Optimum Saving

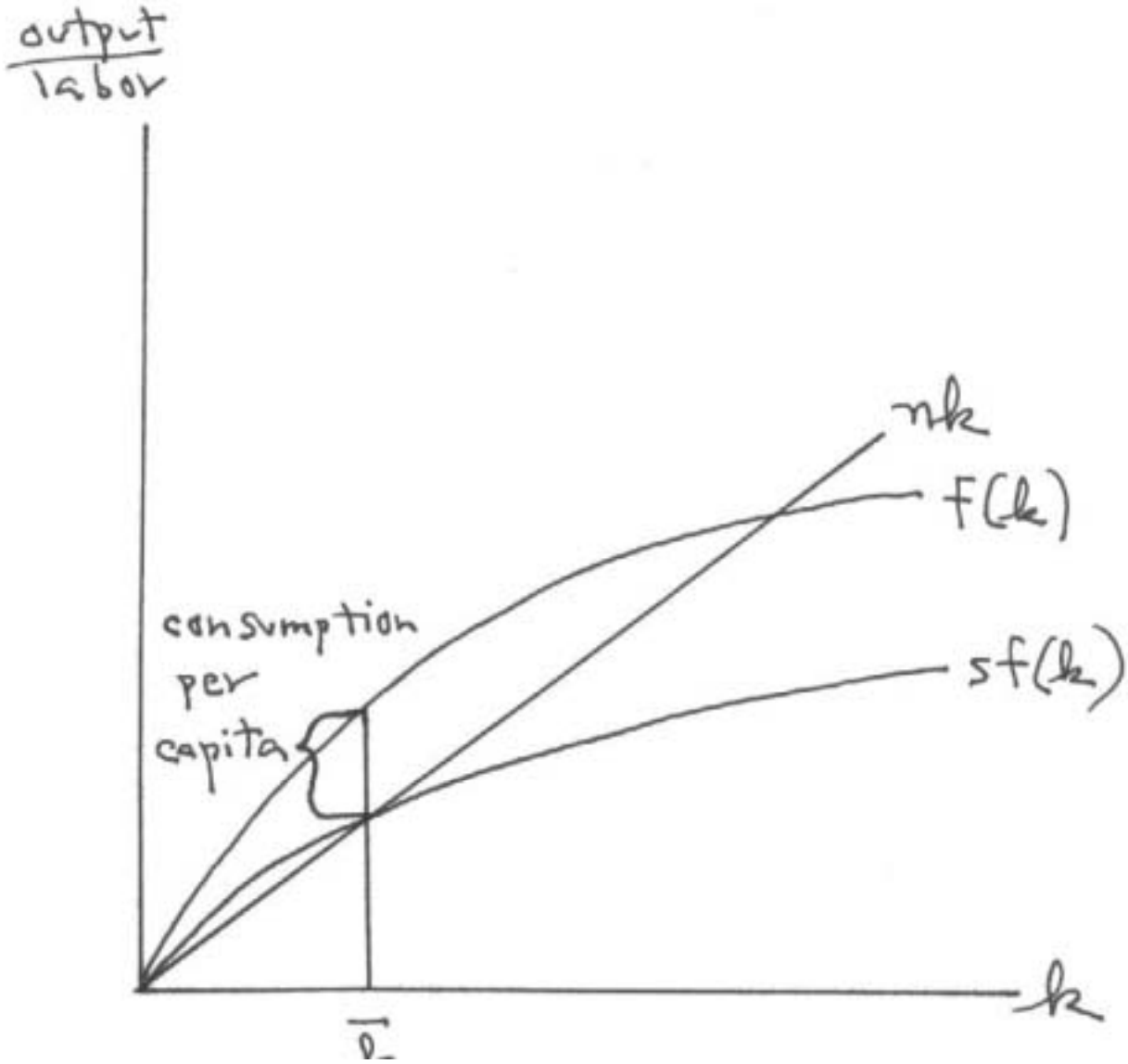
In the Solow growth model, is there an optimum saving rate?

An approach to optimum saving is to find the saving rate that maximizes consumption *per capita* in the steady state.

Steady-State Consumption

Given a rate of saving, the economy converges to steady-state growth in the long run (figure 1). In the steady state, consumption *per capita* is constant.

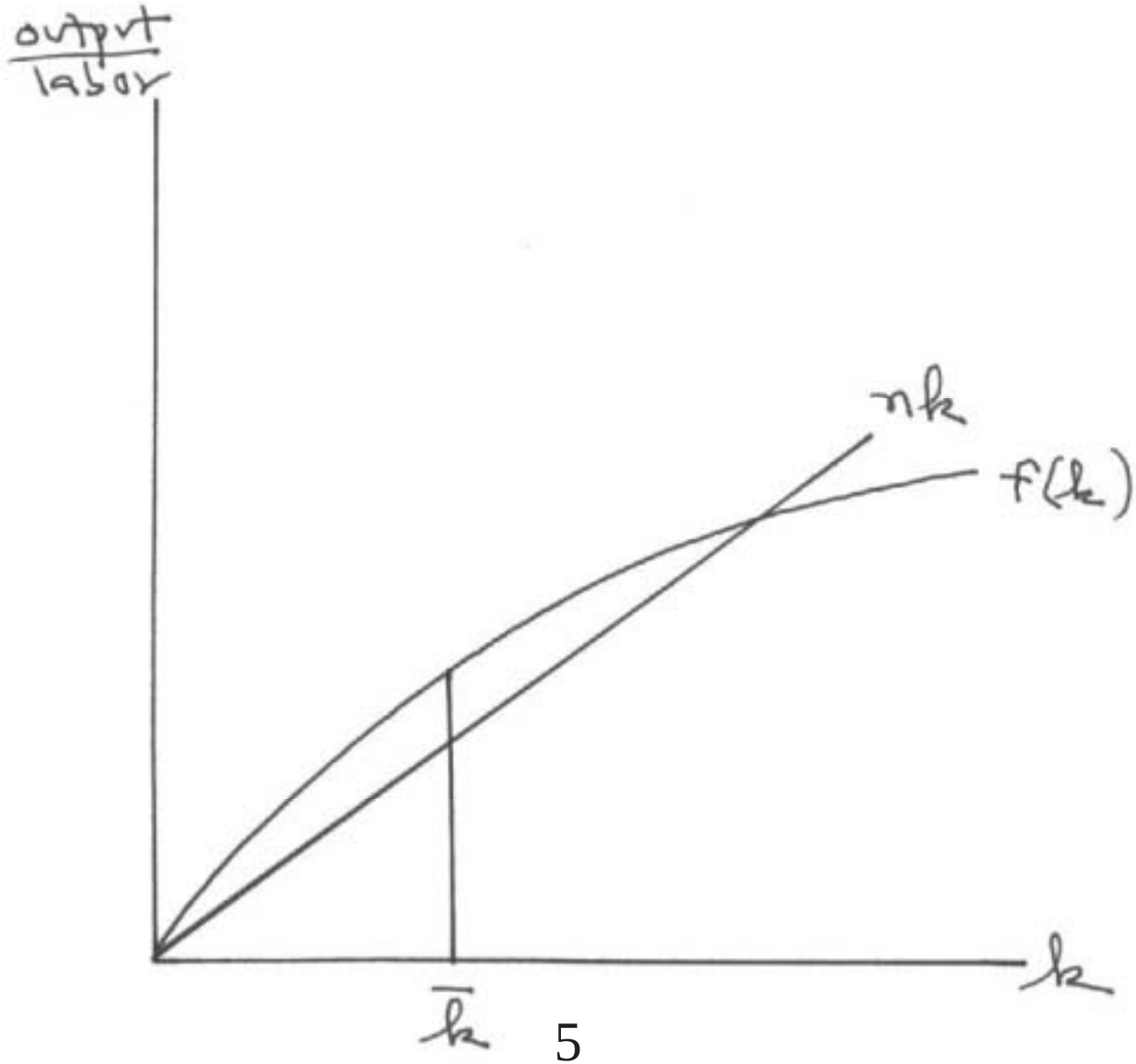
Figure 1: Steady-State Consumption



Maximum Consumption *Per Capita*

To maximize consumption per capita in the long-run steady state, choose the saving rate to make the steady state where the vertical distance between the capital-widening ray nk and the intensive production function $f(k)$ is largest (figure 2).

Figure 2: Golden Rule



Golden Rule of Capital Accumulation

This saving rate is the “golden-rule” saving rate.

Lower Saving Rate

A lower saving rate would reduce long-run steady-state consumption *per capita*, but would imply higher consumption in the short run. Thus the choice of the saving rate is a trade-off of current consumption for future consumption.

Maxim

The name “golden rule” reflects the maxim of Jesus, “do unto others as you would have them do unto you”: the current generation saves at the high golden-rule rate, which benefits the future generation.

One need not see the golden rule of saving as the optimum: consumption in the long run is not all that matters.

Inefficiency

A saving rate higher than the golden-rule saving rate is inefficient. The long-run steady-state consumption per capita is reduced below the golden-rule value, but in addition consumption is lower in the short run.

Equal Slopes

Equivalently, the golden-rule of saving is to make the steady state where the capital-widening ray nk and the intensive production function $f(k)$ have the same slope.

The slope of the capital-widening ray is n. The slope of the intensive production is the marginal product of capital MP_K. In market equilibrium, the marginal product of capital equals the real interest rate r.

Under the golden-rule of saving,

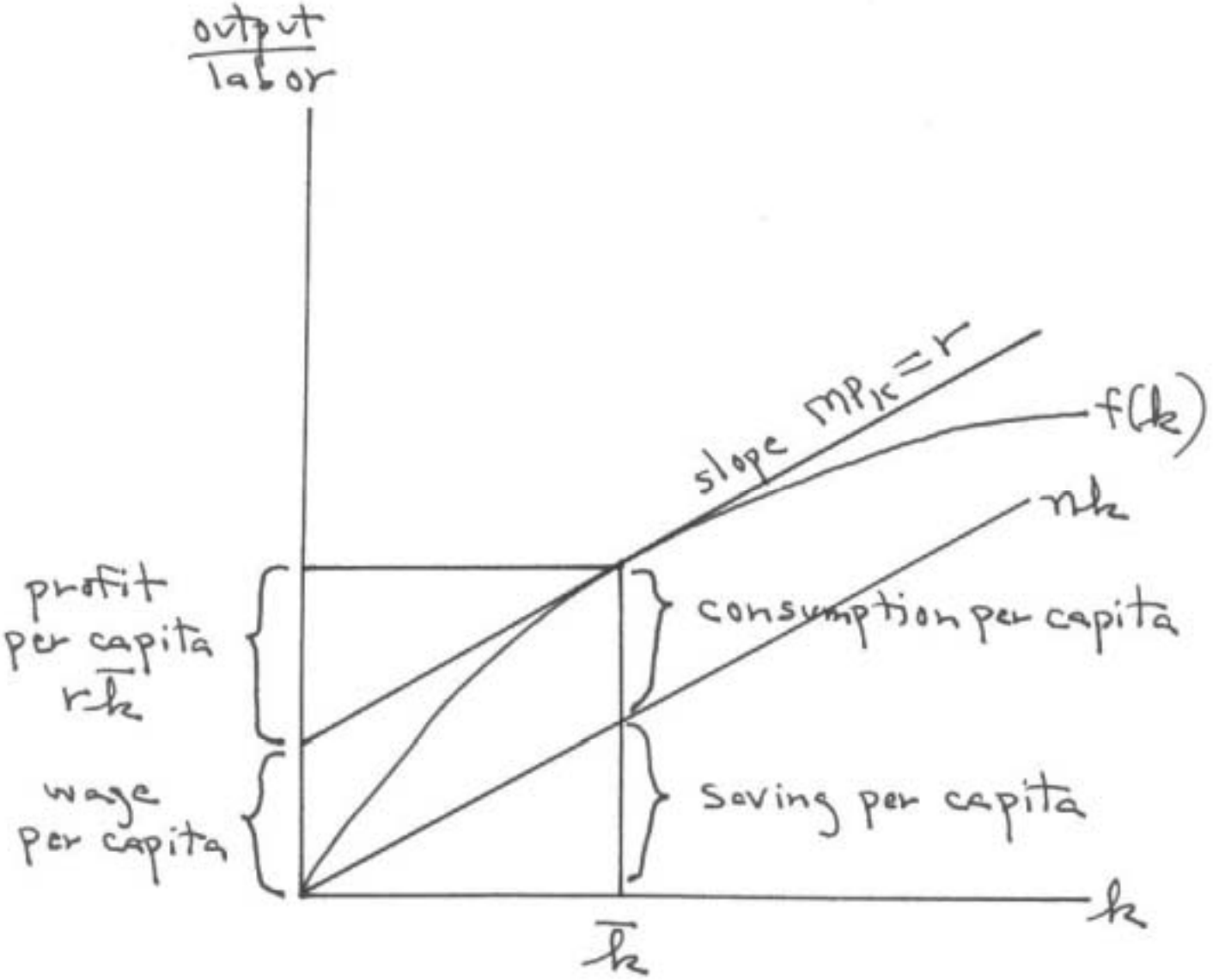
$$r = n;$$

the real interest rate equals the rate of population growth.

Profit *Per Capita*

Under the golden-rule of saving, the upper-left vertical segment in figure 3 shows the income to capital *per capita* (the profit *per capita*). The vertical distance is the slope r times the capital/labor ratio k.

Figure 3: Saving Equals Profit



Saving Equals Profit

In figure 3, the capital-widening ray is parallel to the line tangent to the intensive production function.

This parallelism implies that saving *per capita* equals profit *per capita*. Furthermore, consumption *per capita* equals the wage *per capita*. So to invest all profit and to consume all wages leads to the golden-rule of saving in the long-run steady state.

Lower Rate of Saving

A lower rate of saving leads to a long-run steady state with $r > n$. Profit exceeds saving, and consumption exceeds wages.