Long-Run Economic Growth

The Solow growth model focuses on long-run economic growth.

Saving and Investment

A key component of economic growth is saving and investment.

An increase in saving and investment raises the capital stock and thus raises the full-employment national income and product.

The national income and product rises, and the rate of growth of national income and product increases.

Higher Saving and Investment

An interest of economic policymakers is how to increase saving and investment.

In the short run, higher saving and investment raises the rate of growth of national income and product.

Short Run Versus Long Run

Solow analyzes how higher saving and investment affects long-run economic growth.

In the short run, higher saving and investment does increase the rate of growth of national income and product in the short run.

According to the Solow growth model, in contrast, higher saving and investment has no effect on the rate of growth in the long run.

Solow Growth Model

Solow sets up a mathematical model of long-run economic growth. He assumes full employment of capital and labor. Given assumptions about population growth, saving, technology, he works out what happens as time passes.

The Solow model is consistent with the stylized facts of economic growth.

Constant Population Growth

The labor force L (the population) grows at a constant rate n:

$$\frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}t} = n.$$

For example, $n = .03$ would mean that the population grows 3% per year.

Investment

Net investment I is the change in capital K,

$$I = \frac{\mathrm{d}K}{\mathrm{d}t}.$$

Saving

That saving S equals investment is an accounting identity.

Saving is a constant fraction s of national income Y,

$$S = sY.$$

As an accounting identity, national income equals national product.

Aggregate Production Function

Net national product Y is a function of capital K and labor L,

$$Y = F(K, L).$$

This aggregate production function is fixed; how the product depends on capital and labor does not change as time passes.

Consumption

Consumption C is national income less saving; equivalently consumption is national product less investment:

$$C = Y - S = Y - I.$$

Full Employment

There is full employment. All capital and labor are utilized in production.

Constant Returns to Scale

The production function exhibits constant returns to scale; doubling the capital and labor doubles the output.

Intensive Form

It is useful to express concepts in *per capita* terms; a lower-case letter denotes the *per capita* amount. An expression in *intensive form* is one relating *per capita* amounts.

National income and product *per capita* is

$$y = \frac{Y}{L}.$$

Capital *per capita* is the capital/labor ratio,

$$k = \frac{K}{L}.$$

Consumption *per capita* is

$$c = \frac{C}{L}.$$

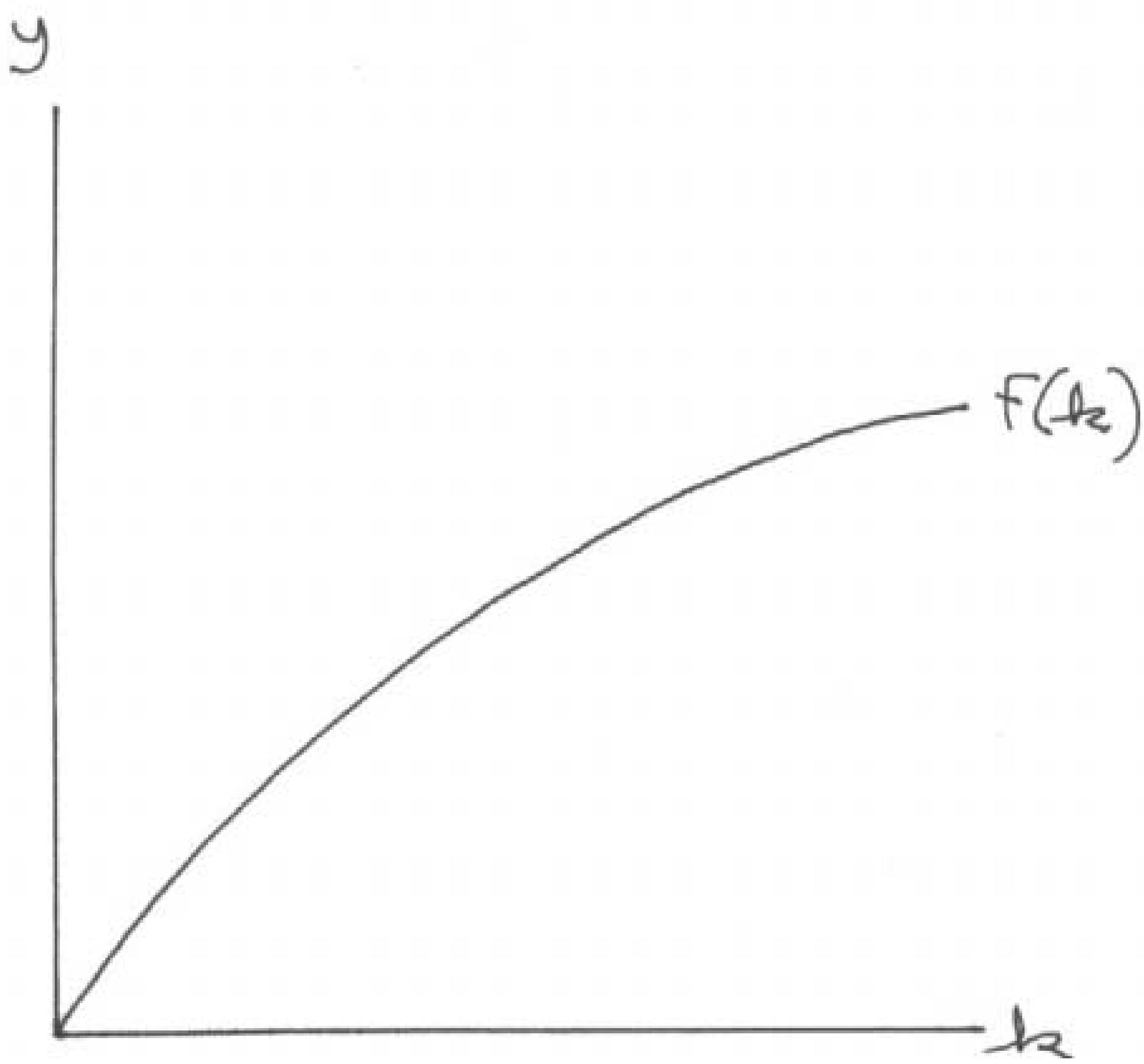
Intensive Production Function

Because returns to scale are constant, output *per capita* can be expressed as a function of the capital/labor ratio,

$$y = f(k).$$

Here $f(k)$ is an increasing function of k (figure 1). By the law of diminishing marginal returns, its slope declines as k rises.

Figure 1: Intensive Production Function



In mathematics,

$$y = \frac{Y}{L} = \frac{F(K,L)}{L} = F\left(\frac{K}{L},1\right) := f\left(\frac{K}{L}\right) = f(k).$$

Here the third equals sign follows from constant returns to scale.

Capital Deepening and Capital Widening

Capital accumulation can be decomposed into *capital deepening* and *capital widening*. Capital deepening is increasing the amount of capital per worker. Capital widening is the equipping of new workers with capital, as the population grows.

The decomposition

$$K = \left(\frac{K}{L}\right) L = kL, \tag{1}$$

expresses capital as the product of the capital/labor ratio k and labor L. Capital accumulation $I = \mathrm{d}K/\mathrm{d}t$ must affect either k or L.

Capital widening refers to the capital accumulation required to keep k constant as L grows. Capital deepening is the capital accumulation that permits k to grow.

Differentiating the decomposition (1) with respect to time, we have

$$\frac{\mathrm{d}K}{\mathrm{d}t} = L\frac{\mathrm{d}k}{\mathrm{d}t} + k\frac{\mathrm{d}L}{\mathrm{d}t} \tag{2}$$

Here total capital accumulation $\mathrm{d}K/\mathrm{d}t$ is the capital deepening $L\,\mathrm{d}k/\mathrm{d}t$ plus the capital widening $k\,\mathrm{d}L/\mathrm{d}t$.

Numerical Example

Consider a numerical example:

$$I = \frac{\mathrm{d}K}{\mathrm{d}t} = 400$$

$$K = 1000$$

$$L = 100$$

$$n = .10.$$

Hence the capital/labor ratio

$$k = \frac{K}{L} = \frac{1000}{100} = 10.$$

The increase in labor is the growth rate of labor times total labor,

$$\frac{\mathrm{d}L}{\mathrm{d}t} = nL = (.10)\,(100) = 10,$$

so capital widening is

$$k\frac{\mathrm{d}L}{\mathrm{d}t} = 10 \times 10 = 100.$$

To equip 10 new workers with capital requires 100 of investment.

Hence the capital deepening must be

$$L\frac{\mathrm{d}k}{\mathrm{d}t} = \frac{\mathrm{d}K}{\mathrm{d}t} - k\frac{\mathrm{d}L}{\mathrm{d}t}$$
$$= 400 - 100$$
$$= 300.$$

Here 300 of investment is used to raise the capital/labor ratio. It must be that

$$\frac{\mathrm{d}k}{\mathrm{d}t} = \frac{300}{100} = 3;$$

the capital/labor ratio is increasing by three per year.

Express the capital deepening and widening *per capita*, by dividing (2) by L,

$$\frac{1}{L}\frac{\mathrm{d}K}{\mathrm{d}t} = \frac{\mathrm{d}k}{\mathrm{d}t} + k\left(\frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}t}\right) = \frac{\mathrm{d}k}{\mathrm{d}t} + kn. \qquad (3)$$

In equation (3), the left-hand side is saving and investment *per capita*:

$$\frac{1}{L}\frac{\mathrm{d}K}{\mathrm{d}t} = \frac{sY}{L} = sy = sf\left(k\right). \qquad (4)$$

Capital Deepening and Capital Widening: Intensive Form

Combining (3) and (4) yields the intensive form

$$s f(k) = \frac{\mathrm{d}k}{\mathrm{d}t} + kn; \tag{5}$$

here saving *per capita* $s f(k)$ equals capital deepening *per capita* $\mathrm{d}k/\mathrm{d}t$ plus capital widening *per capita* kn.

Numerical Example

We continue with the example above. Saving *per capita*

$$sf(k) = \frac{S}{L} = \frac{400}{100} = 4.$$

Capital widening *per capita*

$$kn = 10 \times .10 = 1.$$

Hence capital deepening *per capita* is

$$\frac{\mathrm{d}k}{\mathrm{d}t} = 4 - 1 = 3.$$

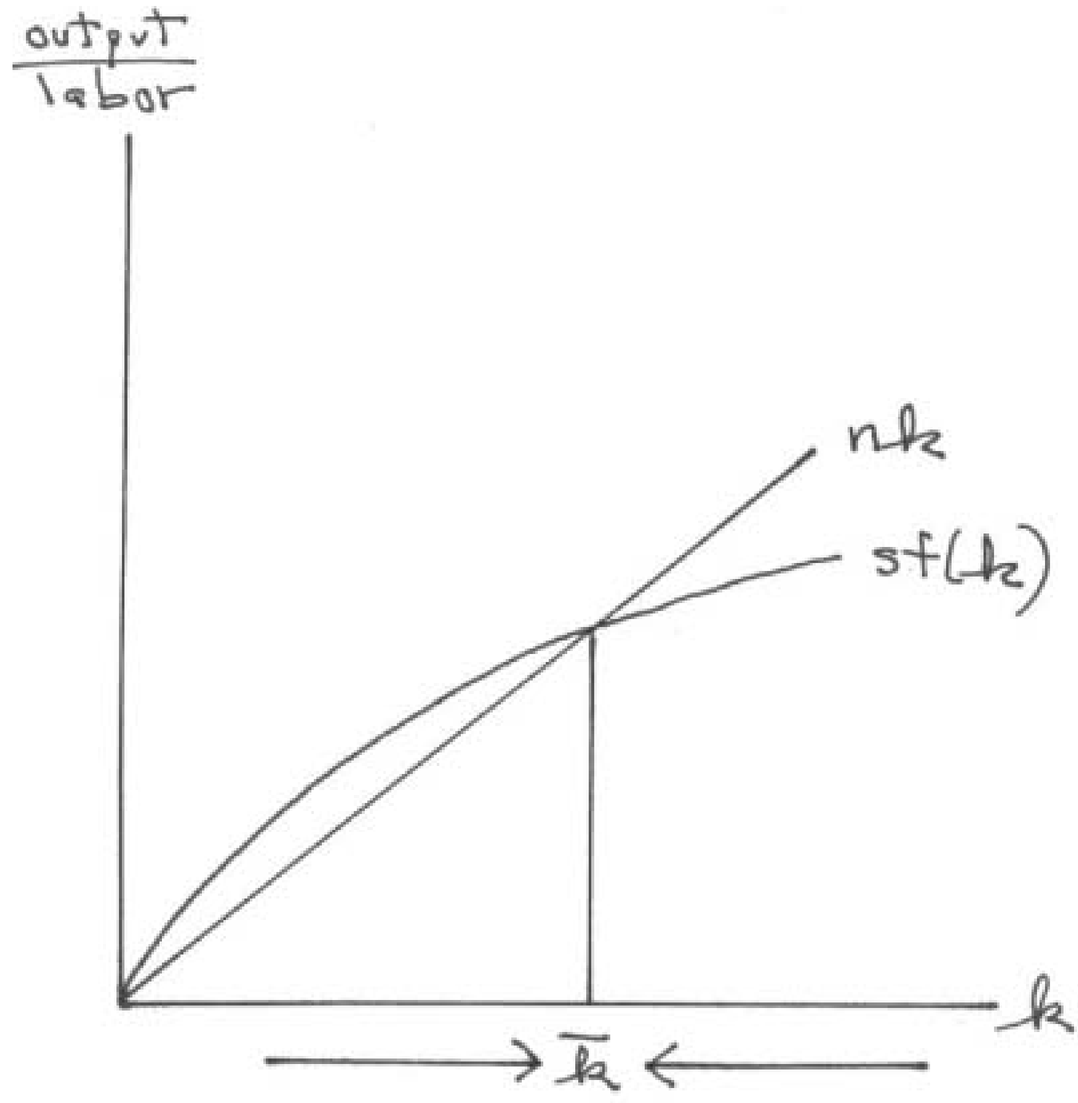
Dynamics

Rearranging (5) gives

$$\frac{\mathrm{d}k}{\mathrm{d}t} = sf(k) - nk.$$

Figure 2 graphs this equation. The curved line is saving *per capita* $sf(k)$, and the straight line is capital widening *per capita* nk. The vertical difference between them is the capital deepening *per capita* $\mathrm{d}k/\mathrm{d}t$.

Figure 2: Long-Run Steady State



For low k, then k increases, rising because there is more than enough saving to equip new workers with capital. Conversely, for high k, then k falls.

In the long run, the capital/labor ratio converges to $\bar{k}$. Saving is just sufficient for capital widening, and there is no investment left over for capital deepening.

Long-Run Steady State

In the long run, there is steady-state economic growth.

Since the capital/labor ratio is constant at $\bar{k}$. As labor grows at rate n, necessarily K grows at rate n. Because returns to scale are constant, national income and product Y, saving and investment $S = I$, and consumption C all grow at rate n.

Income and product *per capita* y and consumption *per capita* c are constant.

Real Interest Rate and Real Wage

If the economy is a competitive market economy, the real interest rate is the marginal product of capital; and the real wage is the marginal product of labor.

Since the capital/labor is constant in the long-run steady state, the marginal products of capital and labor are constant. Hence the real interest rate and the real wage are constant.

A Change in Population Growth

The rate of population growth sets the long-run growth rate of the economy.

If the population growth rate n rises, the capital-widening term nk rises.

Consequently the steady-state capital/labor ratio $\bar{k}$ falls.

Hence the steady-state output *per capita* falls. In the steady state, the real interest rate is now higher, and the real wage is lower.

A Change in the Saving Rate

Although the saving rate *s* does raise the rate of economic growth in the short run, it has no effect on the rate of growth in the long run.

A higher value *s* does raise the steady-state capital/labor ratio $\bar{k}$. Hence the steady-state output *per capita* rises. In the steady state, the real interest rate is now lower, and the real wage is higher.