Macroeconomics I: Mid-Term Exam

Diamond Overlapping generations model with exogenous growth

Time: discrete, infinite horizon

Demography: A mass N of newborns enter in every period (i.e. no population growth). Everyone lives for 2 periods except for the first generation of old people.

Preferences: for the generations born in and after period 1:

$$U_t(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period t and stage i of life. For the initial old generation $\tilde{U}(c_{2,1}) = \ln(c_{2,1})$.

Productive technology: Firms have access to the technology $z_t F(K_t, L_t) = z_t K_t^{\alpha} L_t^{1-\alpha}$ where K_t is the period t capital stock, L_t is period t labor, $\alpha \in (0,1)$ and z_t is the period t total factor productivity (TFP). The gross growth rate of $z_t = \frac{z_{t+1}}{z_t} = 1 + \gamma$ so that γ is the net growth rate of TFP. It will be convenient to use the period t output per worker, $y_t = z_t k_t^{\alpha}$, where k_t is per worker (i.e. per young person) capital stock at the firm.

Endowments: Everyone has one unit of labor services when young. (Old people cannot work so they have to rely on earnings from renting capital.) The first generation of old have k_1 units of capital each.

Institutions: There are competitive markets, for labor, physical capital and consumption goods. Using the consumption good as the numeraire, let the per unit wage in period t be w_t and the gross return on capital rented in period t be R_t.

1. Write out and solve the problems faced by generation t individuals and firms in this economy (ignore inside money).

2. Write down the market clearing condition for capital and define a competitive equilibrium.

   As TFP grows at the rate γ, there is no steady state. Instead, we will look for a "balanced growth path" (BGP). On the BGP all variables we are interested in grow at a fixed rate. Let G_x be the notation for the gross growth rate of variable $x = k$, c_1, c_2 etc. So, for example $G_k = \frac{k_{t+1}}{k_t}$ for all t.

3. In terms of γ, find G_k and show that $G_k = G_y$.

4. Show that G_R, the growth rate of the gross interest rate (or rental rate of capital) equals 1 so that $R_t = \bar{R}$ for all t?

5. Solve for expressions for the per worker capital stock, k_t and $\bar{R}$ as a function of z_t and model parameters.

6. Write down and solve the (per young person) problem faced by a Social Planner who weights all generations equally.

7. Given that on any BGP the growth rates of consumption, G_{c_1} and G_{c_2} are equal to G_y, under what condition on parameters does the first welfare theorem hold?