

## Macroeconomics I: Mid-Term Exam

### Infinite horizon alternating endowment model with government and outside money

**Time:** Discrete, infinite horizon:  $t = 0, 1, 2, \dots$

**Demography:** A single representative infinite lived individual/household of each of two types who differ by their endowment stream: type  $e$  (for even) and type  $o$  (for odd)

**Preferences:** Discounted lifetime utility is given by,

$$U^i = \sum_{t=0}^{\infty} \beta^t u(c_t^i) \quad i = e, o$$

where  $c_t^i \in \mathbf{R}_+$  is period  $t$  consumption by household type  $i = e, o$  and  $\beta \in [0, 1)$  is a constant discount factor. The function  $u(\cdot)$  is strictly increasing, strictly concave and  $\lim_{c \rightarrow 0} u(c) = 0$ .

**Endowments:** Household type  $e$  receives  $y_H$  units of the perishable consumption good in the even periods,  $t = 0, 2, \dots$  and  $y_L < y_H$  in the odd periods,  $t = 1, 3, \dots$ . The type  $o$  household receives  $y_L$  in the even periods and  $y_H$  in the odd periods.

**Institutions:** There is a government which has to meet exogenous spending  $g < 2y_L$  each period. (Note:  $g$  is constant.) It can finance the spending through taxes or by issuing bonds (we will look at both in turn).

There is no inside money. (No enforcement mechanism for individual contracts exists.)

1. Write down and solve the problem of the Planner who weights each household equally. To what extent does the Planner's consumption allocation for each household type vary over time?
2. What will the allocation look like if the government only uses constant taxes,  $\tau = g/2$  levied on both households to cover government spending? (Hint: there are no markets here.)
3. Now consider what happens if the government uses one-period bonds to cover government spending. There will now be a market for bonds. A bond issued in period  $t$  to acquire one unit of the consumption good will repay the owner  $R_{t+1}$  units of the consumption good in period  $t + 1$ . Let the number of bonds issued in period  $t$  be  $b_t > y_H - y_L$ . No Ponzi schemes will be permitted.
  - (a) Write down and solve the problem for either type of household,  $e$  or  $o$ . (They are basically the same.)
  - (b) Write down the government budget constraint and the market clearing conditions.
  - (c) Define a competitive equilibrium.
  - (d) What is the unique value of  $R_t$  that is consistent with a steady state equilibrium?
  - (e) How does the consumption allocation compare with the Planner's solution?
4. Comment on the applicability of Ricardian equivalence in this environment. How might your answer change if inside money was available?