The Greek Debt Crisis: Excusable vs. Strategic Default*

Betty C. Daniel† Jinwook Nam‡

November 1, 2021

Abstract

The Greek debt crisis exposed the need for a sovereign default model for advanced countries. We cannot use the canonical strategic default model to understand the Greek crisis or to predict crises in other advanced economies because the model misses key features of the data. The model misses the behavior of debt leading up to the crisis date and important business cycle characteristics of advanced countries. Additionally, it fails to generate a crisis. We propose an alternative model which replaces the assumption that a sovereign cannot commit to repay, with the assumption that she can commit to repay up to her known ability to repay. When debt exceeds ability to repay and the sovereign repays what she can, the ensuing default is excusable and incurs no punishments. We calibrate ability to pay using the Greek crisis. Our alternative model of excusable default allows values of debt/GDP as large as those in advanced countries, a spike in debt prior to the crisis, counter-cyclical government debt, and consumption volatility less than income volatility. These are characteristics of Greek debt prior to the crisis and of business cycles in advanced countries.

Keywords: Strategic Default, Excusable Default, Fiscal Limits, Ability to Pay, Debt Cyclicality, Consumption Smoothing

JEL Codes: F4

*The authors would like to thank the editor, Martin Uribe, and Diego Perez, Stephie Fried, referees, and seminar participants at the Gabelli School of Business at Fordham University and the University at Albany for helpful discussions and comments on this paper.

†University at Albany, bdaniel@albany.edu

‡University at Albany, jnam@albany.edu

1 Introduction

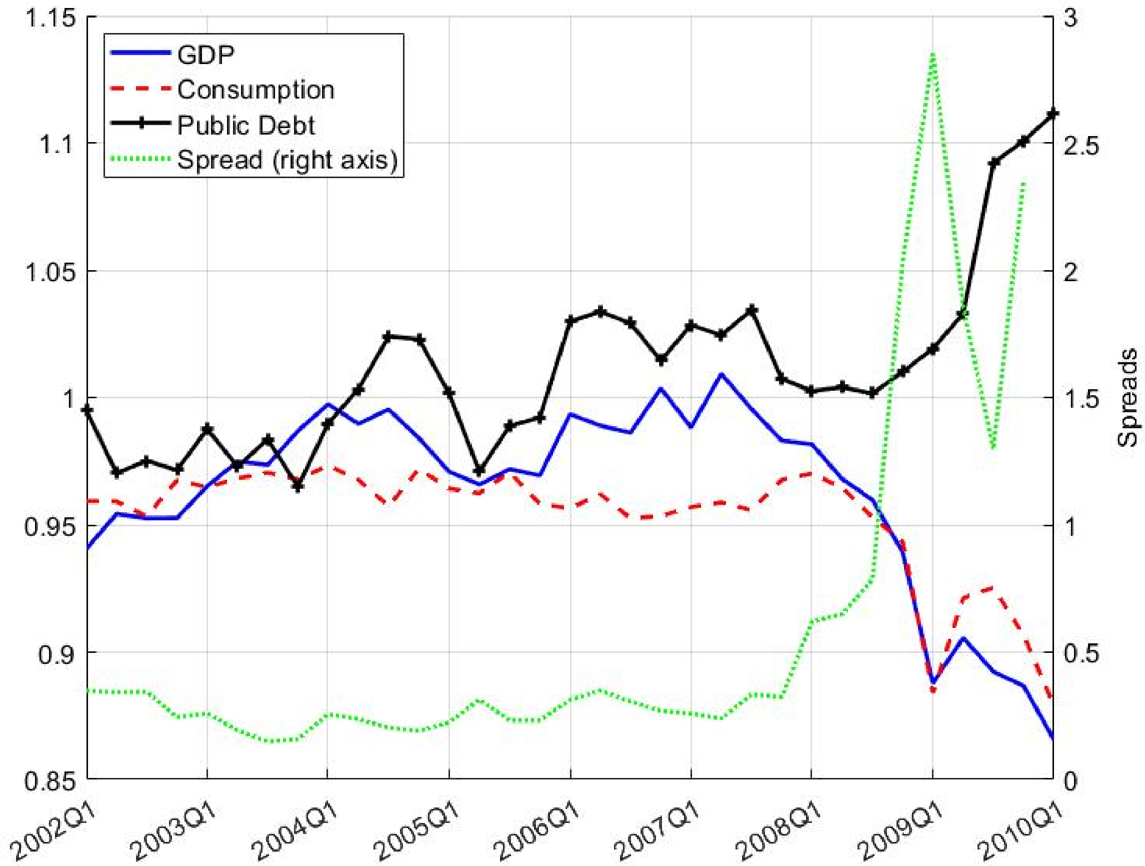
The sovereign debt crisis in Greece has raised the need for an economic model of sovereign default for advanced countries. The strategic default model was developed to explain business cycle moments and debt crises for emerging economies – historically, the crisis countries. The Greek default has demonstrated that advanced countries can also experience crises. Additionally, rising debt/GDP in many countries, in the wake of the global pandemic, raises the question of which advanced countries could succumb next to a debt crisis. Can we take the canonical strategic default model, as presented in Arellano (2008), calibrate it for an advanced economy like Greece, and use it to explain the Greek experience? And by extension, can we use it to assess the probability of future crises in advanced countries which are currently accumulating large quantities of debt? We show that the strategic default model has trouble matching key features of the data leading up to the Greek crisis and offer an alternative model.

Figure 1 illustrates the behavior of GDP, consumption, public debt, and the interest rate spread in the period leading up to the Greek crisis. We cut the data after 2010Q1 due to official intervention beginning in 2010Q2. We want a model consistent with this behavior. Output is below the mean for almost the entire period and takes a sharp downward plunge beginning with the world-wide financial crisis. The behavior of consumption roughly follows that of output, but is less volatile. Debt is high at about 105 percent of GDP and appears counter-cyclical,[1] rising early with output below the median and then spiking once output falls dramatically. Interest rate spreads remain low until they spike, first with a sharp fall in output in 2009Q1 and second, just prior to the first bailout in 2010Q2.

We view 2010Q1 as the crisis date. Although Greece did not have scheduled debt repayments due in 2010Q1, Greece sought external help and began austerity programs, indicating financial stress. Greece received its first bailout in 2010Q2, and this bailout was likely re-

[1] Kaminsky et al. (2004) and Frankel et al. (2013) provide empirical evidence that government spending in developing (advanced) countries is pro-cyclical (counter-cyclical).

Figure 1: Greece's Debt Crisis



Notes: GDP and consumption are detrended and demeaned Greek data from 1960Q1 to 2010Q1. The measure of output (consumption) is the exponentiated difference between log real GDP (consumption) and a linear time trend. Public debt is detrended using the method explained in Section 3.3. The interest rate spread is the difference between interest rates on Greek and German ten-year government bonds.

sponsible for postponing the actual default. Therefore, we argue that the first period in which Greece faced financing difficulties, which could have created default if Greece had had debt due and not received official assistance, was 2010Q1.

We recalibrate the strategic default model to the recent Greek experience in an attempt to use the model to explain the Greek crisis. In the canonical strategic default model, the optimizing sovereign is not committed to repay debt and chooses default whenever the cost of default, primarily loss of output, exceeds the benefits of default, elimination of all debt. Simulations from the calibrated model do not match economic outcomes in Greece leading up to the crisis date. The model does get high values of debt relative to GDP by using high default costs. However, the model cannot get consumption less volatile than income, countercyclical government debt, or a rising path of debt leading up to the crisis date. The model does not generate a crisis.

The model misses due to the incentives embedded in the model for taking on debt and defaulting. In the period following Greece's admission to the EMU, Greek debt is high enough for there to be a small probability of default. This is due to the possibility that output could fall, reducing the cost of default below the value of debt. For debt within this range, a fall in output raises the probability of default, increasing the interest spread. The higher interest rate reduces borrowing, creating procyclical debt and raising the volatility of consumption relative to income, contrary to data. Output peaks in 2007Q4, and thereafter moves persistently downwards leading to the crisis. The fall in output leads to rising spreads and falling debt in the model. A particularly sharp fall in output in 2009Q1 leads to such a sharp increase in the model spread and fall in model debt that the model avoids a crisis.

We propose an alternative model to match the behavior of the data. A fundamental assumption in strategic default is that the sovereign cannot commit to repay. We make an alternative assumption, designed to bring our theoretical model on sovereign default closer to policy literature on sustainable debt and fiscal limits. We assume that there are limits on a country's ability to repay debt and that these limits are increasing in output and are known.[2] A sovereign cannot commit to repay debt above her fiscal limits, but she does commit to repay up to fiscal limits. Specifically, she is committed to fully honor debt unless adverse events create a situation in which full repayment is not politically or economically feasible. Debt becomes "unsustainable" when it is so high that the country cannot be expected to repay, conditional on current and expected future output.

When debt exceeds the sovereign's ability to repay, we assume that default is "excusable" in the sense of Grossman and Van Huyck (1988), and incurs no punishments, as long as the

[2] In fact, these limits are not known and a lot of policy work is devoted to determining their values. We ignore this uncertainty here, leaving that for future work.

sovereign repays what she can. Therefore, consistent with real world behavior, a sovereign in default does not erase her debt. Instead, she repays debt up to her ability, in line with IMF guidance that a defaulting sovereign negotiate debts "in good faith". Therefore, the sovereign commits to abstain from strategic default, which we define as default when she has the ability to repay, but not from default. Excusable default serves as an implicit international risk-sharing agreement, limiting the downside risk of debt for the sovereign in return for an interest spread which accounts for the risk of default.

As in the canonical strategic default model, the sovereign chooses debt to maximize the expected utility of consumption subject to endowment shocks. Her decisions, including the default decision, are optimal, subject to the constraint that she repay debt up to her ability. The sovereign's commitments to shun strategic default and to repay debt up to her ability, imply that the incentives for the sovereign to accumulate debt and to default are fundamentally different from those in strategic default. These different incentives alter debt dynamics leading up to default.

First, the excusable default model produces large values for debt/GDP with calibration of large values for ability to pay. Second, in contrast with strategic default, the possibility of excusable default does not reverse consumption-smoothing incentives.[3] This is because repayment in default up to fiscal limits mitigates the interest rate response to an increase in the probability of default. Third, the small responsiveness of the interest rate to the probability of excusable default implies that, when debt is near the fiscal limit, the sovereign has the incentive to take on even more debt because she will not repay the additional debt in default. This behavior creates a spike in debt.[4]

Related Literature This paper brings together three literatures on sovereign risk and default, those on strategic default, excusable default, and sustainable debt. Eaton and

[3]Arellano (2008) demonstrated that the possibility of strategic default reverses consumption-smoothing incentives, allowing the relatively higher volatility of consumption compared to output, created by procyclical debt.

[4]Stiglitz and Weiss (1981) first demonstrated that the possibility of default cuts off the lower portion of the risk distribution, thereby incentivizing risk-taking.

Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008) are the seminal papers on strategic default. Aguiar and Amador (2014) and Aguiar et al. (2016) provide extensions and survey extensions offered by others. This literature is focused on the implications of strategic default for business cycle moments, particularly in emerging and poor countries. It has grown very large, but with few exceptions, the model has not been applied to advanced countries.

Application to advanced countries requires modifications which allow higher debt/GDP, counter-cyclical government debt, and consumption volatility less than income volatility. Modifications to address each change have been proposed, but to our knowledge there are no modifications which include all changes appropriate to an advanced economy. Increasing equilibrium debt/GDP ratios to match those in emerging markets has been addressed by Uribe and Schmitt-Grohe (2017), who use a recovery value proportional to debt. Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) use long-term bonds, and Hatchondo et al. (2016) add a proportional recovery value. These models do not generate debt as large as that in Greece and often not as large as that in emerging markets.[5] Yue (2010) finds that the model with endogenous recovery rates supports only a moderate level of debt compared to the data. The partial default model by Arellano et al. (2019) has recovery in default, thereby allowing higher debt/GDP ratios, appropriate to those in emerging and developing economies, but smaller than those in Greece.

Paluszynski (2020) creates a model with learning to address the behavior of debt near a crisis, allowing debt to rise early in the crisis, but not later. Paluszynski and Stefanidis (2020) add frictions in the adjustment of government expenditures that allow for "borrowing into debt crises" so that debt rises as a crisis approaches. However, debt in their model remains procyclical overall, and their calibration to Mexican data does not generate debt/GDP as

[5] Chatterjee and Eyigungor (2012) choose to match only 70 percent of outstanding debt for crisis in Argentina in order not to sacrifice matching the other target moments. Hatchondo et al. (2016) match debt/GDP ratio in Spain, but the debt level of Spain is not comparable to that of Greece. Other literature with long-term bonds such as Arellano and Ramanarayanan (2012) match slightly over 50 percent of total debt for the crisis in Brazil.

high as that in Mexico. Bocola et al. (2019) modify the model by introducing a minimal level for public consumption forcing government borrowing when output is low, that is, counter-cyclical debt behavior. This modification does reduce the variance of consumption and creates counter-cyclical government debt. However, their model cannot support the high levels of debt in Eurozone countries and does not replicate the spike in debt prior to a crisis. The strongly pro-cyclical debt pricing policy, embedded in the strategic default model, dominates the incentives to smooth consumption, leading to a high interest rate and decreased debt issuance when output falls.

The second literature we draw from is that on sustainable debt and fiscal limits. This literature is largely empirically-based and asks how high can debt become before it is unsustainable. The premise is that every government faces limits on its abilities to raise revenue and to reduce spending. The fiscal limit is defined as the maximum debt the government can repay in a particular output state. Default occurs when the government cannot pay contractual debt. Bi (2012), Bi and Leeper (2013), Bi and Traum (2014), and Daniel and Shiamptanis (2013, forthcoming) use fiscal limits as explanations of default. Debt/GDP ratios are as large as those in the data, but the behavior of the sovereign is determined either empirically or exogenously, not optimally, as in strategic default and in our model. We use the government's budget constraint and the idea that there is a limit on the amount of debt that a sovereign can repay to calibrate ability to pay, our measure of the fiscal limit. This aligns our strategic default model with literature on debt sustainability. It also reconciles policy work, designed to determine sustainable debt, with theoretical models of default.

The third literature on excusable default (Grossman and Van Huyck (1988)) is based on the fact that shocks can occur, which make full repayment of debt impossible, due to limited ability to pay. When full repayment is impossible, default is excusable and does not trigger punishment, as long as the agent pays what she is able. Adam and Grill (2017) use the concept of excusable default to design debt contracts with state-contingent repayments as risk-sharing behavior on behalf of a fully-committed sovereign.

The paper is structured as follows. Section 2 lays out the 'strategic' default model and the 'excusable' default model. Section 3 calibrates each model to Greece and Section 4 discusses the main quantitative results. Section 5 concludes.

2 Model

In this section, we describe the canonical 'strategic' default model in Arellano (2008) and present our alternative, the 'excusable' default model. Both models share a common risky endowment economy. They differ in the operation of financial markets available to finance consumption, due to the different assumptions about commitment to repay.

2.1 Common Endowment Economy

A small open economy is populated by identical risk-averse households, risk-neutral foreign investors and a benevolent government. The sovereign acts as a central planner for the economy and makes decisions about borrowing, repayment, and default to maximize the expected present value of the utility of consumption given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1, \tag{1}$$

where $\mathbb{E}_0$ denotes the conditional expectation at time zero, t denotes time, β is the discount factor, c_t is consumption at time t, and $u(\cdot)$ is increasing and strictly concave.

Income (y) follows a Markov process with transition matrix $f(y'|y)$, where the prime denotes a one-period-ahead value. Risk-neutral international investors offer one-period non-contingent bonds in international markets. Access to these bonds and their pricing differs across models.

2.2 Strategic Default Model (SD)

The sovereign has outstanding debt, which she is not committed to repay. When the sovereign chooses repayment, she has access to international financial markets to finance consumption. Letting D denote outstanding debt, D' her choice of next period's debt, and $q(D', y)$ the price of next period's debt, the resource constraint in a repayment period is given by

$$c = y - D + q(D', y)D'. \tag{2}$$

When the sovereign chooses not to repay, she is in default. She loses output of $\phi(y)$ and is excluded from international financial markets for a random period of time. The resource constraint in default becomes

$$c = y - \phi(y). \tag{3}$$

Let $v^o(D, y)$ denote the value function of a sovereign with the option to default.[6] She chooses whether to repay her debt or default with the objective of maximizing the utility of households. Her value function depends on initial debt (D) and income (y), and is given by

$$v^o(D, y) = \max\{v^r(D, y), v^d(y)\} \tag{4}$$

where $v^r(D, y)$ is the value of repaying debts and staying in the contract and $v^d(y)$ is the value of a decision to default.

Substituting consumption from the resource constraint in equation (2), the value of repaying debt and continuing to participate in international financial markets is given by

$$v^r(D, y) = \max_{D'} \left\{ u(y - D + q(D', y)D') + \beta \int_{y'} v^o(D', y') f(y'|y) dy' \right\}. \tag{5}$$

If the sovereign chooses default, she faces an immediate fall in income and loses access

[6] The sovereign has the option to default if she repaid her debts last period or if she has been randomly readmitted to financial markets after being in default last period.

to international financial markets for a random period of time. Letting θ be the probability of reentry to international financial markets, the value function for defaulting is given by

$$v^d(y) = u(y - \phi(y)) + \beta \left[(1-\theta) \int_{y'} v^d(y') f(y'|y) dy' + \theta \int_{y'} v^o(0, y') f(y'|y) dy' \right]. \tag{6}$$

A sovereign with the option to default, has value function $v^o(D, y)$ from equation (4), and therefore chooses default when the value of defaulting exceeds the value of repaying. The default rule becomes

$$d = \begin{cases} 1 & \text{if } v^r(D, y) < v^d(y) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where d is an indicator function with $d = 1$ implying strategic default and $d = 0$ implying full repayment. Arellano (2008) proves that if default is optimal at the state given by $(D, \hat{y})$, then it is optimal for all states $(D, y < \hat{y})$. We define an income threshold, $\hat{y}(D)$, as the lowest value for output, for a given value of D, for which the sovereign repays.

International lenders are risk-neutral and require an expected rate of return equal to the risk-free interest rate. This implies an arbitrage equation in which expected returns on sovereign debt must equal the risk-free rate. The arbitrage equation is given by

$$q(D', y) = \frac{1 - F(\hat{y}'|y)}{1 + r^*}, \tag{8}$$

where $F(\hat{y}'|y)$ is the cumulative probability of output up to state $\hat{y}'$ conditional on output initially in state y, equivalently the probability of default. The price of a unit of debt is the present value of the probability of repayment. The price of debt is falling in the probability of default, which is falling in output, due to its autoregressive behavior, and rising in debt, since $\hat{y}$ falls with an increase in debt.

The derivative of the proceeds from the sale of debt with respect to debt is given by

$$\frac{\partial(qD')}{\partial D'} = \frac{1 - F(\hat{y}'|y) - D' f(\hat{y}'|y)\frac{\partial \hat{y}'}{\partial D'}}{1+r^*}. \tag{9}$$

The proceeds from the sale of debt are given by its price adjusted for the fact that an increase in debt raises the output threshold below which default occurs. Therefore, when debt is risky, the proceeds from debt rise by less than the price of debt because the price of debt itself falls.

The recursive equilibrium for the strategic default model is defined as:

A set of policy functions for consumption $c(D, y)$ and debt $D'(D, y)$, threshold values for output determining repayment as a function of debt $\hat{y}(D)$, and a bond price function $q(D', y)$ such that:

(*i*) *With the bond price function given, policy functions for consumption $c(D, y)$ and debt $D'(D, y)$, and threshold values for output $\hat{y}(D)$ determining default satisfy the optimization problem subject to the resource constraint and punishments for default.*

(*ii*) *Bonds prices $q(D', y)$ reflect the government's probability of default and are consistent with international creditors' no-arbitrage conditions.*

In equilibrium, the average length of crises is determined by the re-entry parameter θ, and the value of all haircuts is 100 percent by assumption.

2.3 Excusable Default Model (ED)

The excusable default model differs from strategic default, fundamentally, in the assumption about commitment to repay debt. First, we assume that every sovereign faces fiscal limits on her 'ability to pay'. These limits can be due to a Laffer curve on taxes and to political constraints to increasing taxes and reducing spending on public goods, inclusive of public employee wages. Following Bi (2012) and Davig et al. (2010), we assume that fiscal limits are increasing in current output ($A(y)$) and known. Second, we assume that the sovereign is committed to repay debt up to her ability. Third, when a defaulting sovereign repays as much as she can, default is excusable, as in Grossman and Van Huyck (1988), and incurs no punishment.[7]

These assumptions imply that whenever ability to pay is less than outstanding debt, the optimizing sovereign chooses default with repayment equal to her ability to repay.[8] Otherwise, she fully repays outstanding debt. This implies that the default rule in excusable default can be written as

$$d = \begin{cases} 1, & \text{if } A < D \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

With these assumptions, the resource constraint for the sovereign is given by

$$c = y - \min(D, A) + q(D', y)D'. \tag{11}$$

Since a defaulting sovereign is not excluded from financial markets, we can write a single value function for excusable default. Defining $\bar{y}$ to be the upper support and $\underline{y}$ the lower support in the income distribution, the value function is given by

$$\begin{aligned} v(D,y) = \max_{D'} \Bigg\{ & u(y - \min(D,A) + q(D',y)D') \\ & + \beta \left[\int_{y'=\underline{y}}^{\hat{y}'} v(A(y'),y') f(y'|y) dy' + \int_{y'=\hat{y}'}^{\bar{y}} v(D',y') f(y'|y) dy' \right] \Bigg\}, \end{aligned} \tag{12}$$

where the income threshold $\hat{y}(D)$ is implicitly defined by

$$D = A(\hat{y}(D)), \tag{13}$$

[7]The notion that default is excusable and does not trigger punishment departs from the strategic default literature in which default leads to output loss and a loss of access to international financial markets. Empirically, disentangling cause and effect around default and output loss is difficult. Yeyati and Panizza (2011) demonstrate that countries usually default after the economy has suffered a downturn. Also, countries in default do continue to borrow (Benjamin and Wright (2013)).

[8]The concept of repaying only part of the debt obligation is different from the partial default introduced in Arellano et al. (2019). In Arellano et al. (2019), the model sovereign optimally chooses how much to borrow and the portion of the debt obligation on which to partially default.

such that repayments in the threshold state equal debt.

The price of debt is determined to assure that international creditors expect to receive the risk free interest rate. The price of debt equals the present value of expected future repayments relative to debt, where future repayments include full repayment of debt, multiplied by probability, plus repayments in default states multiplied by the probabilities of those states. We add a deadweight cost of default under the assumption that the sovereign repays her full ability in default, but the investor only receives a fixed fraction, ω, of the repayment. We view this as an operating loss and not as a default punishment. The deadweight loss facilitates the calibration and gives the ED model the same number of parameters to calibrate as the SD model.[9] This yields the price of debt as

$$q(D', y) = \frac{1 - F(\hat{y}'|y)}{1 + r^*} + \frac{\int_{y'=\underline{y}}^{\hat{y}'} \omega A(y') f(y'|y) dy'}{D'(1 + r^*)}. \tag{14}$$

Compared to the price of debt with strategic default (equation (8)) the price of debt in excusable default (equation (14)) contains an additional term. This term reflects repayment the sovereign makes in default, thereby mitigating the fall in the price of debt as default probability rises, due either to an increase in debt or a reduction in output.

The derivative of the proceeds from the sale of debt with respect to debt is given by

$$\frac{\partial(qD')}{\partial D'} = \frac{1 - F(\hat{y}'|y) - [D' - \omega A(\hat{y}')] f(\hat{y}'|y) \frac{\partial \hat{y}'}{\partial D'}}{1 + r^*}, \tag{15}$$

where $[D' - \omega A(\hat{y}')]$ is zero since ability to pay at threshold output equals debt from equation (13).[10] This simplifies the derivative to

$$\frac{\partial(qD')}{\partial D'} = \frac{1 - F(\hat{y}'|y)}{1 + r^*}. \tag{16}$$

[9]Note that the defaulting sovereign is already repaying as much as she can, implying that the only way to reduce the return to creditors is requiring that they receive of a fraction of the sovereign's repayment.

[10]In an abuse of notation, $\omega = 1$ at the point for which repayments equal debt because there is no default and no deadweight loss. For repayments less than debt, $\omega < 1$ because there is default and its associated deadweight loss.

The proceeds from the sale of debt rise by more with an increase in debt (equation (9)) than under SD due to repayments in default.

Recursive equilibrium for the excusable default model is defined as:

A set of policy functions for consumption $c(D, y)$ and debt $D'(D, y)$, threshold values for output determining repayment as a function of debt $\hat{y}(D)$, and a bond price function $q(D', y)$ such that:

(*i*) *Taking as given the bond price function, policy functions for consumption $c(D, y)$ and debt holdings $D'(D, y)$, threshold values for output $\hat{y}(D)$ determining default, and debt repayments satisfy the optimization problem subject to the resource constraint and commitment to repay debt up to ability to pay.*

(*ii*) *Bonds prices $q(D', y)$ reflect the government's probability of default together with expected repayments after default and are consistent with international creditors' no-arbitrage conditions.*

Each default lasts a single period because the sovereign settles her defaulted debt by repaying what she is able. After a default, she chooses next period's debt using the policy function. In the subsequent period, if ability to pay is less than contractual debt, she defaults again, extending the period of default beyond a single period. When defaults are recurrent, we view each period's repayment and reborrowing, not as actual transactions, but as debt renegotiation affecting net repayments over the extended default period. We define crisis duration as the time between entering the crisis with default and exiting the crisis with the first full repayment of contractual debt, which is not followed by another default within one year.

We measure the haircut using the concept of excess return, calculated as actual repayments relative to contractual repayments minus 1. In the absence of default, actual repayments equal contractual repayments implying that excess return is zero. With default, actual repayments are less than contractual repayments implying that excess return is negative. The haircut is the negative of the excess return.

For example, if the default lasts only one period, then actual repayments relative to contractual repayments imply a gross return of $A(y_1)/D_1 < 1$ for the single period, where the subscripts denote the period in default, with 1 denoting the first period. The excess return is $A(y_1)/D_1 - 1 < 0$. For a two-period default, we measure the excess two-period return as the product of the first period and second period gross returns minus 1. In general, for an asset whose period in default is given by n, the haircut is one minus the gross n-period return, yielding

$$HC = 1 - \frac{[A(y_1)] \times [\min(A(y_2), D_2)] \times \cdots \times [\min(A(y_n), D_n)]}{D_1 \times D_2 \times \cdots \times D_n}, \tag{17}$$

where integers in parentheses represent the period in default, and where the default ends once $\min(A(y_{n+1}), D_{n+1}) = D_{n+1}$ and is not followed by another default for four consecutive quarters. Note that net repayments in each default period i are determined by the minimum of the state-specific fiscal limit, given by the ability-to-pay ($A(y_i)$), and contractual debt obligations D_i.

2.4 Incentives to Take on Debt in the Two Models

We can compare the incentives to take on debt in the two models by using the Euler equation, which has an identical form for each.[11] The Euler equation is given by

$$\frac{\partial u(c)}{\partial c} \frac{\partial (qD')}{\partial D'} = \beta \int_{y'=\hat{y}'}^{\bar{y}} \frac{\partial u(c')}{\partial c'} f(y'|y) dy'. \tag{18}$$

Since the choice of debt does not affect the value of repayments, and therefore the value of consumption when $y' < \hat{y}'$ (default states), these terms are missing. The absence of these terms makes the right-hand side of the equation smaller requiring a smaller marginal utility

[11] See Appendix A for derivation.

of current consumption. Current consumption must be larger, requiring a choice of larger debt. This is the classic result by Stiglitz and Weiss (1981), that the possibility of default cuts off the lower portion of the risk distribution, increasing the incentives for risk-taking.

We use the Euler equation to highlight two important differences in behavior across the two models. One is values for income thresholds as a function of D, given by $\hat{y}(D)$. Higher income thresholds allow the sovereign to accumulate more debt without default.

The other is the derivative of borrowing proceeds with respect to debt, given by $\frac{\partial(qD')}{\partial D'}$. For ED, an increase in D' delivers a larger increase in the proceeds from borrowing since q falls less, due to payment of recovery in default (equations (8) and (14)). Even if the models were calibrated to have identical values for β and for the schedule of income thresholds as a function of D, incentives to take on debt would differ due to the different responsiveness of q.

Substituting from equation (9) yields the Euler equation for SD as

$$\frac{\partial u(c)}{\partial c} = \frac{\beta(1+r^*)\int_{y'=\hat{y}'}^{\bar{y}} \frac{\partial u(c')}{\partial c'} f(y'|y)dy'}{1 - F(\hat{y}'|y) - D' f(\hat{y}'|y)\frac{\partial \hat{y}'}{\partial D'}}, \tag{19}$$

while substituting from equation (16) yields the Euler equation for ED as

$$\frac{\partial u(c)}{\partial c} = \frac{\beta(1+r^*)\int_{y'=\hat{y}'}^{\bar{y}} \frac{\partial u(c')}{\partial c'} f(y'|y)dy'}{1 - F(\hat{y}'|y)}. \tag{20}$$

Therefore, the Euler equation requires that $\frac{\partial u(c)}{\partial c}$ relative to $\int_{y'=\hat{y}'}^{\bar{y}} \frac{\partial u(c')}{\partial c'} f(y'|y)dy'$ be lower for the ED model. This requires relatively larger current consumption compared to expected future consumption, requiring a larger D'. The larger D' in turn raises $\hat{y}'$, raising the lower limit on the integral and reducing the right-hand side of the Euler equation for ED. This requires an even smaller relative $\frac{\partial u(c)}{\partial c}$, requiring an even larger D'.

The Euler equation demonstrates that, for values of debt and output for which there is risk of default, the ED model chooses a higher value of debt than the SD model, even if they are calibrated to have identical values income thresholds ($\hat{y}$) and for β. And the higher choice for D' implies higher risk. The difference is due to the different responsiveness of the debt price in the two models to the choice of debt.

3 Calibration

In this section, we calibrate the two models described in Section 2 to the Greek experience prior to its crisis in 2010Q1. We describe parameters calibrated outside of the model first and then turn to model-calibrated parameters.

3.1 Externally-Calibrated Parameters

We assume that utility takes the form of constant relative risk aversion, a common assumption in the literature. Utility is given by

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \tag{21}$$

where the risk aversion coefficient σ is set to two, also standard.

We set the quarterly risk-free interest rate at 0.53 percent, equal to the annualized interest rate of 2.12 percent, calculated as the average of the interest rate on German ten-year bonds net of inflation over the period from 2005 to 2008.

The stochastic income process is estimated from the quarterly series of Greece's real GDP between 1960Q1 and 2010Q1 after log-linear detrending. We set the end date at 2010Q1 because this is the last date before official intervention into the Greek crisis with the first bailout. Our model does not have a role for official intervention, requiring us to omit those periods. The stochastic income process is assumed to be a log-normal AR(1) process given by

$$\ln y_t = \rho_y \ln y_{t-1} + \eta_y \epsilon_t, \text{ with } \epsilon_y \sim \mathcal{N}(0,1) \tag{22}$$

where $\ln y_t$ is log-linearly detrended and demeaned output, ρ_y is estimated to be 0.9666 and

η_y is estimated to be 0.0253.

3.2 Parameters Calibrated within the Models

Patience, given by β, is calibrated within the model for both models. Additional parameters are specific to each model.

3.2.1 Functional Forms with Parameters Specific to Strategic Default Model

A central feature of the strategic default model is the default output-cost function. Following Chatterjee and Eyigungor (2012), we assume that the default cost function $\phi(y)$ takes the form

$$\phi(y) = \max\{0,\ d_0 y + d_1 y^2\}. \tag{23}$$

This quadratic default cost function allows cost to rise more than proportionately with output.[12] This feature is key in assuring that the sovereign chooses default more frequently when output is low, consistent with data. The loss function adds two parameters to calibrate within the strategic default model, d_0 and d_1. We also calibrate the parameter for the probability of reentry after a crisis, θ.

We are explicitly calibrating the output-cost parameters and the probability of re-entry after a crisis instead of setting their values outside the model. We make this choice, since both are hard to pin down with external data. Hatchondo and Martinez (2017) explain that assessing the output cost of default using information on the fall in output around a default mixes causation and correlation, as well as omitting other potential default costs. The reentry parameter is difficult to calibrate externally since we have so few observations of default for Greece and therefore very few observations on reentry.

[12] Arellano (2008) parameterizes a default cost as $\phi(y) = \max\{0, y - \bar{y}\}$. Mendoza and Yue (2012) discuss alternative default cost functions.

3.2.2 Functional Forms with Parameters Specific to Excusable Default Model

The ability to pay plays a central role in the excusable default model, parallel to that of the output loss function in strategic default. To calculate ability to pay, we assume that the sovereign is constrained by a minimum value for consumption. With this constraint, equilibrium requires that debt not exceed the expected present value of output less minimum consumption. However, a sovereign could not be expected to follow a policy which forced consumption to remain at its minimum forever. "Austerity forever" is not politically viable. Therefore, we assume that the ability to use all income above a minimum consumption to repay debt is strongest during negotiations to end default and eventually diminishes over time by factor Ψ^t where $\Psi < 1$. Therefore, the country's ability to pay respects both a minimum level of consumption and a time frame for the country to raise consumption above the minimum. This present-value concept of ability to pay, given by A, is expressed as

$$\begin{aligned} A_t &= y_t - \bar{c} + \mathbb{E}_t \left[\frac{y_{t+1} - \bar{c}}{1+r^*} + \Psi \frac{y_{t+2} - \bar{c}}{(1+r^*)^2} + \Psi^2 \frac{y_{t+3} - \bar{c}}{(1+r^*)^3} + \dots \right] \\ &= y_t - \bar{c} + \left(\frac{1}{1+r^*} \right) \mathbb{E}_t \left[\sum_{t=1}^{\infty} \left(\frac{\Psi}{1+r^*} \right)^{t-1} (y_{t+1} - \bar{c}) \right]. \end{aligned} \tag{24}$$

This specification allows ability to pay to rise more than proportionately with output.

Our concept of ability to pay is a wealth concept, not a liquidity concept. A country, which repays A_t following default, is not required to have the liquidity from current output to make this repayment. The country is allowed to finance its current repayment with borrowing, which is constrained by expected future ability to pay. Therefore, ability to pay can exceed income. Additionally, when output is low, expected future ability to pay is larger than current ability, implying that a defaulting sovereign can choose to borrow more than she repays.

The ability to pay gives us two parameters to calibrate within the model for excusable default, $\bar{c}$ and Ψ. We also have the deadweight cost parameter ω from the bond-pricing function, equation (14).

3.3 Targets for Calibration

We need four targets to calibrate the four parameters in each model. For both models, we need to calibrate the discount factor, β. For the strategic default model, we also need to calibrate the parameters in the default cost function, d_0 and d_1, and the parameter for the probability of reentry θ. For the excusable default model, we need parameters governing the ability to pay, $\bar{c}$ and Ψ, and the deadweight loss parameter, ω.

We choose the same four targets for both models to ensure that the calibrations across the two models are consistent. The standard calibrations of the SD model for emerging markets miss on the Greek experience for the path of debt leading up to a crisis and on the relative volatility of consumption to income. To maximize the chance of our SD model calibration getting these features, we choose to match characteristics of the debt path leading up to the crisis date and the relative volatility of consumption to income.

We want to match the evolution of debt near a crisis, and not just the evolution of debt/GDP. Therefore, we need a detrended measure of real debt from the data. We reject multiplying the data series of debt/GDP by real GDP and log linearly detrending it. Our series on debt relative to GDP begins in 2000Q1, and is, therefore, relatively short. For most of the time period leading up to the crisis date, real GDP is below its mean and debt is rising, as consumption-smoothing would imply. If we log-linearly detrend this measure of real debt, then we remove this model implication. Therefore, we choose an alternative method to detrend.

We assume that debt and output have the same real and nominal trends. This implies that the ratio of detrended real debt to detrended real output equals the ratio of nominal debt to nominal output. We obtain a measure of detrended real debt by multiplying the ratio of nominal debt to nominal GDP in the data by our detrended and demeaned measure of real output. This gives our measure of debt the interpretation of real debt relative to the trend in real output.

Another decision on debt is whether to calibrate to external debt, as in the standard strategic default model, or to government debt. We choose to calibrate to government debt following Bocola and Dovis (2019) and Bocola et al. (2019), who calibrate to government debt in explaining European crises and justify their choice as yielding a better fit.[13]

We have additional reasons for this choice. First, the Greek crisis we want to explain is a government debt crisis. Second, actual governments choose government debt, not external debt. Third, the IMF debt negotiation procedures are for government debt and Greece defaulted on government debt. Fourth, the current account, which governs the accumulation of external debt, is the difference between national savings and investment. The endowment economy has no investment. Therefore, we should not expect to match current account dynamics, and therefore external debt dynamics, without investment. Finally, the original concept of ability to pay is based on the economic and political inability to extract additional tax resources above spending for debt payment in a closed economy, a concept based on government debt.

We have data on government debt only from 2000Q2 after forwarding one period.[14] This was a period with a major policy disturbance for Greece, its admission to the European Monetary Union. Given the short sample, inclusion of these years with a major policy disturbance could have a sizable impact on moments. Therefore, we choose to omit the early years to allow for adjustment after EMU admission, and begin our sample for debt and interest rate with 2002Q4. We end the sample with 2010Q1 due to substantial official intervention beginning in 2010Q2.

[13]We model our choice differently because their model requires a fixed tax rate which limits the response of ability to pay to changes in income. Our alternative justification equates the government budget constraint with the country resource constraint. We assume that the social planner problem represents an economy with a government and households, and that the government borrows on behalf of households to maximize household utility. Households have no incentive to borrow, with the additional assumption that there is a a capital tax on borrowing which makes the bond price the household perceives move in the same way as the bond price the government faces, as in Uribe and Schmitt-Grohe (2017). With these assumptions, external debt and government debt are equivalent.

[14]We forward the debt data by one quarter, because data debt is end-of-period, while model debt is beginning-of-period.

We choose two targets related to government debt. At the beginning of the period, debt was rising slowly, and as output fell, the rate of increase in debt accelerated. We want to capture two characteristics of this path. First, we want to capture the level of debt, and second, the extent of its crisis-triggering increase. We measure the first using the mean over the period 2002Q4-2008Q4, and the second with the value of debt just prior to the crisis in 2010Q1. For the mean, we target the mean of debt/GDP since this is standard in the literature. However, for the crisis value of debt, we target the value of detrended debt because we want to capture the rise in actual debt and not the rise in debt/GDP created by the fall in GDP.

Our third calibration target is the value of the interest rate spread prior to the crisis (2009Q4). We choose this over mean spreads because in the literature, SD models with one-period bonds have done poorly on this measure. In both models, spreads are low while the probability of default is low and jump anticipating the crisis. We want to capture that jump.

Our fourth and final calibration target is the standard deviation of consumption relative to the standard deviation of output. Calibrations of the SD model for emerging markets obtain more volatile consumption than output, consistent with data in emerging markets. Greece is an advanced country with relatively less volatile consumption and we want to match that characteristic.

3.4 Calibration for Parameter Values and Results

We set parameter values and use value function iteration to solve the model for debt policy functions, with a debt grid with 1,000 equally spaced values from 0.0 and 10.0. We use Tauchen (1986)'s method of approximating an autoregressive process as a Markov chain to discretize output into 201 states.[15] We simulate model paths for the variables by setting initial debt at its actual value in 2002Q4, and iteratively feeding in actual values of output

[15] Hatchondo et al. (2010) find that using a fine grid for output is necessary to model spread moments accurately in quantitative strategic default models.

in each period. We compute model moments using these simulated paths. We choose this method over that of generating model moments using a full simulation because output, over the period for which we have full data, was below the mean, implying that the full simulation would not be representative of the data.[16] We compute data moments over the same period and select parameter values to minimize the sum of log-difference distances between sample moments and model-implied moments (simulated method of moments).

While all of the values of the targets in each model are jointly determined by all of the model's parameters, some targets are more important for some parameters than others. We discuss each parameter and its primary targets.

The discount factor, β, is primarily pinned down by the average debt/GDP before debt spikes upwards in 2009Q1. A smaller β implies greater impatience and a stronger upward trend to debt. The parameters in the output cost of default for SD, d_0 and d_1, and those in ability to pay for ED, $\bar{c}$ and Ψ, are important in determining output thresholds as a function of debt and the willingness of the sovereign to let the threshold, which depends on debt, approach the current value for output. The proximity of output to thresholds determines the probability of default, in turn affecting the spread in 2009Q4 and the level of debt in 2010Q1. The spike in debt beginning in 2009Q1 for ED is governed by the value for ω as well as the value for β. Together, all the parameters govern the cost of default and therefore the willingness to take on debt in response to a fall in output, thereby governing the relative volatility of consumption to output.

The values for the calibrated parameters and the method of calibration are displayed in Table 1. It is noteworthy that the (quarterly) discount factor β is calibrated to be 0.8513 for the strategic default model and 0.9939 for the excusable default model. A low discount

[16] The position of output relative to the mean affects the probability of default and therefore the behavior of the interest rate. When output is above the mean, the probability of default is consistently low, implying less interest rate volatility compared with a sample in which output is distributed more evenly about its mean. The response of the interest rate to output affects other moments. Additionally, realizations of output in our sample were in the negative tail, inconsistent with average behavior over a long sample. Subsamples of 201 periods, the number from the beginning of our data in 1960Q1 until the crisis, are also not representative of our data in which output is persistently below the mean. This is because output is generally above the mean for a sample in which the model avoids default for such a long period.

Table 1: Model Parameters

Parameter	Strategic Default Model	Excusable Default Model	Method of Calibration
r^*	0.0212		Data
σ	2.0000		Conventional value
ρ_y	0.9666		Estimates of equation (22)
η_y	0.0253		Estimates of equation (22)
β	0.8513	0.9939	Method of simulated moments
d_0	-0.5607		Method of simulated moments
d_1	0.7928		Method of simulated moments
θ	0.0300		Method of simulated moments
$\bar{c}$		0.5843	Method of simulated moments
Ψ		0.9268	Method of simulated moments
ω		0.9987	Method of simulated moments

factor in the strategic default literature is common. The value for β in Arellano (2008), Chatterjee and Eyigungor (2012), Mendoza and Yue (2012) and in other strategic default models focusing on emerging market economies ranges from 0.72 to 0.96. The impatient government in the model is often justified by arguing that political economy incentives lead government decision makers to display higher rates of time preference (Mendoza and Yue (2012)). These political economy incentives are more likely to apply to emerging markets with volatile political environments than they are to advanced economies with more stable political systems. Bocola et al. (2019) and Daniel (2021), which study European debt crises and counter-cyclical debt, calibrate β to be 0.98 and 0.9938, respectively.

Our calibration of the parameters in ability to pay for the ED model generates an ability to pay relative to output which is increasing in output. Therefore, countries can repay a larger fraction of their output in times when output is high than when it is low.

Table 2 illustrates the quantitative performance of the strategic and excusable default models in matching the Greek targets. We express debt/GDP with GDP at annual rates, as is conventional. Additionally, detrended real debt is relative to the trend in GDP at an annual rate and interest rates are expressed at annual rates. The ED model matches the targets reasonably well, but the SD model does not. Both models match average debt/GDP

Table 2: Model Fit for the Targeted Moments

Statistic	Data	Strategic Default Model	Excusable Default Model
Debt/GDP (2002Q4-2008Q4)	1.024	1.036	1.032
Crisis Period Debt (2010Q1)	1.112	0.898	1.106
Spread Prior to Crisis (End of 2009)	2.352	1.053	2.343
$\sigma(c)/\sigma(y)$	0.644	2.458	0.643

Note: The data moment for $\sigma(c)/\sigma(y)$ is computed using data from 2002Q4 to 2010Q1.

prior to its spike, but ED outperforms SD on all other targets, including targets at the ends of the debt and spread paths, as well as the relative volatility of consumption and output.

As a test of model fit, we compare untargeted model and data moments in Table 3. Both models get spread volatility and its negative correlation with output a little low, and the ED model gets mean spread too low. The SD model misses the sign of the correlation of debt and its forward-looking first difference with output, while the ED model misses the sign of the correlation of the trade balance and output. The SD model does not produce a crisis. Other moments are relatively close for both models.

We chose to leave default frequency as an untargeted moment because default occurs so infrequently that it is difficult to measure frequency. We define a sovereign to be in default if she is not participating in private financial markets (due to exclusion in SD) or if she has not fully settled outstanding debt (with payment of the her renegotiated contractual debt in ED and no return to default for one year) and returned to private markets. We define default frequency as the fraction of time that Greece was in default by either definition. It took nine years after the beginning of the crisis in 2010Q1 for Greece to reissue long-term bonds. Greece was either in a bailout agreement or not participating in financial markets for nine of the sixty years between 1960Q1, the beginning date of the available data, and 2020Q1, equivalent to 15 percent of the time.[17] An alternative measure would be the fraction

[17]We have only one default episode in our sample and the period featured large official intervention,

Table 3: Model Fit for the Untargeted Moments

Statistic	Data	Strategic Default Model	Excusable Default Model
$mean(r-r^*)$	0.611	0.600	0.116
$\sigma(r-r^*)$	0.729	0.574	0.446
$corr(r-r^*,y)$	-0.868	-0.632	-0.639
$corr(TB/y,y)$	-0.279	-0.675	0.927
$corr(D,y)$	-0.587	0.896	-0.720
$corr(\Delta D,y)$	-0.344	0.563	-0.768
Default Frequency	0.150∼0.225	0.156	0.200
Default Timing	2010Q1	No Default	2010Q1
$mean$(Default Duration)	9.0	8.455	9.003
σ(Default Duration)		8.635	9.345
$mean(HC)$	0.590∼0.650		0.531
$\sigma(HC)$			0.270

Notes: The forward looking variables, related to interest rate spreads and the change in debt are computed using data from 2002Q4 to 2009Q4 since the 2010Q1 variable likely anticipates the official intervention the following quarter. The other moments excluding default and haircut related moments are computed using data from 2002Q4 to 2010Q1. The data on default duration and haircut (Zettelmeyer et al. (2013)) are from the recent Greek crisis.

of time that the modern democratic government has spent in default where the beginning date is admission to the European Union in 1981, giving a frequency of 22.5 percent. And Reinhart and Rogoff (2008) argue that using two centuries of data, Greece was in default for 50.6 percent of the time between 1829 and 2006.

Additional untargeted moments associated with default include haircuts and crisis duration. These moments have the same measurement issues as default since we have so few observations. The Greek crisis lasted nine years and Zettelmeyer et al. (2013) estimate the haircut at between 59 and 65 percent. Sturzenegger and Zettelmeyer (2008), Benjamin and Wright (2013), and Cruces and Trebesch (2013) find high variability in the size of haircuts for sample of defaults in emerging and developing countries. Together, these papers claim that most haircuts are within the range of 25-40 percent. Benjamin and Wright (2013) place the mean duration at eight years.

implying that crisis resolution was not likely determined as in our model. However, we use this episode as one measure of the default moment.

We calculate the frequency of default states over a 500,000 period simulation[18]. The frequency of default in both SD and ED models is within bounds consistent with those of Greece. For moments on crisis length, we compute the length of each crisis event in the simulation and average them. In the ED model, we compute the haircut using equation (17) for each crisis event and compute the average and standard deviation. In SD, crisis duration is determined by the calibration of θ and is consistent with the data, but haircuts are one hundred percent by assumption. In ED, crisis duration is identical to that of Greece in the recent crisis and close to mean duration in defaulting emerging markets. The model haircut is close to the estimates for Greece in Zettelmeyer et al. (2013). Both the model crisis duration for SD and the model crisis duration and haircut for ED are consistent with the data. Finally, the SD model does not generate a crisis, while the ED model gets a crisis with the correct timing.

4 Discussion

4.1 Paths for Debt, Interest Spreads, and Consumption

To facilitate understanding the moment matching successes and failures of each model, we plot data and model paths for debt, interest spreads, and consumption, leading up to 2010Q1 in Figures 2, 3, and 4. The values in the final period for the debt and spread paths are targeted, but other values are not. All figures are drawn by feeding in actual values of output from 2002Q4 to 2009Q4 and the value of debt in 2002Q4 to generate model debt, spread, and consumption paths to compare with the corresponding data paths.

The paths provide another measure of model fit. For debt, the fit early in the sample is tighter for SD than for ED due to higher volatility, but the fit deteriorates markedly for SD after 2007Q3, when model debt begins falling and data debt rises. The fit for ED is much closer over these dates. For interest rate spreads, both models have spreads low early with

[18]We exclude the first 10,000 observations from the 510,000 period simulation.

Figure 2: Debt Path

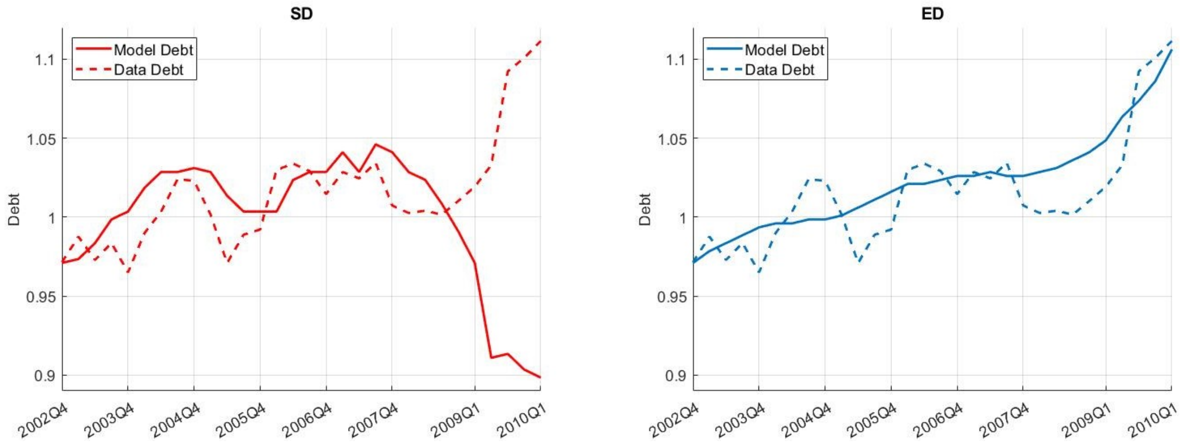


Figure 3: Spreads Path

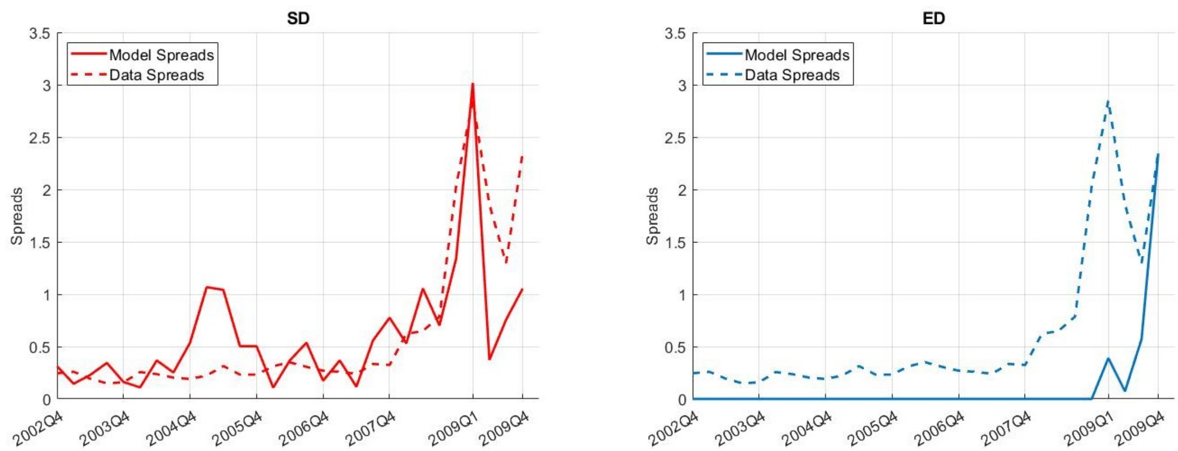
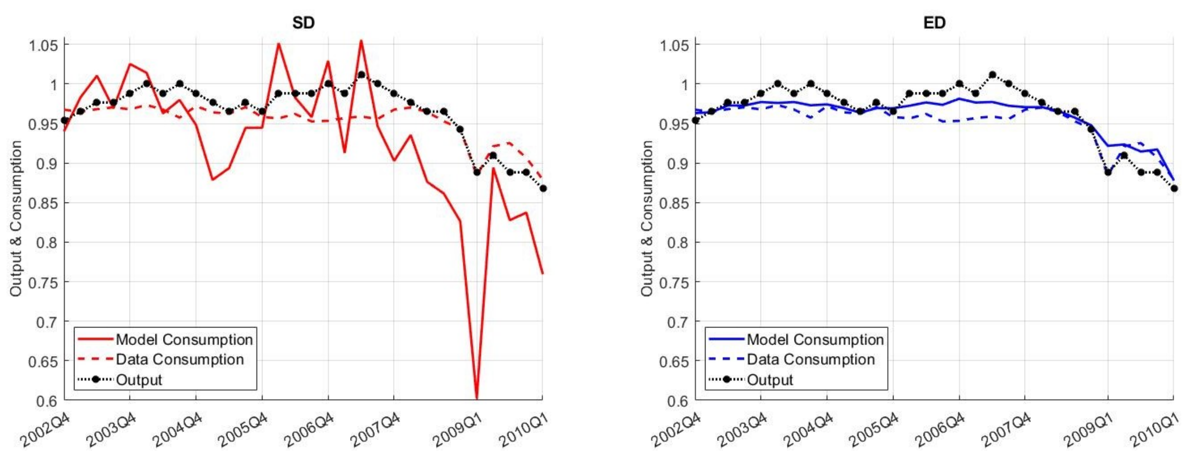


Figure 4: Consumption Path



increases later. Early, the mean and volatility are too high for SD while they are too low for ED. Later, SD matches the spread value in 2009Q1, while ED matches its value in 2009Q4. For consumption, the volatility and downward fall are much too high for SD, while they are about right for ED.

4.2 Data Targets and Related Untargeted Moments

We use these figures to understand why the SD model misses the data targets associated with the crisis, including the debt in 2010Q1, spread in 2009Q4, and the relative volatility of consumption and output, while the ED model matches them. First, compare trends in each variable generated by the behavior of output. Output is relatively flat prior to 2007Q3, but is generally below its mean of unity (Figure 4). The trend in output is negative afterwards.

The trend in debt for both models is positive prior to 2007Q3, due to impatience in SD and borrowing to smooth consumption in ED. Afterwards, debt moves in opposite directions in the two models, with debt falling in SD and continuing to rise in ED. For both models, the trend for consumption follows that for output, while the trend for spreads is positive overall. In summary, the period of falling output is accompanied by falling consumption and rising spreads in both models. The big difference is the sharp divergence in the trends for debt for SD and ED, beginning in 2007Q3. The trend for ED follows the data, while the trend for SD diverges. The incentives in the model, which create this divergence, are responsible for the failure of SD to match the targets.

Trends in debt diverge in the two models with the fall in output in 2007Q3.[19] Consumption-smoothing dictates an increase in debt in response to the fall in output, mitigating the fall in consumption caused by the fall in output. However, the accompanying increase in default probabilities increases the interest rate, with a larger increase for SD than for ED (Figure 3). For ED, the effect of consumption-smoothing dominates, and debt rises, allowing

[19] In an earlier version of the paper, we find that the behavior of debt is similar in a more conventional calibration of the SD model, where we target debt service/GDP, cyclicality of trade balance and default frequency. The downturn in the path of debt in SD is not due to the unconventional calibration, but to incentives in the model.

consumption to fall less than income. In contrast, for SD, the higher interest rate reduces debt, exacerbating the fall in consumption. The downward path for debt in SD is created by repeated falls in output creating interest rate increases which are large enough to offset consumption-smoothing. The upward path for debt in ED reflects smaller increases in interest rates, allowing consumption-smoothing behavior to continue. The SD model misses the targets because model debt moves so far away from data debt. This reversal of consumption-smoothing due to the interest rate behavior also explains why the SD model misses on the relative volatility of consumption and income.

The same reasoning explains matching the untargeted moments of debt counter-cyclicality in ED and trade balance counter-cyclicality in SD. In the endowment economy, the trade balance is income net of consumption. In SD, a fall in income, which creates an increase in default probability, raises the interest rate sufficiently to reduce debt. Consumption falls more than income, increasing the trade balance. Therefore, debt is pro-cyclical and the trade balance is counter-cyclical. For ED, the increase in the interest rate is smaller, yielding an increase in debt and consumption, reducing the trade balance. Therefore, debt is counter-cyclical and the trade balance is pro-cyclical.

Given that income is the only shock, if we match data on consumption volatility being less than income volatility, then we cannot have a counter-cyclical trade balance. The model inconsistency of a counter-cyclical trade balance and consumption volatility less than income volatility is likely due to the fact that the endowment economy has no investment. If the model had strong enough pro-cyclical investment, then the inconsistency would disappear. The endowment model is missing investment and therefore cannot get both a counter-cyclical trade balance and income volatility in excess of consumption volatility.

Our ED model is notable in generating the upward spike in debt just prior to a crisis. Bocola et al. (2019) introduce minimum government spending into the strategic default model. This forces borrowing in bad times, giving the model a generally counter-cyclical debt policy. However, the model misses the upward spike in debt just prior to a crisis. Debt initially rises as output falls, but falls just prior to the crisis. The pressure from the strongly pro-cyclical debt-pricing schedule overwhelms the consumption-smoothing motive, leading to the pre-crisis fall in debt. Debt relative to GDP rises as the crisis approaches, due to the fall in GDP, highlighting the importance of modeling the behavior of debt and not just its ratio to GDP.

4.3 Debt Behavior Near Default

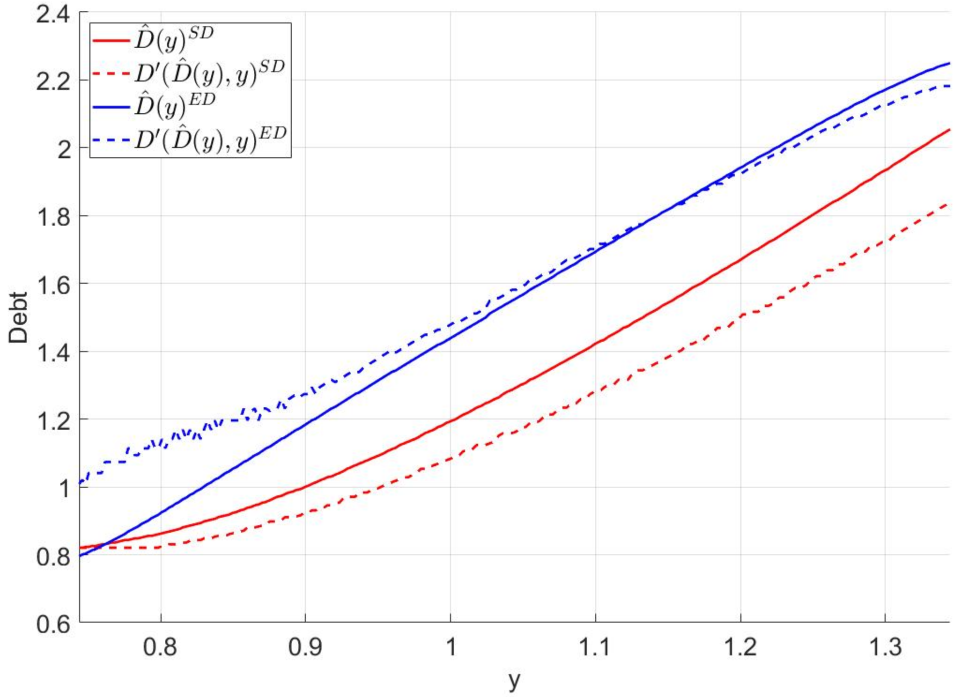
We cannot use SD to explain the Greek default crisis because we cannot generate a crisis with SD. To understand why ED produces a default crisis and SD does not, consider how default occurs. Default next period occurs if output (y') falls below the income threshold of $\hat{y}'(D')$. We can also think about default using the concept of a debt threshold. Define a debt threshold $\hat{D}'(y')$, analogous to the income threshold $\hat{y}'(D')$, as the maximum value of debt, given the value of output, for which the country will repay.

Figure 5 plots debt thresholds for each model as solid lines, together with the largest value of debt the sovereign would take on for each value of output, as dashed lines of the same color. Maximum debt choice at each value of output is given by the choice of next period's debt at the debt threshold $[D'(\hat{D}(y), y)]$. As income rises, debt thresholds rise in both models, but, for most values of income, debt thresholds are higher for ED than for SD. The dashed lines show that maximum debt is below below thresholds for SD, with the distance rising as income rises. In contrast, maximum debt often exceeds thresholds in ED, with a larger distance between the maximum debt choice and the threshold for lower income. This different behavior in the choice of debt near thresholds is due to the different behavior of interest rates, creating different incentives to take on debt. The different thresholds, combined with the interest incentives governing the behavior of debt around the thresholds, contribute to the behavior of debt along the simulated path.

The lower debt thresholds for SD are necessary to match the target for debt/GDP.[20]

[20]Even if we calibrate SD to get higher debt thresholds (not the calibration choice of SMM), we cannot

Figure 5: Debt Thresholds and Debt Choice at Thresholds



Note: The pair of lines with higher debt values is for debt thresholds and maximum debt choices for ED, while the lower pair is for SD.

However, the different behavior of debt near the thresholds illustrates that even if the models were calibrated to have the same debt thresholds, the simulated behavior of debt would differ. As debt nears a threshold, the ED sovereign aggressively takes on debt, while the SD sovereign pulls debt back below the threshold. The aggressive behavior by the ED sovereign is the classic Stiglitz and Weiss (1981) result that bankruptcy cuts off the lower portion of the risk distribution, encouraging additional risk-taking. These differences in behavior lead to different paths of debt near the respective thresholds, with debt rising in ED and falling in SD.

Default occurs if future output (y') falls sufficiently to reduce the debt threshold below the recently chosen value of debt (D'). For the simulation of the SD model, the fall in

match the behavior of debt near the crisis date. This calibration requires higher default costs, raising debt and reducing spreads. Debt/GDP begins too high and falls as output begins falling and takes a large fall in 2009Q1. The probability of default decreases, leading to an even lower value for the value of the spread 2009Q4.

output in 2009Q4 was almost large enough to create default. But since default did not occur, the fall in output led to a severe reduction in debt, as shown in Figure 2. With debt substantially lower, the fall in output in 2010Q1 was not large enough to create default. For ED, the aggressive increase in debt for 2010Q1, together with the large fall in output, triggered default.

4.4 Interest Spreads

We continue to use the simulated paths to understand moments on interest spreads. Both models get the counter-cyclicality of interest rate spreads, although magnitudes are a little low for both. The SD model gets variance approximately correct by getting it too high early, before output begins falling, and too low later, as model and data debt diverge. The variance in the ED model comes entirely after the probability of default has become large enough to generate meaningful interest rate movements. The SD model generates mean spreads close to those in the data, by getting mean spreads early too high, nailing the magnitude of the spread in 2009Q1, and getting spreads after 2009Q1 too low. The ED model generates negligible spreads early while debt is low, misses the magnitude of the spike in the spread in 2009Q1, and gets spreads comparable to the data as the crisis nears.

We can explain part of these misses for the ED model with two real world characteristics, which the model omits. Finland and Austria, EMU countries with excellent credit ratings comparable to those in Germany, have small but positive mean spreads relative to Germany over this period. These small spreads are not likely to reflect default probability, but instead something like a liquidity premium, which is missing from the model. This implies that we should expect a positive small spread between equally credit-worthy countries when one of the countries in the pair (Germany) has a relatively large and liquid bond market. The mean spreads for Austria and Finland over period 2002Q4-2009Q4 were 0.217 and 0.162, respectively. If we adjust the data spreads downwards to reflect only the portion due to default risk, the order of magnitude of the miss on the mean is reduced to approximately thirty basis points for the ED model.[21] A missing liquidity premium enlarges the miss on the mean spread for the SD model.

Second, the ED model misses the magnitude of the first spike in the spread with the fall in output in 2009Q1. In March 2009, the US stock market plunged and international financial markets experienced a short-lived race to quality. Spreads between Germany and virtually riskless countries like Austria and Finland spiked upwards,[22] due to this change in investor preferences. Additional evidence for an omitted shock is that Greek output was in similar states in 2009Q1 and 2009Q4, debt was much higher in 2009Q4, and the spread was higher in 2009Q1. Without an omitted shock, the interest premium would not have been higher in 2009Q1. Neither of our models allows for this change in preferences, implying that we should not fault the ED model for missing the magnitude of this jump.[23] And we question crediting the SD model with matching this jump when it omits this additional real world shock.

At least part of the problem with matching spreads is not specific to either the excusable or strategic default model, but instead is with our use of one-period bonds instead of long-term bonds. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) demonstrate that long-term bonds enable a closer match of spread moments than short-term bonds for the SD model. Long-term bonds price default probability in all future periods within the bond's maturity, while short-term bonds price only one-period-ahead default probability. When default probability is rising, spreads on long-term bonds begin rising earlier than spreads on short-term bonds. Therefore, in this sample, spreads on long-term bonds should be higher and more volatile than spreads on short-term bonds. The spread in the data is for long-term bonds, implying that the mean and volatility of data spreads should exceed those of model spreads. This is true for both models, although negligibly so

[21] A missing small liquidity premium has not been an issue in emerging market since their spreads are larger. When mean spreads are larger, a small liquidity premium does not create a large percentage deviation in the spread.

[22] In March 2009, the spread for Austria was 1.141 and for Finland was 0.791.

[23] The increase was temporary, with larger spreads between Germany and either Austria or Finland substantially diminished by 2009Q4.

for the mean in SD.

We chose to retain one-period bonds, instead of switching to long-term bonds, for several reasons. First, retaining short-term bonds facilitates comparison with original SD models. Second, it is not obvious how the switch to long-term bonds could alleviate problems in the SD model associated with the path of debt prior to a crisis or with the volatility of consumption being less than that of income. Additionally, although the addition of long-term bonds to SD models does seem to match spread mean and volatility better, in our SD calibration, mean spread is basically correct and the miss in volatility is small.[24]

4.5 Average Paths for Debt and Interest Spreads Leading to Crises

The simulated paths for spread and debt path do not allow us to compare the behavior of debt and the spread in advance of a crisis since the SD paths do not lead to a crisis. To compare pre-crisis paths, we plot average paths for debt and the interest spread preceding crises in Figure 6. Following Arellano (2008), we create subsamples of the 201 quarters that result in crisis at the end,[25] where the crisis is defined by default rules, equation (7) for the strategic default model and equation (10) for the excusable default model. From our 500,000 period simulation, we have 485 and 636 sequences that are 201 periods long and end with default for the strategic and excusable default models, respectively. Note decreases in debt preceding crises for strategic default and increases for excusable default, consistent with the results above. However, the fall in GDP, that precedes default, implies that debt/GDP is rising just before default in both, in spite of opposite movements in debt.

Note the qualitatively similar path for spreads for the two models in Figure 7. When probability of default is low, spreads are flat, and they spike upwards when the probability of default rises. Mean spreads are higher for SD, reflecting the difference between models

[24]We have an additional technical reason. Recovery in ED yields such a strong debt dilution incentive that the increase in debt needs to be restrained with an assumption about a maximum debt or interest premium as in Hatchondo et al. (2016). Our experimentation with calibration in the ED model revealed that the behavior of the model is sensitive to this maximum even when the sovereign never chooses it. We do not currently know how to resolve this and leave resolution to future research.

[25]The data sample, 1960Q1:2010Q1 is 201 quarters long.

Figure 6: Average Debt Path Leading to Crisis

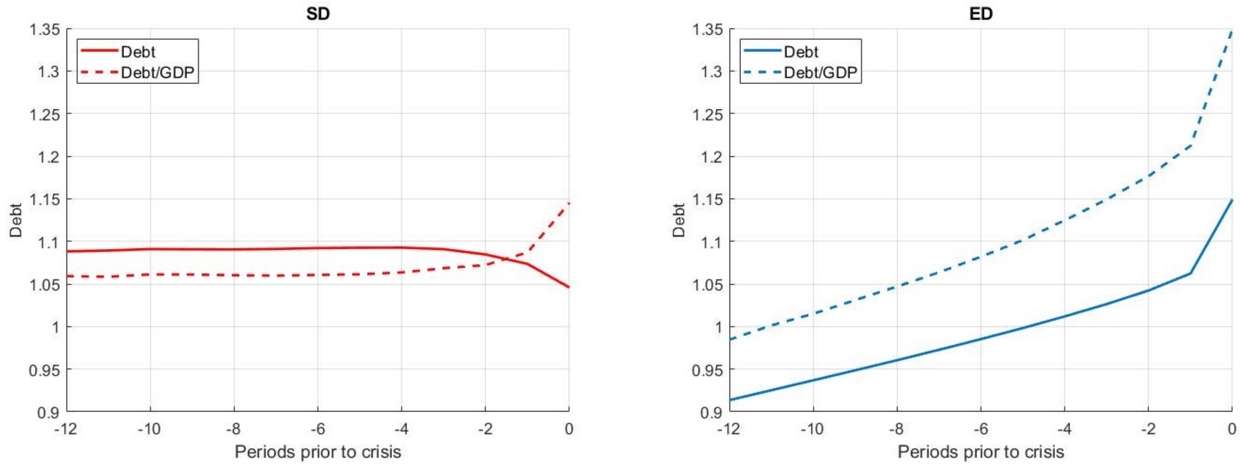
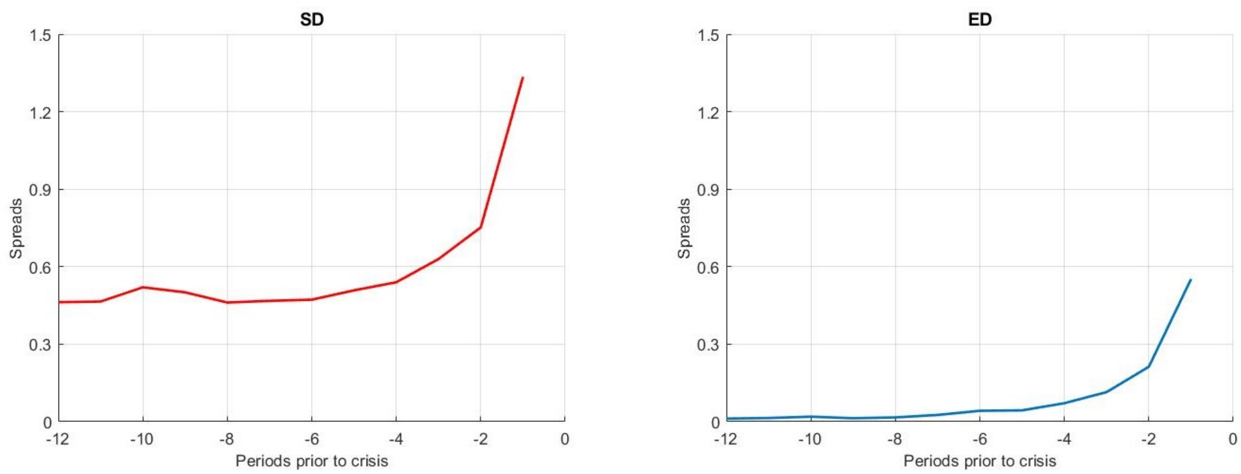


Figure 7: Average Spreads Path Leading to Crisis



with and without repayment in default.

4.6 Fundamental Difference in the Two Models

The strategic default model reduces to the excusable default model if we make three changes in assumptions.[26] First, eliminate output cost of default. Second, eliminate any period of exclusion from financial markets after default. And, third, add recovery value equal to $\min\{D, A(y)\}$. Once we add recovery value determined by ability to pay, internal consistency

[26] We thank Martin Uribe, the editor, for this point.

requires that we also make the additional two changes. We must drop additional default cost (due either to output loss or exclusion) because the sovereign is already reducing the weighted average of expected future consumption by the maximum to make recovery payments. She cannot reduce it further. There is an additional reason we must eliminate exclusion. Since ability to pay can exceed output, a sovereign in default, who is excluded from financial markets and chooses to repay, would not always be able to make the recovery payment.

These changes to the SD model create a fundamentally different model on three counts. The first is in the commitment to repay debt. The first SD model was Eaton and Gersovitz (1981). It was designed to address the question about why a sovereign with no commitment to repay debt ever repays. Costs of default are necessary to assure repayment in some states. The modifications we make to the SD model no longer allow it to answer this question. Instead, the modifications with a commitment to repay up to ability and no costs to default yield a model more like the first excusable default model, Grossman and Van Huyck (1988). A sovereign is committed to repay up to her ability. If she is unable to repay, the default is excusable as long as she repays as much as she can. We view strategic default and excusable default models as conceptually different.

Second, the changes imply different ways to think about the maximum debt a country can accumulate without default. For SD, debt thresholds are determined by the cost of output loss during default, together with the cost of exclusion from financial markets. For ED, debt thresholds are determined by ability to pay. The different determinants of debt thresholds imply different determinants for the value of debt which triggers a crisis.

Third, the different assumptions imply different incentives to accumulate debt in response to a fall in output. The different incentives would yield different results even if the models were calibrated to have identical debt thresholds. Repayment according to ability, even in default, yields implies a smaller response of the interest rate to an increase in default probability compared with no repayment in default. The different response is responsible for the ED model being able to generate a path for debt leading to a default crisis, while the

SD model cannot.

5 Conclusion

The Greek financial crisis raised the need for a crisis model that we can apply to advanced countries. We demonstrate that the canonical strategic default model, calibrated for Greece, does not create a crisis. The excusable default model does. This is because the debt path in SD takes a sharp turn away from the data as output begins a sustained downward trajectory, while the debt path in ED continues to follow the data. The SD model misses in additional key ways that differentiate advanced economies from emerging ones. In the data, consumption is less volatile than output and government debt is counter-cyclical, while the strategic default model reverses both. Other authors have modified the standard model to address some of these differences. We offer an alternative model.

Our model is based on the assumption that a sovereign has limits on her ability to pay and that she can commit to repay up to her ability. This assumption alters the incentives to accumulate debt and to default. Repayment up to ability implies that the sovereign can accumulate debt ratios like those observed in Greece. It also makes the interest rate less responsive to a fall in output, which raises default probability. The increase in default probability raises the interest rate, as in strategic default, but the increase is mitigated by the fact that the sovereign does repay something in default. This repayment allows the model to retain consumption-smoothing behavior, necessary to match stylized facts in advanced countries. We calibrate our alternative model for Greece and generate simulations beginning with the value of debt in 2002Q4 and feeding in realizations of output until 2010Q1. Over this period, consumption is less volatile than output, government debt is counter-cyclical, and government debt ratios are consistent with values observed in advanced countries. Additionally, the path of debt prior to a crisis matches data, with debt spiking upwards. And the end point is a crisis.

Useful extensions of the excusable default model include the addition of long-term bonds and replacement of the endowment economy by a production economy with investment. The addition of long-term bonds would allow the model to do better in matching data on spreads. It would also offer an alternative calibration strategy for a country that has not experienced default but has encountered elevated spreads due to default risk. The addition of investment could allow the excusable default model to match the counter-cyclical trade balance, while retaining the counter-cyclicality of debt and lower volatility of consumption than income.

References

Adam, K. and Grill, M. (2017), 'Optimal sovereign default', *American Economic Journal: Macroeconomics* **9**(1), 128–164.

Aguiar, M. and Amador, M. (2014), Sovereign debt, *in* 'Handbook of International Economics', Elsevier, pp. 647–687.

Aguiar, M., Chatterjee, S., Cole, H. and Stangebye, Z. (2016), Quantitative models of sovereign debt crises, *in* 'Handbook of Macroeconomics', Elsevier, pp. 1697–1755.

Aguiar, M. and Gopinath, G. (2006), 'Defaultable debt, interest rates and the current account', *Journal of International Economics* **69**(1), 64–83.

Arellano, C. (2008), 'Default risk and income fluctuations in emerging economies', *American Economic Review* **98**(3), 690–712.

Arellano, C., Mateos-Planas, X. and Rios-Rull, J.-V. (2019), 'Partial default'. NBER Working Paper 26076.

Arellano, C. and Ramanarayanan, A. (2012), 'Default and the maturity structure in sovereign bonds', *Journal of Political Economy* **120**(2), 187–232.

Benjamin, D. and Wright, M. (2013), 'Recovery before redemption: A theory of delays in sovereign debt renegotiations'. Manuscript, Federal Reserve Bank of Chicago and State University of New York at Buffalo.

Bi, H. (2012), 'Sovereign default risk premia, fiscal limits, and fiscal policy', *European Economic Review* **56**(3), 389–410.

Bi, H. and Leeper, E. M. (2013), 'Analyzing fiscal sustainability', *Bank of Canada Working Paper* **2013-27**.

Bi, H. and Traum, N. (2014), 'Estimating fiscal limits: The case of greece', *Journal of Applied Econometrics* **29**(7), 1053–1072.

Bocola, L., Bornstein, G. and Dovis, A. (2019), 'Quantitative sovereign default models and the European debt crisis', *Journal of International Economics* **118**, 20–30.

Bocola, L. and Dovis, A. (2019), 'Self-fulfilling debt crises: A quantitative analysis', *American Economic Review* **109**(12), 4343–4377.

Chatterjee, S. and Eyigungor, B. (2012), 'Maturity, indebtedness, and default risk', *American Economic Review* **102**(6), 2674–2699.

Cruces, J. J. and Trebesch, C. (2013), 'Sovereign defaults: The price of haircuts', *American Economic Journal: Macroeconomics* **5**(3), 85–117.

Daniel, B. C. (2021), 'Strategic default with an exit decision'. Working Paper, University at Albany.

Daniel, B. C. and Shiamptanis, C. (2013), 'Pushing the limit? Fiscal policy in the European Monetary Union', *Journal of Economic Dynamics and Control* **37**(11), 2307–2321.

Daniel, B. C. and Shiamptanis, C. (forthcoming), 'Identifying countries at risk of fiscal crises: High-debt developed countries', *Canadian Journal of Economics* .

Davig, T., Leeper, E. M. and Walker, T. B. (2010), '"Unfunded liabilities" and uncertain fiscal financing', *Journal of Monetary Economics* **57**(5), 600–619.

Eaton, J. and Gersovitz, M. (1981), 'Debt with potential repudiation: Theoretical and empirical analysis', *The Review of Economic Studies* **48**(2), 289–309.

Frankel, J. A., Vegh, C. A. and Vuletin, G. (2013), 'On gaduation from fiscal procyclicality', *Journal of Development Economics* **100**(1), 32–47.

Grossman, H. I. and Van Huyck, J. B. (1988), 'Sovereign debt as a contingent claim: Excusable default, repudiation, and reputation', *American Economic Review* **78**(5), 1088–1097.

Hatchondo, J. C. and Martinez, L. (2009), 'Long-duration bonds and sovereign defaults', *Journal of International Economics* **79**(1), 117–125.

Hatchondo, J. C. and Martinez, L. (2017), Calibrating the cost of defaulting in models of sovereign defaults, *in* L. Odor, ed., 'Rethinking Fiscal Policy after the Crisis', Cambridge University Press, pp. 414–440.

Hatchondo, J. C., Martinez, L. and Sapriza, H. (2010), 'Quantitative properties of sovereign default models: Solution methods matter', *Review of Economic Dynamics* **13**(4), 919–933.

Hatchondo, J. C., Martinez, L. and Sosa-Padilla, C. (2016), 'Debt dilution and sovereign default risk', *Journal of Political Economy* **124**(5), 1383–1422.

Kaminsky, G. L., Reinhart, C. M. and Végh, C. A. (2004), 'When it rains, it pours: Procyclical capital flows and macroeconomic policies'. NBER Working Paper 10780.

Mendoza, E. G. and Yue, V. Z. (2012), 'A general equilibrium model of sovereign default and business cycles', *The Quarterly Journal of Economics* **127**(2), 889–946.

Paluszynski, R. (2020), 'Learning about debt crises'. Working Paper, University of Houston.

Paluszynski, R. and Stefanidis, G. (2020), 'Borrowing into debt crises'. Working Paper, University of Houston.

Reinhart, C. M. and Rogoff, K. S. (2008), 'This time is different: A panoramic view of eight centuries of financial crises'. NBER Working Paper Series.

Stiglitz, J. E. and Weiss, A. (1981), 'Credit rationing in markets with imperfect information', *American Economic Review* **71**(3), 393–410.

Sturzenegger, F. and Zettelmeyer, J. (2008), 'Haircuts: Estimating investor losses in sovereign debt restructurings, 1998–2005', *Journal of International Money and Finance* **27**(5), 780–805.

Tauchen, G. (1986), 'Finite state markov-chain approximations to univariate and vector autoregressions', *Economics Letters* **20**(2), 177–181.

Uribe, M. and Schmitt-Grohe, S. (2017), *Open Economy Macroeconomics*, Princeton University Press.

Yeyati, E. L. and Panizza, U. (2011), 'The elusive costs of sovereign defaults', *Journal of Development Economics* **94**(1), 95–105.

Yue, V. Z. (2010), 'Sovereign default and debt renegotiation', *Journal of International Economics* **80**(2), 176–187.

Zettelmeyer, J., Trebesch, C. and Gulati, M. (2013), 'The Greek debt restructuring: an autopsy', *Economic Policy* **28**(75), 513–563.

A Appendix: Euler Equation

A.1 Strategic Default Model

The first order condition for a sovereign in good standing, from equation (5), is

$$\frac{\partial u(c)}{\partial c}\frac{\partial qD'}{\partial D'} - \beta \int_{y'=\hat{y}'}^{\bar{y}} \frac{\partial v^r(D', y')}{\partial D'} f(y'|y)\, dy' = 0,$$

where the effect of D' on $v^d(y)$ is missing since $v^d(y)$ does not depend on D'. Substituting from the envelope condition, given by

$$\frac{\partial v^r(D, y)}{\partial D} = \frac{\partial u(c)}{\partial c},$$

yields the Euler equation for the SD model as

$$\frac{\partial u(c)}{\partial c}\frac{\partial qD'}{\partial D'} - \beta \int_{y'=\hat{y}'}^{\bar{y}} \frac{\partial u(c')}{\partial c'} f(y'|y)\, dy' = 0.$$

A.2 Excusable Default Model

The first order condition from equation (12) is

$$\frac{\partial u(c)}{\partial c}\frac{\partial qD'}{\partial D'} - \beta \int_{y'=\hat{y}'}^{\bar{y}} \frac{\partial v(D', y')}{\partial D'} f(y'|y)\, dy' = 0.$$

Using the envelope theorem yields the Euler equation as

$$\frac{\partial u(c)}{\partial c}\frac{\partial qD'}{\partial D'} - \beta \int_{y'=\hat{y}'}^{\bar{y}} \frac{\partial u(c')}{\partial c'} f(y'|y)\, dy' = 0.$$

B Appendix: Data

Interest rate spreads

The interest rate spreads on bonds with a ten-year residual maturity are obtained from FRED. We use interest rates on Greek and German ten-year government bonds. We use monthly series and compute the spread based on the final month in the quarter. The interest rate spreads for Austria and Finland are computed in the same way.

Balance of Payments related Variables

The quarterly real GDP and consumption as well as the trade balance data are obtained from the OECD (VPVOBARSA: US dollars, volume estimates, fixed PPPs, OECD reference year, annual levels, seasonally adjusted). The detrended real GDP data are defined as the difference between log real GDP and a linear time trend. The same method is applied for detrended consumption data. We obtain standardized measures of output by taking the exponential of our demeaned and detrended data.

Debt

The quarterly public debt as a percentage of GDP annually is obtained from BIS Statistics Warehouse (F5: Total credit to the government sector (core debt)). Since debt data is an end-of-period measure and in the model, debt is measured at the beginning of the period, we adjust the data dates by one period. The debt variable in our model is debt relative to mean output as described in Section 3.3.