiscal limit on debt by setting a maximum value for the primary surplus at each level of output and taking its expected value. Instead, to account for the fungibility of surpluses across time, we let the fiscal limit on debt be determined by the expected present value of income.

We assume that the sovereign's ability to pay, equivalently the fiscal limit, is proportional to the expected present value of current and future endowments. Letting A_h denote the the ability-to-pay (equivalently the fiscal limit) in endowment state h, it is given by

$$A_h = \phi_h E_0 \sum_{t=0}^{\infty} (y_t | y_0 = y_h) \left(\frac{1}{1+r^*} \right)^t . \tag{21}$$

The term, y_h, represents current endowment income, and ϕ_h represents the fraction of the expected present value of current and future endowments that the government could raise in primary surpluses in the current endowment state h. This functional form implies that if the current surplus is larger than $\phi_h y_{h,}$ then the largest expected future surpluses must be smaller than $\phi_h E_0 \sum_{t=1}^{\infty} (y_t | y_0 = y_h) \left(\frac{1}{1+r^*} \right)^t$ because future surpluses must be reduced to offset the current, unsustainably-high surplus.

Additionally, we assume that ϕ_h is increasing linearly in y_h, broadly consistent with the maximum tax rate rising with income, as in the Laffer curve approach. Explicitly, the sovereign can raise ϕ of current and expected future endowments in the worst state, with ϕ_h rising linearly in the expected present value of endowments, according to

$$\phi_h = \phi + \delta \left[E \sum_{t=0}^{\infty} (y_t | y_0 = y_h) \left(\frac{1}{1+r^*} \right)^t - E \sum_{t=0}^{\infty} (y_t | y_0 = y_1) \left(\frac{1}{1+r^*} \right)^t \right], \tag{22}$$

where y_1 is the lowest endowment state.

The results of the model do not require this particular functional form for fiscal limits. The results do require two essential elements: 1) the fiscal limit depends on the entire expected present value of expected future income such that it is wealth-based; 2) the fiscal limit/GDP is rising in GDP except for very large values of GDP.[13] Note that the fiscal limit is stochastic because the endowment is stochastic. The functional form in equation (22) introduces two parameters to calibrate, ϕ and δ.

surpluses with the maximum Laffer-curve tax rate to account for the fact that it might not be politically possible to get taxes as high as the Laffer curve.

[13] Once endowments are large, they are expected to fall toward the median, implying that fiscal limits as a fraction of GDP do not necessarily continue to rise.

3.1.2 Utility

We adopt the standard CRRA utility function with

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \qquad \text{for } \sigma > 0 \text{ and } \sigma \neq 1,$$

$$u(c) = \ln c \qquad \text{for } \sigma = 1,$$

yielding one parameter to calibrate, σ.

3.2 Calibration

We calibrate the model quarterly, to match the timing for the beginning of the Greek crisis and the behavior of Greek government debt leading up to the crisis.

We must decide how to measure debt in the model. If we implement the social planner model of the endowment economy by assuming that the agent consumes her income net of lump-sum taxes, then external debt and government debt are identical, while they clearly differ in the data. We choose to match government debt, not external debt, for several reasons.

First, the current account, which governs the accumulation of external debt, is the difference between savings and investment. However, the endowment economy has no investment. Therefore, we should not expect to match debt, determined by current account dynamics without investment.[14] Second, the concept of the fiscal limit is based on the economic and political inability to extract additional resources for debt payment in a closed economy, a concept based on government debt. Additionally, actual governments choose government debt, not external debt, and Greece defaulted on government debt. Bocola, Bornstein, and Dovis (2018) show that government debt is more closely correlated with interest premiums than external debt and show that under assumptions that the private financial market is highly efficient, the debt in the strategic default model can be interpreted as total government debt. An alternative way to justify calibration to government debt would be to replace consumption in utility with government consumption, as in Bocola and Dovis (2016), and reinterpret the budget constraints in terms of government income and spending.

[14] In particular, note that the covariance of the current account with income equals the covariance of savings with income minus the covariance of investment with income. If we generate a counter-cyclical current with counter-cyclical savings, then the cyclicality of the current account exceeds that of investment. This is unlikely to hold in any country's data since investment tends to be the component of GDP with the greatest cyclicality.

3.2.1 Data

Output is measured with quarterly OECD data from 1960Q1 to 2008Q2. For the external interest rate (r^*) we use the average annual value of the German ten year bond rate net of inflation over the period 2005-2008, yielding an annual rate of $r^* = 0.0212$. We use Eurostat data on quarterly values of debt relative to GDP, beginning in 2006Q1, and annual values for 2001-2005.[15] We convert these values to our measure of debt, which is detrended real debt relative to detrended real GDP in the median state, by multiplying the Eurostat data on debt/GDP by actual detrended real GDP relative to detrended mean real GDP, computed using OECD data on real GDP.[16] This measure does not change with changes in real GDP. We refer to our data measure simply as "debt," consistent with the definition of debt in the model.[17]

The calculations are at a quarterly frequency and therefore use debt relative to output measured at quarterly rates. We report debt ratios with output measured at annual rates, as is conventional.

3.2.2 Standard and Estimated Parameter Values

The coefficient of relative risk aversion (σ) takes on its standard value, yielding $\sigma = 2$. For output, we estimate the autoregressive parameter for real Greek GDP. We linearly detrend and demean the log of the quarterly data and obtain values of $\rho = 0.965$ and $\sigma_\epsilon = 0.028$, yielding an unconditional standard deviation of output of 0.0917.

3.2.3 Deadweight Cost of Default

A deadweight cost to default is a financial market friction, which reduces the value of payments creditors receive in default, without reducing what debtors pay. Our model has no deadweight loss in default, implying no financial market frictions. Its addition would imply a larger response of the interest rate to an increase in debt, an adjustment

[15] The measure of debt/GDP in the data measures GDP at an annual rate. The model value measures GDP at a quarterly rate. For purposes of calibrating and simulating the model, we make the two consistent by multiplying the data value of debt/GDP by four to obtain the ratio with GDP expressed at a quarterly rate. We report values of debt/GDP and debt/mean GDP by reconverting to the more commonly used measures, which value GDP at annual rates. Additionally, we adjust data debt forward by one quarter since data debt is end-of-period and model debt is beginning-of-period.

[16] This requires that we use quarterly interpolations of the annual debt data for 2005, the first year of our sample. To justify this calculation, assume that GDP has trends due to a nominal and a real component and that debt shares these trends. Therefore, we can view the ratio of debt/GDP as the ratio of real detrended debt to real detrended GDP. We obtain real detrended debt relative to real detrended GDP by multiplying debt/GDP by real detrended GDP relative to the mean of real detrended GDP.

[17] In the model, there is no trend and the mean value of output is unity.

which seems necessary in the calibration. Therefore, we add a small deadweight loss to default, allowing the increase in the interest rate to offset some of incentive to take on additional risk (and debt) in the neighborhood of default. We view this deadweight loss as the administrative cost of the default and not as an explicit punishment to default.

In the model, the sovereign already pays the maximum she is able in default, implying that we cannot add anything to these payments. Therefore, in default, the sovereign continues to pay her full ability, but the lender receives only a fraction ω of this payment. The deadweight loss reduces the price of debt and raises the interest rate, requiring revision of equation (13) to yield

$$q = \frac{D' \left[1 - F\left(\hat{\jmath}|h\right)\right] + \int_{j=1}^{\hat{\jmath}} \omega A_j f\left(j|h\right) dj}{\left(1 + r^*\right) D'}. \tag{23}$$

With the deadweight loss, the interest premium becomes

$$r - r^* = \frac{1}{q} - \left(1 + r^*\right) = \frac{\left(1 + r^*\right) \int_{j=1}^{\hat{\jmath}} \left(D' - \omega A_j\right) f\left(j|h\right) dj}{D' - \int_{j=1}^{\hat{\jmath}} \left(D' - \omega A_j\right) f\left(j|h\right) dj}.$$

This revision changes the derivative of the price of debt and current borrowing with respect to the face value of debt, equations (15) and (17), and the Euler equation (20), to yield

$$\frac{\partial q}{\partial D'} = -\frac{\int_{j=1}^{\hat{\jmath}} \omega A_j f\left(j|h\right) dj}{\left(1 + r^*\right) \left(D'\right)^2} - \left[1 - \omega\right] \frac{f\left(\hat{\jmath}|h\right)}{1 + r^*} \frac{\partial \hat{\jmath}}{\partial D'} < 0 \tag{24}$$

$$\frac{\partial \left(qD'\right)}{\partial D'} = \frac{\left[1 - F\left(\hat{\jmath}|h\right)\right] - f(\hat{\jmath}|h) A_{\hat{\jmath}} \left(1 - \omega\right) \frac{\partial \hat{\jmath}}{\partial D'}}{1 + r^*} \geq 0, \tag{25}$$

$$\frac{\partial u\left(c\right)}{\partial c} = \beta \left(1 + r^*\right) \left[\frac{\int_{j=\hat{\jmath}}^{\bar{\jmath}} \left(\frac{\partial u(c')}{\partial c'}\right) f\left(j|h\right) dj}{\left[1 - F\left(\hat{\jmath}|h\right)\right] - f(\hat{\jmath}|h) A_{\hat{\jmath}} \left(1 - \omega\right) \frac{\partial \hat{\jmath}}{\partial D'}}\right], \tag{26}$$

where we have used $A_{\hat{\jmath}} = D'$. With $\omega < 1$, repayments in default received by creditors, per unit of debt, are lower, implying a lower price of debt from equation (23). Since increases in next period's debt raise the value of $\hat{\jmath}$ $\left(\frac{\partial \hat{\jmath}}{\partial D'} > 0\right)$, reducing the number of states with full repayment, deadweight loss ($\omega < 1$) implies that the price of debt falls with the interest premium rising (equation 23). Additionally, the proceeds from additional borrowing (qD') do not rise as much due to the larger increase in the interest rate as debt rises (equations 15 and 25).

The deadweight cost of default adds an additional parameter, ω, for calibration.

3.2.4 Remaining Parameters

We calibrate the parameters determining the fiscal limit, ϕ and δ in equations (21) and (22), as well as values for ω, and β to match four features of the Greek debt data leading up to the crisis. To generate model values we solve for the debt policy functions, using value function iteration over a debt grid with 1000 equally spaced values from 0.0 to 2.0.[18] These values of debt are relative to mean detrended GDP, the measure of debt in the model.[19] We approximate the behavior of the income data using a discrete approximation with fifty-one output states based on Tauchen's (1986) method of approximating an autoregressive series with a Markov chain. We simulate the model to predict the path for debt generated by actual values for output.

Our model defines the crisis date as the first period in which Greece's fiscal limit is less than debt. In 2010Q1, Greece suffered a reduction in output, reducing the fiscal limit. Greece did not have scheduled debt payments in this period, implying that there were no observations on payments, either missed or made. However, Greece began austerity programs and the ECB softened rules on collateral for ECB loans, implying that Greece expected financing difficulties once maturity dates arrived. This evidence implies that the first period in which Greek debt exceeded the fiscal limit was 2010Q1, leading us to use this date as the first period of the crisis.

We must choose a start date for the simulated time series. Our data on debt begins in 2001, the same year in which Greece successfully gained admission to the EMU after a period of austerity. Greece was in recession, and our model implies that the sovereign would choose to raise debt. However, Greece kept debt relatively stable for several years, probably a choice influenced by EMU admission. Our model omits the EMU milestone, implying that our start date should be several years after admission, allowing possible transitory effects of entry to diminish. We choose the start date as 2004Q3, the peak of the boom following the recession in 2001, and find that results are robust to small

[18] Calculations use debt ratios with debt at quarterly rates, implying an upper bound of 8.

[19] The upper bound on the debt grid is larger than any sovereign ever chooses, but the lower bound is a constraint. Aiyagari, Marcet, Sargent, and Seppala (2002) show that precautionary savings in response to stochastic income creates a downward drift to the optimal behavior of government debt. They suggest imposing a lower bound to match behavior of actual sovereigns, who do not accumulate large assets. Battaglini and Coate (2008) and Barseghyan, Battaglini and Coate (2013), create political economy models with endogenous lower bounds on debt. Although our lower bound on debt is exogenous, it serves to prevent the sovereign from accumulating large amounts of assets in good times, consistent with empirical evidence, and with political economy models.

We also calibrated with a debt grid which had a lower bound of -10.00, something no country has ever reached. This country swings through periods with very large assets. The calibrated parameters are slightly different, but the fundamental characteristics of the model near a debt crisis are the same. Moments like mean debt are sensitive to the lower bound, and we therefore omit these moments.

modifications.[20]

Our calibration strategy requires that the sovereign choose debt over a period of twenty-three quarters such that: **1)** the path for debt is consistent with the **timing** of the crisis; **2)** the **value for debt on eve of the crisis** (2009Q4) matches the data; **3)** the **value for debt on the crisis date** (2010Q1) matches the data; and **4)** the **average value of debt,** over the period 2004Q3 - 2008Q3, matches the data, where the end date is the quarter before debt spikes sharply upwards. For the path of debt to be consistent with the timing of the crisis, the sovereign must choose values for debt which are below the realized fiscal limit for twenty-two quarters, and one which is above for the final, twenty-third quarter.

Our first step in matching model values with the data is to narrow the choices of the parameter values to those which exactly match the timing of the crisis. This places restrictions on fiscal limits and therefore on parameters determining the fiscal limits, ϕ and δ. We restrict these parameters such that the fiscal limit prior to the crisis, when the economy was in state 15 (A_{15}), exceeds actual debt, while the fiscal limit in the crisis period with output in state 14 (A_{14}) falls short of debt. This requires that ϕ and δ be calibrated such that

$$A_{15} > 1.105 \qquad A_{14} < 1.099,$$

where we have expressed debt relative to mean output at annual rates, as is conventional. Our procedure is to consider alternative values for ϕ, with δ calculated to minimize the sum of the difference between the implied ability to pay and data debt in these two periods.

We consider alternative choices for ϕ, δ, ω, and β until the model delivers the correct timing. Then, we continue to refine parameter choices until the model delivers values close to the data targets. Parameter values and the sources for their calibration are given in Table 1.

[20] The ability to match crisis timing, as well as values for debt just before and on the crisis, are robust to choices for the start date, as early as 2003Q1. However, for earlier start dates, the model chooses too much debt in the early part of the period, but makes correct choices as the crisis approaches. Start dates within one year on either side of 2004Q3 provide very similar debt choices to the date we use.

Table 1: Parameter Values

Parameter	Value	Source
σ	2	standard value
r^*	0.0053	German ten year bond yield (quarterly value)
σ_ϵ	0.028	regression estimate using real GDP data (1960Q1:2008Q2)
ρ	0.965	regression estimate using real GDP data (1960Q1:2008Q2)
ϕ	0.0153	crisis timing and three data targets
δ	0.0024	crisis timing and three data targets
β	0.9943	crisis timing and three data targets
ω	0.99805	crisis timing and three data targets

Consider how the four different parameter values affect the model values of our targets. All are important in matching crisis timing. The role for ϕ and δ in matching the fiscal limit around the crisis is discussed above. Additionally, they determine how much debt the sovereign wants to take on over the business cycle with the knowledge that ability to pay is higher with higher output. The value for β partially determines the sovereign's propensity to take on debt in alternative states and is important in determining the average value of debt. The values for ω and for δ are important in determining the sharp increase in borrowing leading up to the crisis. The closer ω is to unity, the smaller the deadweight loss and the steeper is the increase in debt leading up to the crisis due to greater Stiglitz-type risk-taking. The larger is δ the steeper the fiscal limit schedule is in output, and the greater the incentive to increase debt in bad states, in anticipation of higher future fiscal limits.

Our calibration does not require an impatient sovereign. The calibrated value of β is very close to the inverse of the gross risk-free interest rate (0.9948). The spike in debt just prior to the crisis is due to the Stiglitz-type risk-taking, created with the high value of ω. Impatience to obtain a strong upward trend in debt is unnecessary.[21]

Table 2 compares model values with those targeted in the data. Values for debt are measured relative to mean (detrended) GDP, as in the model. We report debt ratios with GDP expressed at annual rates to conform with standard presentations. The model fits the data well, matching the value for average debt relative to mean GDP and for crisis debt within 0.02 percentage points, and for pre-crisis debt within 2.8 percentage points.

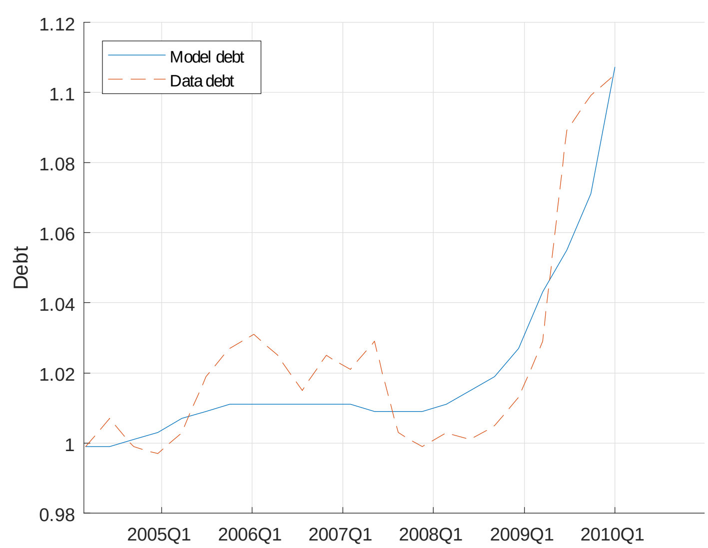
[21] Bocola and Dovis (2016) and Bocola, Bornstein and Dovis (2018) also calibrate a large beta with a modification which incentivizes the government to accumulate debt when output is low.

Table 2: Model Fit

	Timing	Average Debt	Pre-crisis Debt	Crisis Debt
Model	2010Q1	1.009	1.071	1.107
Data	2010Q1	1.011	1.099	1.105

Figure 1 plots the actual time path for debt, defined as debt relative to mean GDP[22], together with the model-generated time path, as a test of model fit. Model debt moves similarly to the data, rising during the initial five quarters of recession, becoming flat as income rises toward the mean, eventually falling, and then rising sharply with the declining output which accompanies the world-wide financial crisis. The sharp rise in debt, as the crisis nears, is produced by incentives to take on more risk, in anticipation of smaller payments in default states. This behavior reflects Stiglitz-type risk taking due to the large probability of default.

Figure 1: General Fit of Model



Model replicates data well.

Our calibration method relies on observing a debt crisis which creates default. The

[22] This deflates the value of debt for secular growth. Cyclical movements of output do not affect the denominator. If, alternatively, we used a measure of debt/GDP, then the measure would not distinguish between movements of debt and movements of GDP.

values for ϕ and δ, which determine fiscal limits, are set to deliver debt dynamics for Greece together with crisis timing. For a country which has not experienced default, we know that ϕ and δ are large enough to have prevented default at realized values for income and debt, establishing lower bounds for ϕ and δ, but we do not know how large they are. Therefore, a limitation of our calibration strategy is that we cannot set the parameters determining fiscal limits without observing a default.

4 Quantitative Results on Debt Dynamics and Crises

We use the sovereign's policy functions, together with simulations based on the policy functions, to: 1) describe the sovereign's incentives to take on additional debt, creating countercyclical debt and its spike as a crisis approaches; 2) explain the evolution of the Greek crisis in terms of the model; 3) measure the severity of default crises using endogenous default duration and haircuts, and 4) compute mean values for output, debt and the interest premium in the neighborhood of a crisis.

4.1 Countercyclical Debt and Spike in Debt

Given random shocks to the endowment, the sovereign uses debt to smooth consumption. The moderate degree of impatience implies little incentive for consumption tilting. Standard consumption-smoothing incentives apply, yielding countercyclical debt. As debt rises toward the fiscal limit, the probability of default rises. As the probability of default rises, the sovereign's incentives to take on additional debt rise since there is falling probability that she will repay the additional debt. Once the probability of default is large enough, these incentives create the spike in debt.

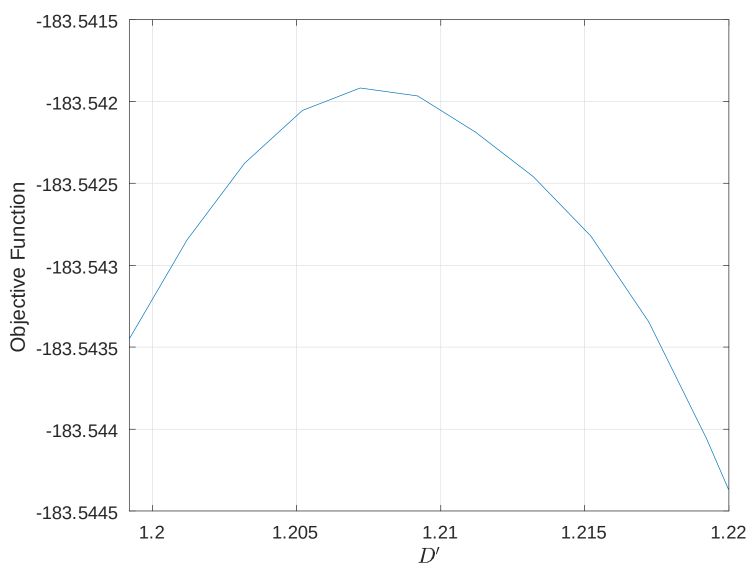
4.1.1 Debt is Low Relative to the Fiscal Limit

First, consider the case in which income is below the median and debt is initially low relative to the fiscal limit. For this case, the probability of default remains low for modest increases in debt. The sovereign's objective function, given by current utility plus the discounted continuation value, is concave in D'. The consumption-smoothing incentive is dominant, and the sovereign chooses a modest increase in debt.

Figure (2) illustrates this decision for a country in state 20, 0.7 standard deviations below the median with initial debt equal to 1.199, the starting value in Figure (2). This implies that the initial value of real detrended debt is 119.9 percent of mean real detrended

GDP. The fiscal limit in state 20 is higher at 1.295 (not on graph). The peak of the objective function occurs with debt a little higher than its current value, but still well below the fiscal limit in the current state. Debt is counter-cyclical, which is generally consistent with fiscal policy in advanced countries, and opposite to fiscal policy in many developing countries. (Frankel et al 2013).

Figure 2: Sovereign Chooses Modest Increase in Debt



Objective function is a smooth, concave function of D'.

4.1.2 Debt is High Relative to the Fiscal Limit

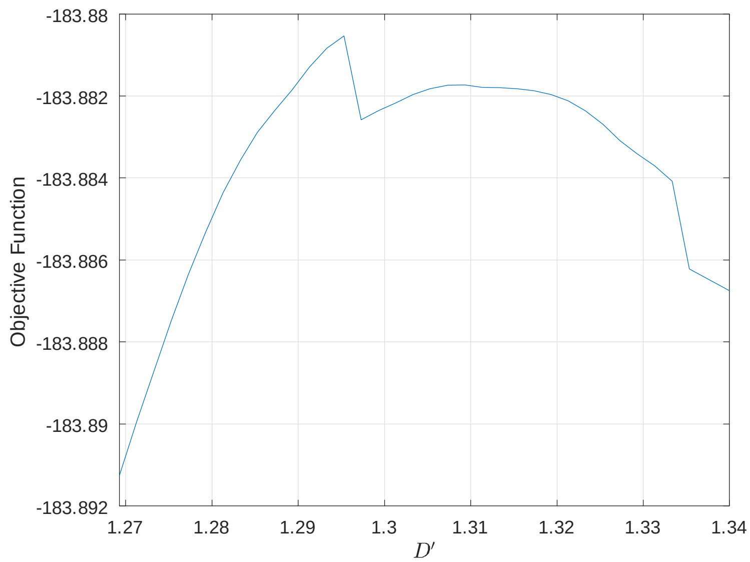
When debt is closer to the fiscal limit, either due to higher debt or a lower fiscal limit (based on lower current income), the probability of default plays more of a role in the choice of debt. A larger probability of default has two effects on the sovereign's incentives to accumulate debt. The first is an incentive to take on more risk (debt) since there are more states in which the smaller fiscal limit replaces contractual debt payments. The second is the opposite. As debt rises, the default risk premium on the interest rate rises, reducing the incentive to take on debt. These two opposing incentives are reflected in the sovereign's objective function no longer being a smooth concave function, but instead a function with sharp local peak(s).

Figure 3 illustrates the sovereign's choice of debt, holding income constant in state 20, but letting initial debt rise to 1.269, closer to the value of the fiscal limit of 1.295. The starting value on the horizontal axis is the current value of debt. The objective function has a sharp peak at the fiscal limit in the current state (1.295). For an increase in next period's debt beyond the peak, the objective function takes a discrete downward jump.

The jump is a consequence of two factors. First, the small increase in debt beyond the state-dependent fiscal limit triggers a downward jump in the price of debt (upward jump in the interest rate), due to increased probability of default and the deadweight cost (equation 24). The reduced price of debt mitigates the increase in current consumption created by additional borrowing. Second, since the small increase in debt leaves debt almost identical to the fiscal limit, default would provide little debt relief. These two factors create the downward jump in the objective function and incentivize the sovereign to keep debt at the fiscal-limit marker.

On the other hand, the sovereign can obtain substantial debt relief in default, if it raises debt towards the fiscal limit in the next state. This is the Stiglitz risk-taking incentive to take on more debt because additional debt increases consumption without increasing the debt burden in default states. For this case, the disincentive to borrow, due to the rise in the interest rate, together with little debt relief in default, keeps the sovereign from going over the first sharp peak. We refer to the value of debt at this sharp peak as "safe-in-state" since the sovereign would not default if output remained in the same state next period. Fiscal policy remains countercyclical, with debt rising with income below the mean, and there is no sharp increase in debt.

Figure 3: Sovereign Chooses D' at Safe-in-State Value



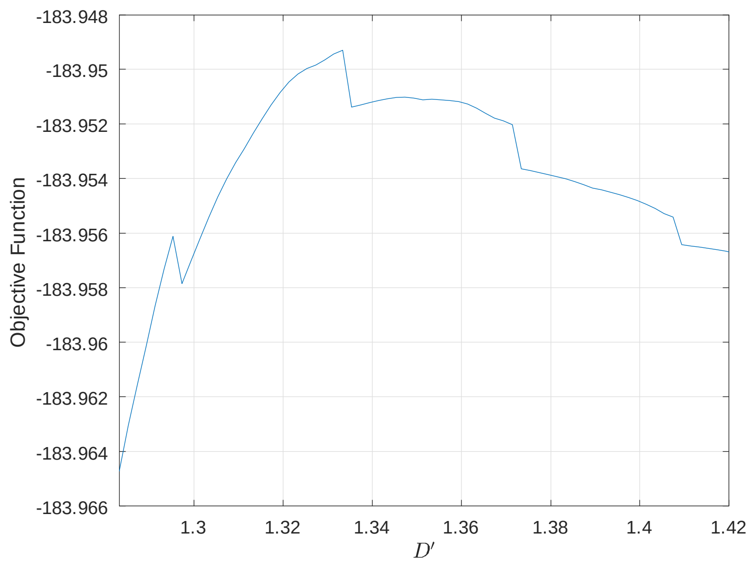
First sharp peak occurs with D' at safe-in-state value.

Figure 4 illustrates the same decision for a sovereign still in state 20, but with even higher debt. Initial debt is given by 1.283, the initial value on the horizontal axis, and below the fiscal limit in state 20 of 1.295, the value of debt at the first sharp peak. The objective function in this case has multiple sharp peaks.

In general, when initial debt is relatively close to the fiscal limit in the current state, the sovereign's objective function has multiple sharp local peaks, located at values of debt equal to the fiscal limit in nearby states. We label a sharp peak with debt equal to the fiscal limit in a lower state, "safe-in-lower-state" value, a peak with debt equal to the fiscal limit in the same state "safe-in-state" value, and a peak with debt equal to the fiscal limit in a higher state "safe-in-higher-state" value. For the lower type of peak, default does not occur even if output falls by one state, while the higher peak requires output to rise to avoid default. As the choice of debt increases from "safe-in-lower-state" through "safe-in-state," to "safe-in-higher-state," the sovereign is taking on more risk.

For this case, in which initial debt is 1.283, the first sharp local peak is at "safe-in-state" debt and the second at "safe-in-higher-state" debt, where the higher state is one higher at 21. The second peak is higher, implying that the sovereign now has the incentive to choose an increase in debt large enough for debt to be safe only if output rises by one state in the future. Stiglitz-type risk-taking dominates the risk-moderating effect of the increase in the interest rate. Debt is not only countercyclical, but spikes upwards because the probability of not fully repaying the debt in the future is so high.

Figure 4: Sovereign Chooses D' at Safe-in-Higher-State Value



Global peak is at safe-in-higher-state debt.

4.1.3 Summary

In all cases, when income is low and debt is below its endogenous limit, debt rises, consistent with counter-cyclical fiscal policy. Additionally, the sovereign has the incentive to take on more debt, the closer is initial debt to the fiscal limit in the current state, equivalently the higher the probability of default. This creates the spike in debt as crisis probability rises.[23]

4.2 Greek Crisis

Consider how the Greek crisis unfolded, using the calibrated model. Over the period 2004Q3 to 2008Q3, after which debt accelerated quickly, our measure of Greek debt, real

[23] This section illustrates changing incentives to take on debt as debt rises with income constant.Similar results obtain as income falls with debt constant. Both narrow the distance between the state-dependent fiscal limit and debt.

detrended debt relative to mean real detrended GDP, averaged slightly more than one hundred percent. The peak in the business cycle (2004Q3) was followed by five quarters of recession with output averaging 0.4 standard deviations below the mean and debt rising, and ended with six quarters of boom with output near trend (output averaged 0.2 standard deviations below trend) and debt falling. This behavior of debt is consistent with standard consumption-smoothing. The boom ended with a relatively mild fall in output in 2007Q3. However, output continued to fall, reaching a value 0.7 standard deviations below trend by 2008Q4. Greek debt increased over the period of recession, as it had in the previous recession, consistent with consumption-smoothing.

The difference is that, this time, after six quarters of moderate but worsening recession, output fell dramatically in 2009Q1, reaching 1.2 standard deviations below trend, instead of rising back toward trend, as it had, following the previous five-quarter recession. Output remained in recession throughout 2009, averaging 1.1 standard deviations below trend. Over this period, debt was increasing and output was falling, both movements pushing debt closer to fiscal limits. The increase in debt accelerated beginning in 2008Q4, as the accumulation of debt and the worsening recession pushed debt even closer to fiscal limits, triggering Stiglitz risk-taking.

In 2010Q1, output fell further, reaching 1.4 standard deviations below trend. Output this low, together with Greece's debt, accumulated over ten quarters of worsening recession, meant that the fiscal limit was lower than debt, dating the crisis.[24] Therefore, Greece experienced a crisis with output 1.4 standard deviations below trend and with real detrended debt equal to 110 percent of mean real detrended GDP. We do not model Greece after 2010Q1 because the data is no longer market-determined, as required by our model. Greece received its first official bailout in May 2010, providing additional debt that the private sector might not have provided and likely affecting the timing and magnitude of actual default.

The model can also explain the spike in the interest spread between Greek and German ten-year bonds in the period preceding the crisis.[25] In September 2009, the premium on ten year bonds for Greece relative to Germany was 130 basis points. It rose to 235 basis points by December 2009. Our model, calibrated to allow a substantial degree of Stiglitz risk-taking, produces a similar spike. The 2009Q3 interest premium is 42 basis

[24] Greece did not actually have scheduled debt repayments in this period, implying that there were no observations on repayments, either missed or made. However, Greece began austerity programs and the ECB softened rules on collateral for ECB loans, implying that Greece expected financing difficulties once maturity dates arrived.

[25] Our data on the Greek interest rate premium is the difference between the interest rate on ten year government bonds for Greece and Germany from the ECB Statistical Data Warehouse.

points, and it rises to 196 basis points in 2009Q4, a substantial increase. The magnitudes are not directly comparable due to the different bond maturities in the data and in the model. The interest premium on the one-quarter bond in the model reflects the probability of a crisis in one quarter, while the interest premium on the ten-year bond in the data reflects the probability of a crisis over ten years. In 2009Q3, the probability of a near-term crisis was low relative to the probability of a crisis over a ten-year horizon, consistent with the difference in interest premia in the model and the data. Additionally, the sharp rise in the probability of a crisis one quarter ahead by 2009Q4 could explain the substantial narrowing of interest premia between the one-quarter bond and the ten-year bond, with the remaining difference reflecting the fact that the ten-year probability of a crisis remained higher than the one-quarter probability.

The explanation in the model for the spike is as follows. The premium in 2009Q3, is due to the sovereign's choice of debt for the next period at a value which would be safe even if output fell by two states. This implies that a crisis requires that output fall by at least three states, an event with a 10 percent probability. The interest rate premium spikes in the model in 2009Q4 because, given the low output, the sovereign chooses a value for debt which would be safe only if output did not fall, raising the one-quarter probability of default to 34 percent.

This scenario makes the Greek crisis sound entirely like bad luck instead of bad policy, and the bad luck of the financial crisis did contribute. We compute the probability of crisis for Greece over the next ten years from our start date of 2004Q3, and obtain a probability of 14 percent. This high probability of a crisis is due to the value of debt relative to fiscal limits, enabling a period of strong recession to send the country into crisis. Either higher fiscal limits or parameters delivering lower debt would have been required to reduce Greek risk in the period preceding the financial crisis.

A model of strategic default does not fit the Greek crisis. Debt ratios were much higher than the value of standard default costs. Debt was countercyclical over this period, not pro-cyclical as in strategic default models. Greece was accumulating debt at a rapid rate prior to the crisis, when output was low and the probability of a crisis one quarter ahead was substantial, peaking at 34 percent in 2009Q4. The model of strategic default would have Greece reducing debt since the probability of a crisis one period ahead was positive. The fall in debt as a crisis approaches in the strategic default model is a theoretical result, not a calibrated result,[26] implying that we cannot calibrate the canonical strategic default model to deliver a spike in debt as the crisis approaches.

[26] See Footnote 1.

Attributing the Greek default to an inability-to-pay instead of to a strategic decision carries a policy implication for creditors. With strategic default, Greece would be refusing to repay when she is able. Threats of additional costs would be warranted to force Greece to repay something closer to her ability. However, if the Greek default is due to inability-to-pay, additional threats and/or additional costs can extract no additional repayments.

4.3 Crisis Severity: Duration and Haircuts

Crisis severity depends on values for output and debt following a default. First, consider incentives to take on additional debt after default, determining debt in default.

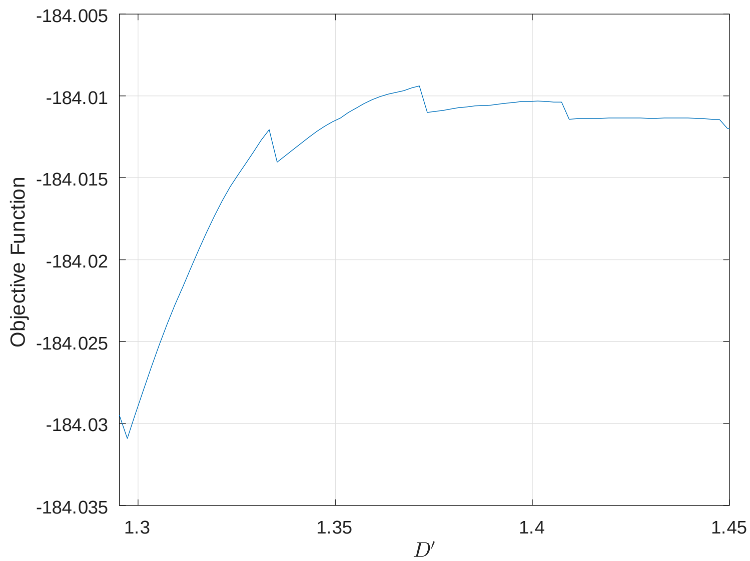
4.3.1 Borrowing after Default

When debt exceeds the fiscal limit, the sovereign defaults, resetting debt equal to the fiscal limit. Afterwards, the international lender is willing to lend again as long as she receives an interest rate high enough to yield an expected return equal the risk-free rate.

Figure 5 illustrates the sovereign's decision on debt after default when the economy is in state 20. Prior to default, debt exceeded the fiscal limit. With default, debt is reset at the fiscal limit given by 1.295, the initial value on the horizontal axis. The decision on next period's debt contains lots of sharp local peaks. The peak with debt at the fiscal limit two states ahead in state 22 (debt at 1.371) is the optimum.

The choice of debt following default is given by the endogenous debt limit, which is the lowest value of debt (D') which maximizes the proceeds from borrowing (qD'). Any larger increase in debt would cause a proportionate fall in its price such the proceeds from borrowing would not change. Therefore, after default, the sovereign chooses maximum risk. In this case, she borrows to yield default next period unless output rises by two states, an unlikely event. The sovereign has the incentive to increase debt by this amount, knowing that if future output is low enough for default, the increase in debt has no cost. Creditors are willing to make this loan since they receive repayment equal to the fiscal limit in default, yielding an expected return equal to the risk-free rate. When output is below the median, the sovereign is able to borrow more than the fiscal limit because output is expected to rise in the future, raising expected future ability-to-pay above the current fiscal limit.

Figure 5: Debt Choice after Default



Global peak is at debt which is safe if output is two states higher.

In general, following default, the sovereign raises debt to its endogenous state-dependent debt limit. Defining χ as the number of states for which output must rise for the debt limit to yield full repayment, χ is falling as the output state rises. Therefore, very low values of output yield debt limits which require output to rise by a substantial number of states to generate full repayment, compared with higher values of output. This implies that the probability of a one-period-ahead default, following a default, and hence of a default persisting for an additional period, is higher the lower is the output state.

4.3.2 Modeling Haircuts and Duration of Defaults

In the model, defaults last a single period because the sovereign settles her debt by either repaying what she is able or repaying the contractual obligation. However, in many cases, the initial default is immediately followed by a succession of future defaults. Therefore, to bring the model to the data, we view the period in which the country is engaged in successive defaults as a period of renegotiation. Exit from this default period requires that the sovereign meet two criteria. First, the sovereign must make a full repayment. And second, she must not default again for the next four quarters. If she defaults again within the one-year time-frame, we aggregate the two crises into one with the exit date of the second. This timing concept aggregates successive single-period defaults in the model.

During an aggregated default episode, behavior continues as in the model. That is, following default, whereby the sovereign repays her current fiscal limit in lieu of contractual debt, the sovereign borrows maximum debt consistent with the state. In the subsequent period, if the fiscal limit is less than contractual debt, she defaults again. We view each period's repayment and reborrowing during the period of default as debt renegotiation. Crisis duration is the time between entering the crisis with default and exiting the crisis with the first full repayment of contractual debt, which is not followed by another default within one year.

There are two ways for a default event to persist even after debt is settled with the sovereign repaying what she is able. First, if she is in a state with a debt limit determined by current the fiscal limit, a default event persists only if output falls. This is because the sovereign reborrows to her safe-in-state debt limit, implying that she will not default again as long as output does not fall. And since, with output below the mean, the probability of output falling is lower than the probability that it remains constant or rises, these defaults tend to resolve quickly.

Alternatively, when the sovereign is in a state for which the debt limit is safe-in-higher-state, the sovereign reborrows to the safe-in-higher-state debt limit, implying that she cannot fully repay and exit default unless output rises. Output rising is less probable than output remaining constant or falling, implying that these defaults tend to be prolonged.

Given that a period of default includes repayments as well as additional lending, it is not obvious how to measure the haircut. We use the concept of excess return, measured by actual repayments relative to contractual repayments minus 1. In the absence of default, actual repayments equal contractual repayments implying that excess return is zero. With default, actual repayments are less than contractual repayments implying that excess return is negative. The haircut is the negative of the excess return.

For example, if the default lasts only one period, then actual repayments relative to contractual repayments imply a gross return of $A_{h(1)}/D(1) < 1$ for the single period. The excess return is $A_{h(1)}/D(1) - 1 < 0$. For a two-period default, we measure the excess two-period return as the product of the first period and second period gross returns minus 1. In general, for an asset whose default period is given by n, the haircut is one minus the gross n-period return, yielding

$$HC = 1 - \frac{\left[A_{h(1)}\right] \times \left[\min\left(A_{h(2)}, D(2)\right)\right] \times .... \times \left[\min\left(A_{h(n)}, D(n)\right)\right]}{D(1) \times D(2) \times ... \times D(n)},$$

where integers in parentheses represent the period of default, and where the default ends once $\min\left(A_{h(n+1)}, D(n+1)\right) = D(n+1)$ and is not followed by another default for four consecutive quarters. Note that net repayments are given by the minimum of the state-specific fiscal limit, give by the ability-to-pay $\left(A_{h(i)}\right)$, and contractual debt obligations $D(i)$.

When the sovereign defaults to safe-in-state debt and does not borrow additionally, default duration is short, implying that haircuts are small. However, when the sovereign defaults to safe-in-state debt and reborrows to the higher safe-in-higher-state limit, default duration tends to be longer, and the magnitude of the haircut grows over time. Therefore, the longer a default lasts, the larger the haircut tends to be, consistent with evidence in Benjamin and Wright (2008).

In summary, haircuts and default duration both depend on the type of debt limit the sovereign chooses in default. Safe-in-higher-state debt limits imply greater crisis severity, as measured by duration and the size of haircuts. As the value for output falls, the sovereign chooses increasingly risky debt limits, those which are safe in increasingly higher states. Therefore, a crisis is more severe, the lower is income when the crisis occurs. And if income falls during the crisis, its severity increases.

4.3.3 Simulations for Haircuts and Default Duration

To describe average haircuts and default duration, we create a time series of 5,010,000 values for output based on our calibrated model. We use a random number generator to create values between zero and unity, and then use the transition matrix, generated from the Tauchen approximation for output with parameters ρ and σ_ϵ, to place each value into one of the fifty-one output states. Beginning with an initial value of debt, we use the calibrated solution of the model to solve for optimal decisions on default and repayment and on next period's debt, conditional on output and on the preceding value of debt. We drop the first 10,000 observations. We collect instances of default, together with experience prior to and after the default. When we have a period between defaults of one year or less, we aggregate the subsequent crises into a single longer one. After aggregation, we have a total of 2,895 separate default-crisis events.

The above discussion suggests that the duration of defaults and value of haircuts should be highly skewed and variable, and simulations confirm this. About five percent of defaults resolve in a single period with a modest haircut. However, there are defaults which last a long time. The mean and standard deviation for default duration and haircuts are given in Table 3. The result that the standard deviations are large relative to the means reflects the fact that many crises are settled quickly with small haircuts, while some take a long time to settle and involve large haircuts. The high means and volatility for duration and magnitude of haircuts are similar to those in the data on historical defaults. However, these moments are not directly comparable because most countries with default histories are developing countries which are likely to lack a commitment against strategic default.[27]

Table 3: Default

	duration (years)	haircut
mean	6.73	0.51
standard deviation	6.80	0.26

4.4 Crisis Moments

We generate crisis moments using the simulated series described in the preceding section. With the modest amount of impatience in our calibration, debt tends to accumulate when output is below trend, and tends to fall with output above trend (unless debt is already very high). Therefore, debt is high only following a period of low output. A default occurs when a sovereign has borrowed so much that the realized fiscal limit is below debt. This cannot happen with low debt and is more likely when output is low. In our calibration, only 0.87 percent of crises occur with output at or above the median. Tomz and Wright (2007) find that most (but not all) historical defaults have occurred with low output.

Using our simulated crises we find that the mean value of output in the period prior to a crisis is 1.09 standard deviations below trend, and the mean value of debt relative to trend GDP is 123 percent. The interest premium in the period prior to a crisis averages 311 basis points. The Greek crisis occurred with output 1.38 standard deviations below the mean, and with debt relative to trend GDP of 110 percent. Therefore, for Greece output was even lower than the mean but debt was not as high. The Greek interest premium was 235 basis points, representing heightened risk.[28]

[27] Sturzenegger and Zettelmeyer (2008), Benjamin and Wright (2008), and Cruces and Trebesch (2011) find high variability in the size of haircuts for sample of defaults with primarily developing countries. Together, these papers claim that most haircuts are within the range of 25 - 40 percent. Benjamin and Wright (2008) place the mean duration at 8 years.

[28] Assets are different, with the model asset being a one-quarter bond and the data asset a ten-year bond, implying that the premia are not directly comparable. Both data and model measures imply some default risk.

4.5 Other High-Debt European Countries

All rich European countries experienced rising debt/GDP as the Great Recession reduced output, implying counter-cyclical fiscal policy. Italian debt/GDP eventually surpassed the value which caused the Greek crisis, yet Italy has not experienced default. Although we cannot calibrate the parameters determining fiscal limits for a country which has not experienced default, we can comment on orders of magnitude. The rich-country default model implies that these countries have been able to accumulate large debt/GDP both because they have committed to repay up to fiscal limits and because their fiscal limits exceed the values of debt/GDP they have experienced. Since Italian debt/GDP has surpassed the value of Greek debt/GDP associated with the crisis, the values of parameters governing fiscal limits for Italy must be larger than those governing fiscal limits for Greece, but we do not know how much larger without a crisis. A comparison of Italy's experience with debt is consistent with this hypothesis. Italy was able to bring debt down from a peak of over 120% of GDP in 1995, whereas Greece has had no history of reducing debt using primary surpluses. Even so, Italian debt did reach another peak of 136% of GDP in 2015Q2, implying that we might learn soon how high its fiscal limits are.

5 Conclusion

The strategic default model was designed to explain default in emerging markets and does not explain large accumulations of debt without default in most European countries. Additionally, it cannot explain three key features of Greek debt leading up to the recent crisis: 1) much larger values for debt/GDP than implied by standard default costs; 2) countercyclical debt in the period leading up to the crisis; and 3) the spike in debt just prior to the crisis. We are aware of no modifications to the strategic default model which can explain either the spike in debt just prior to a crisis or the other two characteristics jointly. We develop a stochastic general equilibrium model of default for a rich country, which is consistent with all three characteristics.

In our rich-country default model, the sovereign cares about global contracts, which require trust to have value. Repudiation of one contract would destroy the trust on which all contracts are based, rendering them valueless. A large enough value of maintaining these global contracts yields a reputational equilibrium in which the sovereign always chooses to repay debt up to her ability. Failure to fully repay, when circumstances render full repayment impossible, is excusable and does not trigger loss of reputation, as long as the sovereign repays what she is able. Default occurs only when shocks render the sovereign unable to fully repay and is excusable. In default, the sovereign repays what she is able, yielding endogenous values for the haircut and the duration of default.

These modifications change the motive for default and alter the equilibrium debt dynamics to match those preceding the Greek crisis. The result, that the possibility of default does not reverse consumption-smoothing incentives, is fundamental to matching the alternative dynamics. The ability of the strategic default model to reverse consumption-smoothing incentives, consistent with the data in emerging markets, is a major data success for strategic default (Arellano 2008). However, stylized facts for rich countries imply consumption-smoothing, supporting the need for an alternative model for rich countries.

We calibrate the model to the Greek crisis, matching crisis timing, average debt over the preceding business cycle, and debt on pre-crisis and crisis dates. Large debt/GDP is supported by the commitment to repay up to ability. The sharp spike in debt is due to Stiglitz risk-taking. Our calibrations imply that the probability of a Greek crisis over the ten years beginning in 2004Q3 was about 14 percent. In 2004Q3, debt was already high relative to the fiscal limit. The crisis was due to additional debt accumulation in response to multiple reductions in Greek GDP, following the world-wide financial crisis.

The model offers an answer to why other rich European countries experienced sharp increases in debt/GDP as output fell, but did not suffer financial crises. The debt accumulation followed standard consumption-smoothing incentives. For these countries to have avoided default, debt must have been lower than fiscal limits in all periods, including the years of the Great Recession. This implies that rich countries with high values for debt/GDP also have high values of fiscal limits relative to GDP, implied by high ability-to-pay.

Finally, we use the calibrated model to describe general characteristics of excusable default crises. We find that most crises occur when output is low and debt is high. As a crisis approaches, debt is rising, unless it has reached an endogenous debt limit. However, crises and recoveries do not all look alike. The simple one-shock model is able to deliver a large variety of crises. Some have long durations and large haircuts while others do not. The model implies that the sovereign tends to take on more risks, creating crises with longer duration and larger haircuts, the lower is output when the crisis strikes.

6 Data Appendix

Eurostat (gov_10q_ggdebt) provides quarterly data on government debt as a percentage of GDP annually from 2000Q4 and quarterly from 2006Q1. We use linear interpolation to estimate quarterly values for Greece for the earlier years. The OECD (VPVOBARSA: US dollars, volume estimates, fixed PPPs, OECD reference year, annual levels, seasonally adjusted) provides quarterly values for real GDP from 1960Q1 to 2016Q2. We use data up to 2008Q2 to estimate behavior of GDP prior to the financial crisis. We obtain demean and detrend the data by regressing logged values on a constant and a linear time trend. We estimate the autocorrelation coefficient and standard error by regressing the residuals on one lag. We obtain standardized measures of output by taking the exponential of our demeaned and detrended data

The debt variable in our model is debt relative to mean output. In our demeaned and detrended data, mean output is unity. In the data, debt is government debt relative to actual GDP, not relative to mean GDP. We convert data to our model variable by multiplying the data variable by real actual GDP relative to mean real GDP, using the OECD data. Additionally, debt data is an end-of-period measure. In the model, debt is measured at the beginning of the period, requiring that we adjust the data dates by one period.

Data for interest rate premiums is the difference between interest rates on Greek and German ten-year government bonds net of CPI inflation. We obtained data from FRED, and original sources are the World Bank for long-term government bond yields (IRLTLT01DEA156N) and Main Economic Indicators from the OECD for inflaton (FPCPITOTLZGDEU).

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