AECO601 Fall 2009

# Macroeconomics I: Mid-Term Exam

1. **Overlapping generations model with storage technology**

**Time:** discrete, infinite horizon

**Demography:** A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people.

**Preferences:** for the generations born in and after period 0;

$$U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \beta u(c_{2,t+1})$$

where $c_{i,t}$ is consumption in period $t$ and stage $i$ of life, $u(.)$ is increasing strictly concave and twice differentiable $\lim_{c \to 0} u'(c) = \infty$, $\lim_{c \to \infty} u'(c) = 0$. For the initial old generation $\tilde{U}(c_{2,0}) = u(c_{2,0})$.

**Storage technology:** For every unit of the consumption good that a person "buries" at the end of the fist period of life they obtain $\gamma$ units of the consumption good in their second period.

**Endowments:** Every person receives $e$ units of the consumption good in their first period of life. Unless the person uses the storage technology the the good is perishable. There is zero endowment of the good in the second period of life.

**Institutions:** Each period there are competitive markets for the consumption good and inside money there is therefore only one price per period. Let $R_{t+1}$ be the number of consumption goods payable to holder of private bonds (inside money) issued in period $t$.

(a) Write out and solve the problem faced by the members of generation $t$. Let $s^G_{t+1}$ be the amount of the good stored (i.e. buried) in period $t$ for consumption in period $t+1$. Let $s^I_{t+1}$ be the period $t$ savings in inside money.

(b) Write down the market clearing conditions and define a competitive equilibrium. What is the value of $R_{t+1}$? Explain why $s^G_t$ must be a constant, $s^G$, for all $t$.

(c) Write down and obtain first order conditions for the Planner's problem for this economy.

(d) If $\gamma < 1+n$ how much saving (via the storage technology) will the Planner do? (Assume that $\lambda_t$ the multiplier on the period $t$ resource constraint is constant.) Use this to comment on the efficiency of competitive equilibrium.

(e) If $u(x) = \ln(x)$ (natural logarithm) solve for the competitive equilibrium values of $c_{1t}$, $c_{2t}$ and $s^G$ in terms of the other parameters.

(g) (BONUS) What does the Planner do if $\gamma > 1+n$? ($\lambda_t$ may not be constant in this case)

2. **Consider the following non-linear first-order one dimensional system in $x_t$:**

$$x_{t+1} = 2x_t(2 - x_t)$$

(a) Identify the all steady states, $x^* \geq 0$ of $x_t$.

(b) Identify the stability properties of each steady state (i.e. stable/unstable, oscillatory/monotone)

(c) Provide a sketch of the dynamic system