Two Equivalent Conditions

The traditional theory of present value puts forward two equivalent conditions for asset-market equilibrium:

Rate of Return The expected rate of return on an asset equals the market interest rate;

Present Value The asset price equals the present value of expected future payments.

We explain these two conditions and show that they are equivalent—either condition implies the other.

Market Interest Rate

The rate-of-return condition says just that all assets share a common expected rate of return. The "market interest rate" refers to the expected rate of return common to all assets.

We assume that the market interest rate $R > 0$ is constant.

Notation

Consider an asset with payment $\$_t$ at time t. For a stock, the payment would be the dividend. For a bond, the payment would be interest or principal.

Let P_t denote the asset price at time t.

Expected Rate of Return

Definition 1 (Return) *The return is the profit divided by the amount invested.*

Definition 2 (Expected Rate of Return) *The expected rate of return is the expected return divided by the length of the time period.*

Disequilibrium

If the expected rate of return were greater than the market interest rate, the security would be seen as a "good buy." Investors would like to buy the security; those holding the security would not want to sell it. Demand would exceed supply. The reverse inequality would lead to excess supply.

Present Value

Definition 3 (Present Value) *The present value of a payment to be received in the future is the dollars attainable now by borrowing against the future payment.*

Definition 4 (Discount Factor) *The present value is the future payment multiplied by the discount factor.*

With compound interest, a dollar borrowed at time 0 will require a repayment of $(1+R)^t$ at time t, the principal plus interest.

Theorem 5 (Present Value) *The present value at time* 0 *of* $\$_t$ *dollars at time t is*

$$\frac{\$_t}{(1+R)^t}$$

dollars. The discount factor is $1/(1+R)^t$.

The present-value equilibrium condition asserts that the asset price at time 0 equals the present value of expected payments,

$$P_0 = \frac{\mathrm{E}_0(\$_1)}{1+R} + \frac{\mathrm{E}_0(\$_2)}{(1+R)^2} + \frac{\mathrm{E}_0(\$_3)}{(1+R)^3} + \cdots.$$

Simple Example of Equivalence

Consider an asset paying P_1 at time 1 and paying nothing at other times. Suppose that the interest rate is R. What would be a fair price P_0 to pay for the asset at time 0?

Rate-of-Return Condition

Using the rate-of-return condition, what would be a fair price P_0 to pay for the asset at time 0? Setting the rate of return equal to the market interest rate gives

$$\frac{P_1 - P_0}{P_0} = R;$$

the profit is the capital gain. Solving for the price gives

$$P_0 = \frac{P_1}{1+R}. \tag{1}$$

Present-Value Condition

For this asset, the present-value condition says that the market price equals the present value of expected payments,

$$P_0 = \frac{P_1}{1+R}.$$

But this condition is identical to (1), obtained from the rate-of-return condition.

Equivalence

In general the present-value condition implies the rate-of-return condition.

As

$$P_t = \frac{\mathrm{E}_t(\$_{t+1})}{1+R} + \frac{\mathrm{E}_t(\$_{t+2})}{(1+R)^2} + \frac{\mathrm{E}_t(\$_{t+3})}{(1+R)^3} + \cdots,$$

also

$$P_{t+1} = \frac{\mathrm{E}_{t+1}(\$_{t+2})}{1+R} + \frac{\mathrm{E}_{t+1}(\$_{t+3})}{(1+R)^2} + \cdots.$$

As the present value is discounted to time $t+1$ rather than time t, the exponent on each term is less by one.

Thus

$$\begin{aligned}
\mathrm{E}_t\left(P_{t+1}\right) &= \mathrm{E}_t\left[\frac{\mathrm{E}_{t+1}\left(\$_{t+2}\right)}{1+R}\right]+\mathrm{E}_t\left[\frac{\mathrm{E}_{t+1}\left(\$_{t+3}\right)}{(1+R)^2}\right]+\cdots \\
&= \frac{\mathrm{E}_t\left(\$_{t+2}\right)}{1+R}+\frac{\mathrm{E}_t\left(\$_{t+3}\right)}{(1+R)^2}+\cdots \\
&= (1+R)\,P_t-\mathrm{E}_t\left(\$_{t+1}\right).
\end{aligned}$$

since $\mathrm{E}_t\left[\mathrm{E}_{t+1}\left(\cdot\right)\right]=\mathrm{E}_t\left(\cdot\right)$, for any random variable.

Expected Rate of Return

Therefore the expected rate of return

$$\begin{aligned}
&\frac{\mathrm{E}_t\left(\$_{t+1}\right)+\left[\mathrm{E}_t\left(P_{t+1}\right)-P_t\right]}{P_t} \\
&= \frac{\mathrm{E}_t\left(\$_{t+1}\right)+\left[(1+R)\,P_t-\mathrm{E}_t\left(\$_{t+1}\right)\right]-P_t}{P_t} \\
&= R,
\end{aligned}$$

the market interest rate.

General Equivalence

One can work backwards to show that the rate-of-return condition implies the present-value condition.

In general, the two conditions for equilibrium are equivalent. If the price equals the present value at every moment, then the rate of return equals the market interest rate at every moment; and *vice versa*.

The rate-of-return condition underpins many economic models.

For example, a typical macroeconomic model contains only a single interest rate, since by assumption all assets yield the market interest rate.