

## MINI-WORKSHOP: VERTEX ALGEBRAS IN MATHEMATICS AND PHYSICS

Department of Mathematics - University at Albany, SUNY (**Earth Science, Room 147**)  
Saturday April 13- Sunday April 14, 2019

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### Saturday 4/13

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9:00-10:00 **K. Ono**, *Can't you just feel the Moonshine?*  
10:0-10:30 Coffee break (discussion)  
10:30-11:30 **D. Adamović**, *On the representation theory of affine vertex algebras at collapsing level*  
11:30-12:30 **S. Harrison**, *3d Modularity*  
12:30-1:00 Discussion (Vertex Algebras and 4d N=2 SCFT, **T. Creutzig, A.Milas,...**)  
1:00-3:00 Lunch break  
3:00-3:30 **M. Rapčak**, *Different Construction of the Corner VOAs*  
3:30-4:00 **B. Williams**,  *$L_\infty$  conformal algebras*  
4:00-4:30 Coffee break (discussion)  
4:30-5:00 **S. Kanade**, *Universal two-parameter even spin  $W_\infty$  algebra*

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### Sunday 4/14

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8:30-9:30 **T. Creutzig**, *Stable equivalence of vertex algebras*  
9:30-10:30 **Y.Z. Huang**, *A construction of twisted modules for grading-restricted vertex (super)algebras*  
10:30-11:00 Coffee break (discussion)  
11:00-11:30 Discussion (Logarithmic Tensor Product Theory, **J. Lepowsky**)  
11:30-12:00 **R. McRae**, *Commuting actions on vertex operator algebras from a tensor category perspective*  
12:00-12:30 **C. Nazaroglu**, *A Modular Completion for False Theta Functions*  
12:30- 1:00 **C. Calinescu**, *Algebraic and combinatorial properties of principal subspaces of higher level standard  $A_2^{(2)}$ -modules*  
1:00- Lunch and departure of participants

### Abstracts

Dražen Adamović (University of Zagreb), On the representation theory of affine vertex algebras at collapsing level

**Abstract:** In the first part of the talk we recall recent results on affine vertex algebras and minimal affine  $W$ -algebras. A level  $k$  is called collapsing when the simple minimal affine  $W$ -algebra at level  $k$  associated with simple Lie superalgebra is isomorphic to its affine vertex subalgebra. We present a proof of complete reducibility of affine vertex algebras at these levels which uses the representation theory of affine algebras and concepts in conformal embeddings (joint project with V. Kac, P. Moseneder-Frajria, P. Papi and O. Perše). The existence of indecomposable and logarithmic modules will be also discussed.

Ken Ono (Emory and University of Virginia), Can't you just feel the Moonshine?

**Abstract:** Richard Borcherds won the Fields medal in 1998 for his proof of the Monstrous Moonshine Conjecture. Loosely speaking, the conjecture asserts that the representation theory of the Monster, the largest sporadic finite simple group, is dictated by the Fourier expansions of a distinguished set of modular functions. This conjecture arose from astonishing coincidences noticed by finite group theorists and arithmetic geometers in the 1970s. Recently, mathematical physicists have revisited moonshine, and they discovered evidence of undiscovered moonshine which some believe will have applications to string theory and 3d quantum gravity. The speaker and his collaborators have been developing the mathematical facets of this theory, and have proved the conjectures which have been formulated. These results include a proof of the Umbral Moonshine Conjecture, and Moonshine for the first sporadic finite simple group which does not occur as a subgroup or subquotient of the Monster. The most recent Moonshine yields unexpected applications to the arithmetic elliptic curves thanks to theorems related to the Birch and Swinnerton-Dyer Conjecture and the Main Conjectures of Iwasawa theory for modular forms. This is joint work with John Duncan, Michael Griffin and Michael Mertens.

Thomas Creutzig (University of Alberta), Stable equivalence of vertex algebras

**Abstract:** A natural notion of stable equivalence of vertex algebras will be introduced, motivated and illustrated.

Sarah Harrison (McGill University), 3d Modularity

**Abstract:** I will discuss a variety of modularity-related symmetries in problems of 3-manifold topology and physics of 3d  $N=2$  theories where such structures a priori are not manifest. These modular structures include: mock modular forms,  $SL(2, \mathbb{Z})$  Weil representations, quantum modular forms, and chiral algebras of logarithmic CFTs.

Robert McRae (Vanderbilt), Commuting actions on vertex operator algebras from a tensor category perspective

**Abstract:** Several recent results suggest that when a commuting pair of algebras acts on a simple vertex operator algebra, one should obtain tensor equivalences, with braidings reversed, between certain module categories for the commuting algebras. The commuting actions could come from an automorphism group and the vertex operator subalgebra of fixed points, or from a vertex operator subalgebra and its commutant, the coset vertex operator subalgebra. Under suitable conditions, the tensor categories of modules for the commuting algebras that are generated by the irreducible modules occurring in the larger vertex operator algebra will be equivalent. I will discuss some of these results and conditions and give some applications in the case of automorphism groups and fixed-point subalgebras. Some of the results discussed are from joint work with Thomas Creutzig and Shashank Kanade.

Yi-Zhi Huang (Rutgers), A construction of twisted modules for grading-restricted vertex (super)algebras

**Abstract:** We give a general, direct and explicit construction of (generalized) twisted modules for a grading-restricted vertex (super)algebra  $V$  associated to an automorphism  $g$  of  $V$ . Even in the case that  $g$  is of finite order, finding such a construction has been a long-standing problem in the representation theory of vertex operator algebra and orbifold conformal field theory. Besides twisted vertex operators, one crucial ingredient in this construction is what we call the "twist vertex operators" or "twist fields." Assuming that a graded vector space  $W$  equipped with a set of twisted fields and a set of twist fields satisfy a weak commutativity for twisted fields, a generalized weak commutativity for one twisted field and one twist field and a number of other properties that are relatively easy to verify, we define a twisted vertex operator map for  $W$  and prove that  $W$  equipped with this twisted vertex operator map is a (generalized)  $g$ -twisted  $V$ -module. We also give an explicit construction of such a space  $W$ , a set of twisted fields and a set of twist fields and thus an explicit construction of a (generalized)  $g$ -twisted  $V$ -module satisfying a universal property.

Shashank Kanade (Denver University), *Universal two-parameter even spin  $W_\infty$  algebra*

**Abstract:** In this talk, I will explain the construction of a vertex algebra over the ring of polynomials in two variables that is freely generated in fields of weights  $2, 4, 6, 8, \dots$  and generated by weight 2 and 4 fields. I will mainly focus on the computational aspects. Many interesting families of VOAs are related to this universal even spin algebra. Time permitting, I will explain how a careful analysis of the parameters leads to interesting coincidental isomorphisms between well-known vertex operator algebras. This is a joint

work with A. Linshaw and builds upon his previous work on the universal two parameter  $W_\infty$  algebra freely generated by fields of weights 2,3,... and generated by weight 2 and 3 fields.

Miroslav Rapčák (Perimeter Institute) , *Different Construction of the Corner VOAs*

**Abstract:** I will discuss a large class of vertex operator algebras appearing naturally in two (mutually dual) 4d gauge theory setups. First, they can be identified with algebras of local operators at junctions of interfaces in maximally supersymmetric Yang-Mills theory. Secondly, they are algebras corresponding to spiked instanton configurations associated to divisors in Calabi-Yau three-folds recently introduced by Nekrasov. After sketching the two configurations for motivational purposes, I will review four conjecturally equivalent construction of the algebras in terms of (1) quantum Hamiltonian reductions (2) truncations of the  $W_\infty$  algebra (3) kernel of screening charges and (4) generalized Miura transformation.

Corina Calinescu (CUNY), Algebraic and combinatorial properties of principal subspaces of higher level standard  $A_2^{(2)}$ -modules

**Abstract:** In this talk we discuss presentations and graded dimensions of the principal subspaces of level  $k$  standard modules for  $A_2^{(2)}$ . As a consequence of the presentations, we obtain a set of recursions satisfied by the graded dimensions of the principal subspaces. Although this is not a complete system to allow us to solve for the graded dimensions, we conjecture a formula for a specialized graded dimension, given by the Nahm sum of the inverse of the tadpole Cartan matrix. When  $k$  is even, this graded dimension is related to Gollnitz-Gordon-Andrews identities. This talk is based on joint work with Michael Penn and Chris Sadowski.

Caner Nazaroglu (Cologne), A Modular Completion for False Theta Functions

**Abstract:** False theta functions are functions that closely resemble theta functions, which despite this similarity do not have the modular properties that theta functions possess. They appear, for example, in the context of link invariants and  $W$ -algebras and also are closely related to mock modular forms. This last relation and the recent development of the theory of mock modular forms has led to a number of interesting results on false theta functions, especially on the real line where they give examples of quantum modular forms. In this talk, I will describe a modular completion for false theta functions allowing us to efficiently go beyond asymptotic results near the real line and work out examples where this completion is used as a tool both to reproduce some known results in the literature and also to obtain new results.

Brian Williams (Northeastern),  $L_\infty$  conformal algebras

**Abstract:** Lie conformal algebras are to vertex algebras as Lie algebras are to ordinary algebras. Moreover, many vertex algebras arise as deformations of universal vertex algebras associated to Lie conformal algebras. Our starting point is a formulation of a multivariable generalization of Lie conformal algebras, with a goal of studying higher dimensional vertex algebras. Naively, a drawback is in the lack of existence of nontrivial examples. Motivated by a class of symmetries in higher dimensional QFT, we present a derived version of Lie conformal algebras that we argue captures a wide class of familiar examples ported to this higher dimensional situation. We will discuss future work in formulating the definition of a “multivariable vertex algebra”, for which L-infinity conformal algebras provide a window into studying.