Macroeconomics I

Solution to Part 1 of Assignment 2

(a) Households solve

\[
\max_{c_1, c_2, s} c_1^{\frac{1}{2}} c_2^{\frac{1}{2}}
\]

s.t. \[c_1 = e_1 - s\]
\[c_2 = e_2 + Rs\]

or

\[
\max_{s} (e_1 - s)^{\frac{1}{2}} (e_2 + Rs)^{\frac{1}{2}}
\]

F.O.C. implies

\[e_2 + Rs = R(e_1 - s)\]

Market clearing implies \(s = 0\) so equilibrium interest rate,

\[R^* = \frac{e_2}{e_1}\]

(b) With storage technology, \(\gamma\) households solve

\[
\max_{c_1, c_2, s, k} c_1^{\frac{1}{2}} c_2^{\frac{1}{2}}
\]

s.t. \[c_1 = e_1 - s - k\]
\[c_2 = e_2 + Rs + \gamma k\]

where \(k\) is the amount of the first period good stored. Substituting out \(c_1\) and \(c_2\) the first order conditions for \(s\) and \(k\) respectively yield

\[e_2 + Rs + \gamma k = R(e_1 - s - k)\]
\[e_2 + Rs + \gamma k = \gamma (e_1 - s - k)\]

For \(k = s = 0\), \(\gamma = R^*\) so endowments have to be such that \(\frac{e_2}{e_1} = \gamma\).

Now if new storage technology, \(\tilde{\gamma} > \gamma\) becomes available, market clearing conditions for goods become

\[c_1 = e_1 - k\]
\[c_2 = e_2 + \tilde{\gamma} k\]

Into 1st order conditions yield

\[k = \frac{e_1 (\tilde{\gamma} - \gamma)}{2\tilde{\gamma}}\]