

Macroeconomics I: Mid-Term Exam

Robustness of Ricardian Equivalence in Two-period endowment model

Time: discrete, two periods.

Demography: There is one representative (i.e. price taking) household of each of two types who differ according to their endowments. We will label them E for “early” and L for “late”. Both households live for both periods.

Preferences: Both household types have the same preferences:

$$U(c_1^i, c_2^i) = u(c_1^i) + \beta u(c_2^i)$$

where c_t^i is consumption by type $i = E, L$ in period $t = 1, 2$. The function $u(\cdot)$ is increasing, strictly concave, $\lim_{c \rightarrow 0} u'(c) = \infty$.

Endowments: The Early household has an endowment of the non-storable consumption goods in period 1 of $e + d$ and in period 2 of e . The Late household has endowment e in the first period and $e + d$ in the second.

Institutions: there is a government which has to make expenditures $2g_t$ where $g_t < e$, in period $t = 1, 2$. The government can issue bonds, b , and can tax individuals, τ_t in period $t = 1, 2$. (Taxes do not depend on the household type.) Because there is no way to enforce loan contracts between individuals, there is **no** market for inside money.

1. First assume $d = 0$ so there is no difference between the two household types. Furthermore, assume that the government cannot issue bonds (i.e. $b = 0$) Write down the government and households’ budget constraints for each period. Write down the allocation for each type in terms of exogenous variables.

The households’ budget constraints are

$$c_t^i = e - \tau_t \quad \text{for } i = E, L \quad \text{and } t = 1, 2$$

The government budget constraints are

$$2g_t = 2\tau_t \quad \text{for } t = 1, 2$$

The allocation is

$$c_t^i = e - g_t \quad \text{for } i = E, L \quad \text{and } t = 1, 2$$

2. Now, still with $d = 0$ we will allow b to be positive. To focus on a particular example set $\tau_1 = 0$.

- (a) Write down and solve the problem faced by the households. Write down the government budget constraint and all market clearing conditions. Define a competitive equilibrium and solve for a characterization.

Households' problems are:

$$\max_{s_i} u(e - s_i) + \beta u(e - \tau_2 + R s_i) \quad \text{for } i = E, L$$

Solution satisfies

$$u'(e - s_i) = R\beta u'(e - \tau_2 + R s_i) \quad \text{for } i = E, L$$

Government budget constraints

$$\begin{aligned} 2g_1 &= b \\ 2g_2 &= 2\tau_2 - Rb \end{aligned}$$

Market clearing:

$$\begin{aligned} c_t^E + c_t^L + 2g_t &= 2e \quad \text{for } t = 1, 2 \\ b &= s_E + s_L \end{aligned}$$

Definition: A competitive equilibrium is an allocation c_t^i for $i = E, L$, $t = 1, 2$, an interest rate, R , and taxes, τ_2 , such that given R and τ_2 , the allocation solves the households' problems, the government budget constraints hold and markets clear.

By symmetry, $c_t^E = c_t^L$ for $t = 1, 2$. The equilibrium is characterized by

$$c_t^i = e - g_t \quad \text{for } i = E, L \quad \text{and } t = 1, 2$$

and

$$u'(e - g_1) = R\beta u'(e - g_2) \quad \text{for } i = E, L$$

- (b) Does Ricardian equivalence hold? Briefly explain.

Yes. The allocation is invariant to the way the government obtains its revenue.

3. Now we will set $d > 0$ and repeat the exercises in parts 1. and 2.

- (a) With $b = 0$ write down the allocation for each type in terms of exogenous variables.

The allocation is

$$\begin{aligned} c_1^E &= e + d - g_1, & c_1^L &= e - g_1, \\ c_2^E &= e - g_1, & c_2^L &= e + d - g_1 \end{aligned}$$

- (b) With $\tau_1 = 0$ and $b > 0$: write down and solve the households' problem for each type (assume that there are no corner solutions), write down the government budget constraints and market clearing conditions. (No need to redefine equilibrium or characterize it.)

Households' problems are:

$$\begin{aligned} \max_{s_E} u(e + d - s_E) + \beta u(e - \tau_2 + R s_E) \\ \max_{s_L} u(e - s_L) + \beta u(e + d - \tau_2 + R s_L) \end{aligned}$$

FOCs:

$$\begin{aligned} u'(e + d - s_E) &= R\beta u'(e - \tau_2 + Rs_E) \\ u'(e - s_L) &= R\beta u'(e + d - \tau_2 + Rs_L) \end{aligned}$$

Government budget constraints

$$\begin{aligned} 2g_1 &= b \\ 2g_2 &= 2\tau_2 - Rb \end{aligned}$$

Market clearing:

$$\begin{aligned} c_t^E + c_t^L + 2g_t &= 2e + d \quad \text{for } t = 1, 2 \\ b &= s_E + s_L \end{aligned}$$

- (c) With $d > 0$ does Ricardian equivalence hold? (Hint: is $s_E = s_L$?) Briefly compare your answer to that in part 2b.

The allocations for parts 3a and 3b will not be the same. The FOCs imply

$$\frac{u'(e + d - s_E)}{u'(e - \tau_2 + Rs_E)} = \frac{u'(e - s_L)}{u'(e + d - \tau_2 + Rs_L)}$$

which cannot be true if $s_E = s_L$. In part 3a late households are liquidity constrained. They would like to borrow but cannot. When the government issues bonds, the two household types are free to differ in their consumption patterns by acquiring different amounts of bonds. Type L acquire more and type E acquire less.

4. How do you think the results would differ if inside money was able to circulate? Briefly explain your answer.

If inside money could circulate then Ricardian equivalence would hold in part 3 as well. The inside money would be a perfect substitute for government bonds. The latter would simply crowd out the former.