Macroeconomics I: Mid-Term Exam

Diamond Overlapping generations model with log utility

Time: discrete, infinite horizon

Demography: A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people.

Preferences: for the generations born in and after period 1;

$$U_t(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period t and stage i of life. For the initial old generation $\tilde{U}(c_{2,1}) = \ln(c_{2,1})$.

Productive technology: Firms have access to the technology $F(K, L) = zK^\alpha L^{1-\alpha}$ where K is the capital stock, L is labor and and $\alpha \in (0, 1)$. (This means that period t output per worker will be zk_t^α where k_t is per worker capital stock at the firm.)

Endowments: Everyone has one unit of labor services when young. (Old people cannot work so they have to rely on earnings from renting capital.) The first generation of old have $(1+n)k_1$ units of capital each.

Institutions: There are competitive markets, for labor, physical capital and consumption goods. Using the consumption good as the numeraire, let the per unit wage in period t be w_t and the gross return on capital rented in period t be R_t.

1. Write out and solve the problems faced by generation t individuals and firms in this economy (ignore inside money).

   Individuals solve:

   $$\max_{s_{t+1}} \{\ln(w_t - s_{t+1}) + \beta \ln(R_{t+1}s_{t+1})\}$$

   FOC:

   $$\frac{-1}{w_t - s_{t+1}} + \frac{\beta R_{t+1}}{R_{t+1}s_{t+1}} = 0$$

   so

   $$s_{t+1} = \frac{\beta w_t}{1+\beta}$$

   For firms,

   $$\max_{k_t} \{zk_t^\alpha - R_t k_t - w_t\}$$

   so that for any interior solution, $R_t = \alpha z k_t^{\alpha-1}$ and $w_t = (1-\alpha)zk_t^\alpha$.

2. Comment on the relative magnitude of the income and substitution effects (i.e. which one dominates?).

   AS R_{t+1} does not enter s_{t+1}, the substitution and income effects exactly balance off.

3. Write down the market clearing conditions and define a competitive equilibrium.

   Market clearing:

$$\text{Capital:} \qquad (1+n)k_{t+1} = s_{t+1}$$

$$\text{Goods:} \qquad F(K_t, L_t) = N_{t-1}c_{2\,t} + N_t\left(c_{1\,t} + s_{t+1}\right)$$

$$\text{Labor:} \qquad L_t = N_t$$

   Definition: A competitive equilibrium is an allocation $\{c_{1\,t}, c_{2\,t}, k_t\}$ and prices $\{R_t, w_t\}$ such that given prices the allocation solves the individuals' and the firms' problems and, markets clear.

4. Solve for the (non-trivial) steady state level of the capital stock.

   The law of motion for k_t is

$$(1+\beta)(1+n)k_{t+1} = \beta(1-\alpha)zk_t^{\alpha}$$

   Steady state capital is then given by

$$k^{*(1-\alpha)} = \frac{\beta(1-\alpha)z}{(1+\beta)(1+n)}$$

5. What are the dynamic properties of the steady state?

   From the law of motion for k_t,

$$(1+\beta)(1+n)\frac{dk_{t+1}}{dk_t} = \beta\alpha(1-\alpha)zk_t^{\alpha-1}$$

   At $k_t = k^*$, $\frac{dk_{t+1}}{dk_t} = \alpha$. So the steady state is monotone sable.

6. Write down and solve the problem faced by a Social Planner who weights all generations equally.

   As in class notes

7. Under what condition on parameters does the first welfare theorem fail to hold?

   The first welfare theorem will fail to hold whenever $R^* < 1+n$, where R^* is the steady state gross rate of return on savings. Thus, $R^* = \alpha z k_t^{*(\alpha-1)}$. After substitution the condition becomes

$$\frac{\alpha(1+\beta)}{(1-\alpha)\beta} < 1.$$