Erratic Price, Smooth Dividend

Shiller [1] argues that the stock market is inefficient: stock prices fluctuate too much.

According to economic theory, the stock price should equal the present value of expected dividends. However dividends are very stable; they fluctuate very little about an upward trend. Expected dividends should therefore also fluctuate little, and consequently stock prices should be stable.

In fact, stock prices fluctuate wildly.

Variance Bound

Shiller shows how the variability of the dividend sets an upper bound to the variability of the stock price.

Present Value

Let $p_t$ denote the stock price at time $t$, and let $d_t$ denote the dividend during period $t$ (the dividend is actually paid at time $t+1$). According to economic theory, the price is the present value of expected dividends,

$$p_t = \frac{E(d_t)}{1+r} + \frac{E(d_{t+1})}{(1+r)^2} + \cdots.$$  \hfill (1)

Ex Post Rational Price

Define the “ex post rational price” $p_t^*$ as the present value of actual dividends,

$$p_t^* = \frac{d_t}{1+r} + \frac{d_{t+1}}{(1+r)^2} + \cdots.$$  \hfill (2)

Economic theory distinguishes between ex post—meaning “after”—and ex ante—meaning “before.” The ex ante (rational) price is just the actual price, based on the present value of expected dividends.

Standard and Poor’s Composite Stock-Price Index

In figure 1[1, p. 422], the price is the Standard and Poor’s Composite Stock-Price Index for 1871-1979, detrended by an exponential growth factor. The ex post rational price is calculated from the dividend.

Whereas the ex post rational price is stable, the actual price is erratic.

Figure 1: Standard and Poor’s Composite
**Forecast**

By (1) and (2), the price is the forecast of the \textit{ex post} rational price,

\[ p_t = \mathbb{E}(p^*_t). \]

We write

\[ p^*_t = p_t + e_t, \] \hspace{1cm} (3)

in which \( e_t \) denotes the forecast error.

**Rational Forecasting**

Consider a scatter diagram of the \textit{ex post} rational price plotted against the price. If forecasting is rational, the data are scattered randomly about a 45° line from the origin.

**Variance Decomposition**

In (3), \( p_t \) and \( e_t \) are uncorrelated. Consequently

\[ \text{Var}(p^*_t) = \text{Var}(p_t) + \text{Var}(e_t), \] \hspace{1cm} (4)

since the variance of the sum of two uncorrelated random variables is the sum of the variances. Expression (4) decomposes the variance of the \textit{ex post} rational price into the variance of the forecast and the variance of the error.

**Variance Bound**

The inequality

\[ \text{Var}(p^*_t) \geq \text{Var}(p_t) \]

is a variance bound. The variance of the \textit{ex post} rational price is an upper limit (upper bound) on the variance of the price.

**Variance-Bound Test**

Unfortunately the variance bound is violated. Figure 1 shows that the variance of the price is \textit{much} larger than the variance of the \textit{ex post} rational price.

Hence the stock market is inefficient.
**Dividend Yield**

This inefficiency means that one can forecast the rate-of-return on stocks from the dividend yield (the dividend/price ratio). A profitable trading rule is to buy when the dividend yield is high (because the price is then too low) and to sell when the dividend yield is low (because the price is then too high).

**Long-Run Autocorrelation**

For some investment, let $x_t$ denote the total excess holding-period return from time $t$ to time $t + T$. If the market is efficient, the correlation of $x_t$ and $x_{t-T}$ must be zero. Many successful tests of the random-walk theory test for a zero correlation when $T$ is short, such as a day or a week.

**Long Period**

When $T$ is long (such as five years), however, one finds a negative correlation. Thus a low return tends to be followed by a high return, and vice versa.

**Statistical Problem**

This negative correlation is not, however, statistically significantly different from zero, because the number of independent observations is small. For Shiller’s data period 1871-1979, the 5-year time period means that the 109 years of data amount to only 22 independent observations. With so few observations, one cannot reject a zero correlation with any confidence.

It is possible to show that this same problem also afflicts the forecasting of the rate of return from the dividend yield.

**Influence of the Great Depression**

In the Great Depression, stock prices fell greatly but ultimately bounced back. This price movement was so great that this single episode dominates statistical testing. Omitting the period of the Great Depression eliminates the negative correlation of long-run returns.

**References**