Macroeconomics I: Mid-Term Exam

A. **Overlapping generations model with discriminatory land rights**

Time: discrete, infinite horizon

Demography: Each period 2 newborns enter the economy and live for two periods. There is no population growth. The newborns comprise one of type A and one of type B. Type A households are allowed to own land and type B are not. The type A household acquires the land (of size A) at the beginning of their second period of life by inheritance from the previous generation of type A individual who just died. They can rent the land to a farm but cannot rent it to type B individuals directly. There is a single farm owned by the type A households.

Preferences: for the generations born in and after period 1:

$$U_t^J(c_{1,t}^J, l_{1,t}^J, c_{2,t+1}^J, l_{2,t+1}^J) = u(c_{1,t}^J, l_{1,t}^J) + \beta u(c_{2,t+1}^J, l_{2,t+1}^J) \quad \text{for } J = A, B$$

where $c_{i,t}^J$ is consumption in period t and stage i of life for a type $J = A, B$ household and $l_{i,t}^J$ is leisure in period t and stage i of life for a type J household. For the initial old generation $\tilde{U}^J(c_{2,1}^J, l_{2,1}^J) = u(c_{2,1}^J, l_{2,1}^J)$ for $J = A, B$. The utility function is twice differentiable, increasing in both arguments and strictly concave. It has a positive cross-partial (i.e. $u_{12} > 0$). Moreover, $\lim_{c\to 0} u_1(c, l) = \infty$, $\lim_{l\to 0} u_2(c, l) = \infty$, $\lim_{c\to\infty} u_1(c, l) = 0$, $\lim_{l\to 1} u_2(c, l) = 0$. The first 3 assumptions here are standard but the last one is new.

Productive technology: The farm has access to a technology so that output, $y_t = f(a_t, n_t)$ where a_t is the amount of land used and n_t is the amount of labor hired in period t. The production function satisfies the standard Inada conditions, is twice differentiable, increasing in both arguments, concave and exhibits constant returns to scale.

Endowments: Everyone has one unit of time in both periods of life. The young can work to earn income; the old cannot. Old type A households receive A units of land. As old people cannot work, they have to rely on savings (using inside money) and, for type A only, earnings from renting land.

Institutions: There are competitive markets, for labor, land and consumption goods. Using the consumption good as the numeraire, let the per labor unit wage in period t be w_t and the per unit rental rate of land be q_t. There are also enforceable IOU contracts so that inside money circulates. Let the gross rate of return on inside money contracts that pay in period t be R_t.

1. Write out and solve the problems faced by generation t individuals of type A and type B and, the farm in this economy.

   Type A:

$$\begin{aligned} & \max_{c_{1t}^A, c_{2t+1}^A, l_t^A} u(c_{1t}^A, l_t^A) + \beta u(c_{2t+1}^A, 1) \\ \text{s.t.} \quad c_{1t}^A &= w_t(1 - l_t^A) - s_{t+1}^A \\ c_{2t+1}^A &= R_{t+1} s_{t+1}^A + A q_{t+1} \end{aligned}$$

   Sub for c_{1t}^A and c_{2t+1}^A. FOCs:

$$\begin{aligned} l_t^A &: \quad -u_1(c_{1t}^A, l_t^A) w_t + u_2(c_{1t}^A, l_t^A) = 0 \\ s_{t+1}^A &: \quad -u_1(c_{1t}^A, l_t^A) w_t + \beta R_{t+1} u_1(c_{2t+1}^A, 1) = 0 \end{aligned}$$

   Type B:

$$\begin{aligned} & \max_{c_{1t}^B, c_{2t+1}^B, l_t^B} u(c_{1t}^B, l_t^B) + \beta u(c_{2t+1}^B, 1) \\ \text{s.t.} \quad c_{1t}^B &= w_t(1 - l_t^B) - s_{t+1}^B \\ c_{2t+1}^B &= R_{t+1} s_{t+1}^B \end{aligned}$$

   Sub for c_{1t}^B and c_{2t+1}^B. FOCs:

$$\begin{aligned} l_t^B &: \quad -u_1(c_{1t}^B, l_t^B) w_t + u_2(c_{1t}^B, l_t^B) = 0 \\ s_{t+1}^B &: \quad -u_1(c_{1t}^B, l_t^B) w_t + \beta R_{t+1} u_1(c_{2t+1}^B, 1) = 0 \end{aligned}$$

   Farm:

$$\max_{a_t, n_t} f(a_t, n_t) - w_t n_t - q_t a_t$$

   So

$$\begin{aligned} f_1(a_t, n_t) &= q_t \\ f_2(a_t, n_t) &= w_t \end{aligned}$$

2. Write down the market clearing conditions for land, labor and inside money and define a competitive equilibrium.

   Market Clearing conditions are:

$$\begin{aligned} \text{Land} &: \quad a_t = A \\ \text{Labor} &: \quad n_t = 2 - l_t^A - l_t^B \\ \text{Inside money} &: \quad s_{t+1}^A = -s_{t+1}^B \\ \text{Consumption Goods} &: \quad f(a_t, n_t) = c_{1t}^A + c_{1t}^B + c_{2t}^A + c_{2t}^B \end{aligned}$$

   Definition: A competitive equilibrium is an allocation, $\{c_{1t}^A, c_{1t}^B, c_{2t}^A, c_{2t}^B, a_t, n_t\}$ and prices $\{R_t, w_t, q_t\}$ such that given prices the allocation solves the households' and firm's problems and markets clear.

3. Now, focussing on a steady state (so that we can drop the time subscripts on variables) solve for a system of 4 equations that characterize the steady state equilibrium. What are the four steady state variables that your system can be used to solve for? **Note:** if you drop arguments, (e.g. from the production function to reduce notation) please indicate at least once, that you know what those arguments are.

   In steady state:

$$\begin{aligned} u_1(f_2(1-l^A)-s^A,l^A)f_2-u_2(f_2(1-l^A)-s^A,l^A) &= 0\\ u_1(f_2(1-l^A)-s^A,l^A)-\beta R u_1(Rs^A+Af_1,1) &= 0 \end{aligned}$$

   and

$$\begin{aligned} u_1(f_2(1-l^B)+s^A,l^B)f_2-u_2(f_2(1-l^B)+s^A,l^B) &= 0\\ u_1(f_2(1-l^B)+s^A,l^B)-\beta R u_1(-Rs^A,1) &= 0 \end{aligned}$$

   where $f_i = f_i(A, 2-l_t^A-l_t^B)$, $i = 1, 2$. The four variables are l^A, l^B, R, s^A.

4. Now, suppose that preferences are such that second period consumption is the same for both types of household. What does that mean for the amount of work done by the type A young people relative the type B young people?

   Dividing the type A equilibrium equations yields

$$f_2 = \frac{u_2(f_2(1-l^A)-s^A,l^A)}{\beta R u_1(Rs^A+Af_1,1)}$$

   Dividing the type B equilibrium equations yields

$$f_2 = \frac{u_2(f_2(1-l^B)+s^A,l^B)}{\beta R u_1(-Rs^A,1)}$$

   If second period consumption is the same for both types then the denominators are the same. As the LHS of each equation is the same too, the numerators must be the same which can only mean that type A work less. They know that in order to eat in the second period of life, the type B people need to lend to them in the first period. So, type A people do not need to work as hard as the type B people.

5. What is the advantage in this model of assuming that $\lim_{l\to 1} u_2(c,l) = 0$?

   Assuming this, rules out the corner solution where the type A people do not work at all in the first period of life. AS the marginal utility of leisure goes to zero when they don't work at all they may as well work no matter how low the wages. This gives meaning to the previous question and any comparative statics about work effort levels.

6. Write down but *do not solve* the Planners's problem for this economy.

$$\begin{aligned} \max_{\{c_{1t}^A,c_{1t}^B,c_{2t}^A,c_{2t}^B,l_t^A,l_t^B\}} & \sum_{J=A,B} u(c_{2,1}^J) + \sum_{t=1}^{\infty}\sum_{J=A,B} u(c_{1t}^J,l_t^J)+\beta u(c_{2,t+1}^J,1)\\ \text{s.t.} \quad & f(A,2-l_t^A-l_t^B) = c_{1t}^A+c_{1t}^B+c_{2t}^A+c_{2t}^B \end{aligned}$$

B. Comparative statics in simple linear technology model

System:

$$\begin{aligned} u(c,l) &= h \\ zu_1(c,l) &= u_2(c,l). \end{aligned}$$

Where $u(.,.)$ is a standard utility function, c is consumption, l is leisure and, both z and h are arbitrary constants. What is $\dfrac{dc}{dh}$?

$$\begin{pmatrix} u_1 & u_2 \\ zu_{11} - u_{12} & zu_{12} - u_{22} \end{pmatrix} \begin{pmatrix} \frac{dc}{dh} \\ \frac{dl}{dh} \end{pmatrix} = - \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

So

$$\frac{dc}{dh} = \frac{-\begin{vmatrix} -1 & u_2 \\ 0 & zu_{12} - u_{22} \end{vmatrix}}{\begin{vmatrix} u_1 & u_2 \\ zu_{11} - u_{12} & zu_{12} - u_{22} \end{vmatrix}} = \frac{zu_{12} - u_{22}}{u_1 \left(zu_{12} - u_{22}\right) - u_2 \left(zu_{11} - u_{12}\right)}$$