Macroeconomics I: Mid-Term Exam

1. **Redistributive effects of monetary policy in the Overlapping generations model.**

Time: discrete, infinite horizon, $t = 1, 2, 3...$

Demography: A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people. Within the population there are two types of household A and B who differ according to their endowments (see below). The population is split exactly in half between the groups: $N_t^A = N_t^B = N_t/2$

Preferences: for the generations born in and after period 0;

$$U_t(c_{1,t}^i, c_{2,t+1}^i) = \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i) \qquad i = A, B$$

where $c_{s,t}^i$ is consumption in period t and stage s of life for type $i = A, B$ individuals. For the initial old generation $\tilde{U}(c_{2,0}^i) = \ln(c_{2,0}^i)$ for $i = A, B$

Endowments: Each type $i = A, B$ person receives e^i units of the perishable consumption good in their first period of life. There is zero endowment of the good in the second period of life. In period 0 the first generation of old are endowed with H_0 units of money spread equally among them which can be stored but provides no utility in consumption. The money supply grows each period so that the aggregate money supply in period t is $H_0(1+\sigma)^t$. The new money is delivered by helicopter drop (i.e. lump sum) to each old person at the beginning of the period in which they are old.

Information: There is complete information with perfect foresight.

Solution concept: Competitive equilibrium. Each period there are markets for the consumption good and money. Let, p_t, be the price for goods in terms of money in period t which is taken as given by every participant.

(a) Write out and solve the problem faced by the members of generation t. Use $M_t^{d,i}$ to represent the nominal money demand of each type $i = A, B$ individual born in period t. (Hint: If p_{t+1} drops out of your first order condition equation don't worry about it.)

(b) Write down the market clearing conditions and define a competitive equilibrium.

(c) Let $1 + g_t = \dfrac{p_t}{p_{t+1}}$ so that in steady state $(1+g) = (1+n)/(1+\sigma)$. Obtain steady state $c_{1,t}^i = c_1^i$ and $c_{2,t+1}^i = c_2^i$ and in terms of model parameters and the steady state value of $\frac{\Delta M_t}{p_t}$. (Note: it is possible to obtain these in terms of model parameters alone but the algebra is messy.)

(d) (No math required.) If $e^A = e^B$ we are back to the version of the model we covered in class (with log utility). In that case what is the optimal monetary policy? Now if $e^A \neq e^B$ (as above) and the Social Welfare Function is equal weight utilitarian, why might optimal policy differ from the $e^A = e^B$ case?

2. **Consider the following non-linear first-order one dimensional system in x_t:**

$$x_{t+1} = x_t^3 - 7x_t^2 + 13x_t$$

(a) Identify the all steady states, $x^* \geq 0$ of x_t.

(b) Identify the stability properties of each steady state (i.e. stable/unstable, oscillatory/monotone)

(c) Provide a sketch of the dynamical system