AECO601 Fall 2009

Macroeconomics I: Mid-Term Exam

1. **Overlapping generations model with storage technology**

Time: discrete, infinite horizon

Demography: A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people.

Preferences: for the generations born in and after period 0;

$$U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \beta u(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period t and stage i of life, $u(.)$ is increasing strictly concave and twice differentiable $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$. For the initial old generation $\tilde{U}(c_{2,0}) = u(c_{2,0})$.

Storage technology: For every unit of the consumption good that a person "buries" at the end of the fist period of life they obtain γ units of the consumption good in their second period.

Endowments: Every person receives e units of the consumption good in their first period of life. Unless the person uses the storage technology the the good is perishable. There is zero endowment of the good in the second period of life.

Institutions: Each period there are competitive markets for the consumption good and inside money there is therefore only one price per period. Let R_{t+1} be the number of consumption goods payable to holder of private bonds (inside money) issued in period t.

(a) Write out and solve the problem faced by the members of generation t. Let s^G_{t+1} be the amount of the good stored (i.e. buried) in period t for consumption in period $t+1$. Let s^I_{t+1} be the period t savings in inside money.

$$\max_{c_{1t}, c_{2t+1}, s^G_{t+1}, s^I_{t+1}} \{u(c_{1t}) + \beta u(c_{2t+1})\}$$

$$\begin{aligned} \text{s.t. } c_{1t} &= e - s^G_{t+1} - s^I_{t+1} \\ c_{2t+1} &= \gamma s^G_{t+1} + R_{t+1} s^I_{t+1} \end{aligned}$$

after substituting out c_{1t} and c_{2t+1} the first order conditions are

$$\begin{array}{lcl} s^G_{t+1} & : & -u'(c_{1t}) + \beta\gamma u'(c_{2t+1}) = 0 \\ s^I_{t+1} & : & -u'(c_{1t}) + \beta R_{t+1} u'(c_{2t+1}) = 0 \end{array}$$

SOC for a max follows from concavity of $u(.)$.

(b) Write down the market clearing conditions and define a competitive equilibrium. What is the value of R_{t+1}? Explain why s^G_t must be a constant, s^G, for all t.

Market clearing:

$$\begin{array}{rcl} \text{goods} & : & (1+n)c_{1t} + c_{2t} + (1+n)s^G_{t+1} = (1+n)e + \gamma s^G_t \\ \text{inside money} & : & s^I_{t+1} = 0. \end{array}$$

Definition 1 *A competitive equilibrium is an allocation* $\{c_{1t}, c_{2t}, s^G_{t+1}, s^I_{t+1}\}_{t=1}^{\infty}$ *and sequence of prices,* $\{R_{t+1}\}_{t=1}^{\infty}$*, such that given prices the allocation solves the individuals' problems and markets clear.*

$$R_{t+1} = \gamma \text{ for all } t$$

s^G_{t+1} is constant because substitution into the first order condition yields,

$$u'(e - s^G_{t+1}) = \beta\gamma u'(\gamma s^G_{t+1})$$

all other components in the equation are constant.

(c) Write down and obtain first order conditions for the Planner's problem for this economy.

$$\max_{\{c_{20}c_{1t}, c_{2t}, s_t\}} u(c_{20}) + \sum_{t=0}^{\infty} u(c_{1t}) + \beta u(c_{2t+1})$$
$$\text{s.t.} \qquad (1+n)c_{1t} + c_{2t} + (1+n)s_{t+1} = (1+n)e + \gamma s_t$$

FOC's:

$$\begin{aligned}
c_{20} &: \quad u'(c_{20}) - \lambda_1 = 0 \\
c_{1t} &: \quad u'(c_{1t}) - \lambda_t(1+n) = 0 \\
c_{2t} &: \quad \beta u'(c_{2t}) - \lambda_t = 0 \\
s_t &: \quad -(1+n)\lambda_{t-1} + \gamma\lambda_t = 0
\end{aligned}$$

(d) If $\gamma < 1+n$ how much saving (via the storage technology) will the Planner do? (Assume that λ_t the multiplier on the period t resource constraint is constant.) Use this to comment on the efficiency of competitive equilibrium.

If λ_t is constant and $\gamma < 1+n$ then the Planner will not save using the storage technology. S/He will simply transfer funds to the old from the young. As long as $\gamma < 1+n$ the planner can improve on the competitive equilibrium allocation so the fist welfare theorem will not hold.

(e) If $u(x) = \ln(x)$ (natural logarithm) solve for the competitive equilibrium values of $c_{1t,}$ c_{2t} and s^G in terms of the other parameters.

$$s^G = \frac{\beta e}{1+\beta}, \quad s^I = 0, \quad c_{1t} = \frac{e}{1+\beta}, \quad c_{2t+1} = \frac{\gamma\beta e}{1+\beta}, \quad R_{t+1} = \gamma.$$

(g) (BONUS) What does the Planner do if $\gamma > 1+n$? (λ_t may not be constant in this case)

The planner can grow the economy by slowly accumulating extra goods each period. The economy will grow without bound for ever.

2. Consider the following non-linear first-order one dimensional system in x_t:

$$x_{t+1} = 2x_t(2 - x_t)$$

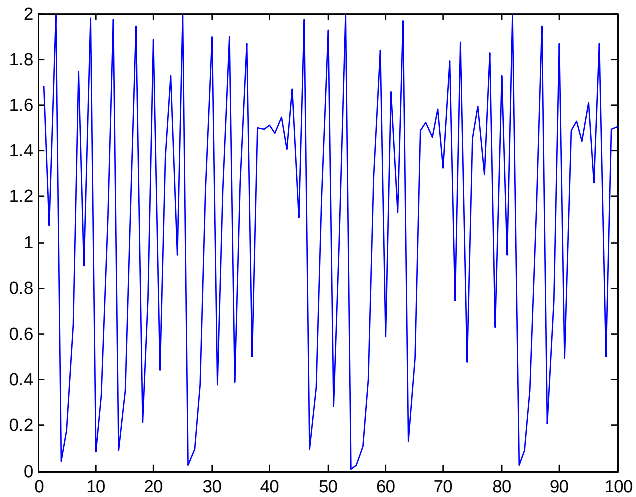
(a) Identify the all steady states, $x^* \geq 0$ of x_t.

The steady states are $x^* = 0$ and $x^* = \frac{3}{2}$.

(b) Identify the stability properties of each steady state (i.e. stable/unstable, oscillatory/monotone)

In general

$$\frac{dx_{t+1}}{dx_t} = 4(1-x)$$



At $x^* = 0$:

$$\left.\frac{dx_{t+1}}{dx_t}\right|_{x_{t+1}=x_t=0} = 4$$

Steady state is monotone unstable

At $x^* = \frac{3}{2}$:

$$\left.\frac{dx_{t+1}}{dx_t}\right|_{x_{t+1}=x_t=\frac{3}{2}} = -2$$

Steady state is unstable oscillatory.

Actually the complete system does not seem to converge to any thing. Below is a plot of the first 100 values starting from 1.4. It gets close to 1.5 on occasion but then moves away as predicted. Even after 10,000 iterations it looks pretty much the same.

(c) Provide a sketch of the dynamic system

It's a parabola that goes through (0,0), (1,2) and (2,0).