# Macroeconomics I: Mid-Term Exam

1. **Overlapping generations model with money and specified utility functions**

**Time:** discrete, infinite horizon

**Demography:** A mass $N_t \equiv N_0(1+n)^t$, $n > 0$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people who only live for one period.

**Preferences:** for the generations born in and after period 1;

$$U_t(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) + \beta c_{2,t+1}$$

where $c_{i,t}$ is consumption in period $t$ and stage $i$ of life and $\beta > 0$ is the . For the initial old generation $\tilde{U}(c_{2,1}) = c_{2,1}$.

**Endowments:** Every person receives 1 unit of the perishable consumption good in their first period of life. There is zero endowment of the good in the second period of life. In period 0 the first generation of old are each endowed with $H$ units of money which can be stored but cannot be created.

**Institutions:** Each period there are competitive markets for the consumption good and money. There is, therefore, only one price per period, $p_t$, the price of goods in terms of money.

(a) Write out and solve the Planner's problem for this economy.

(b) For the decentralized economy, write out and solve the problem faced by the members of generation $t$. Use $M_t^d$ to represent the nominal money demand of each individual born in period $t$ and solve for the real money demand, $\frac{M_t^d}{p_t}$ as a function of $1 + g_t \equiv \frac{p_t}{p_{t+1}}$

(c) Write down the market clearing conditions and define a competitive equilibrium.

(d) Under what conditions is the competitive equilibrium a Pareto optimal allocation?

(e) Solve for the law of motion for $g_t$ (it should take the form $g_{t+1} = f(g_t)$) and show that there are two steady states, $g = n$ and $g = \frac{1-\beta}{\beta}$.

(f) Is either steady state oscillatory? Under what circumstances is each of them stable or unstable?