Macroeconomics I: Mid-Term Exam

Infinite horizon alternating endowment model with government and outside money

Time: Discrete, infinite horizon: $t = 0, 1, 2, ...$

Demography: A single representative infinite lived individual/household of each of two types who differ by their endowment stream: type e (for even) and type o (for odd)

Preferences: Discounted lifetime utility is given by,

$$U^i = \sum_{t=0}^{\infty} \beta^t u(c_t^i) \qquad i = e, o$$

where $c_t^i \in \boldsymbol{R}_+$ is period t consumption by household type $i = e, o$ and $\beta \in [0, 1)$ is a constant discount factor. The function $u(.)$ is strictly increasing, strictly concave and $\lim_{c \to 0} u(c) = 0$.

Endowments: Household type e receives y_H units of the perishable consumption good in the even periods, $t = 0, 2, ...$ and $y_L < y_H$ in the odd periods, $t = 1, 3, ...$ The type o household receives y_L in the even periods and y_H in the odd periods.

Institutions: There is a government which has to meet exogenous spending $g < 2y_L$ each period. (Note: g is constant.) It can finance the spending though taxes or by issuing bonds (we will look at both in turn).

There is no inside money. (No enforcement mechanism for individual contracts exists.)

1. Write down and solve the problem of the Planner who weights each household equally. To what extent does the Planner's consumption allocation for each household type vary over time?

$$\max_{\{c_t^e, c_t^o\}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t^e) + u(c_t^o)\right]$$
$$\text{subject to} \quad : \quad y_H + y_L = c_t^e + c_t^o + g$$

Writing out Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{\beta^t \left[u(c_t^e) + u(c_t^o)\right] + \lambda_t \left[y_H + y_L - c_t^e - c_t^o - g\right]\right\}$$

FOCs:

$$c_t^i : \qquad \beta^t u'(c_t^i) - \lambda_t = 0 \qquad i = e, o$$

So $c_t^e = c_t^o$ and from the resource constraint, both are constant. The Planner will smooth their consumption at $(y_H + y_L - g)/2$.

2. What will the allocation look like if the government only uses constant taxes, $\tau = g/2$ levied on both households to cover government spending? (Hint: there are no markets here.)

   Consumption will be $y_H - g/2$ in good periods and $y_L - g/2$ in bad periods. No consumption smoothing!

3. Now consider what happens if the government uses one-period bonds to cover government spending. There will now be a market for bonds. A bond issued in period t to acquire one unit of the consumption good will repay the owner R_{t+1} units of the consumption good in period $t+1$. Let the number of bonds issued in period t be $b_t > y_H - y_L$. No Ponzi schemes will be permitted.

   (a) Write down and solve the problem for either type of household, e or o. (They are basically the same.)

$$\max_{\{c_t^i, s_{t+1}^i\}} \sum_{t=0}^{\infty} \beta^t u(c_t^i)$$
$$\text{subject to} \quad : \quad c_t^i = y_t^i - s_{t+1}^i + R_t s_t$$

   After substituting for c_t^i we obtain the FOC,

$$s_{t+1}^i : \qquad -\beta^t u'(c_t^i) + \beta^{t+1} u'(c_{t+1}^i) R_{t+1} = 0.$$

   So,

$$\beta R_{t+1} = \frac{u'(c_t^i)}{u'(c_{t+1}^i)}$$

   (b) Write down the government budget constraint and the market clearing conditions.

$$\begin{aligned} \text{GBC} \quad &: \quad b_{t+1} = R_t b_t + g \\ \text{Bonds} \quad &: \quad b_t = s_t^o + s_t^e \\ \text{Goods} \quad &: \quad y_H + y_L = c_t^e + c_t^o + g \end{aligned}$$

   (c) Define a competitive equilibrium.
   A competitive equilibrium is an allocation, $\{c_t^e, c_t^o, s_t^o, s_t^e\}$ and a price sequence, $\{R_t\}$ such that:
   (i) Given prices the allocation solves the households' problems
   (ii) Markets clear
   (iii) GBC holds.

   (d) What is the unique value of R_t that is consistent with a steady state equilibrium (indeed with any equilibrium)?
   In equilibrium $R_t = R = 1/\beta$ for all t. This has to be true because it is the only way both household FOC's can hold simultaneously.

   (e) How does the consumption allocation compare with the Planner's solution?
   With $R = 1/\beta$, consumption is constant and from the goods market clearing condition the allocation is the same as the Planner's solution.

4. Comment on the applicability of Ricardian equivalence in this environment. How might your answer change if inside money was available?

   Ricardian equivalence (RE) does not hold. When the government uses bonds it create a market by which consumers can smooth their consumption. That is not possible using taxes. Indeed they have to have enough bonds ($b_t > y_H - y_L$) or there will always be some limitation to consumption smoothing even when they use both taxes and bonds.

   If inside money is introduced, you get perfect capital markets and RE will apply again. With inside money the economy will always achieve a Pareto optimum.