Proportional income tax in simple dynamic model with a government.

Consider the following economy:

Time: Discrete; infinite horizon

Demography: Continuum of mass 1 of (representative) consumer/worker households, and a large number of profit maximizing firms, owned jointly by the households.

Preferences: the instantaneous household utility function is $u(c)$ where c is household consumption and $u(.)$ is strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology for which labor is the only input so that a firm that hires h units of labor produces zh units of output.

Endowments: Each household has 1 unit of time per period to allocate how ever they like between work and leisure.

Institutions: There is a government that has to meet an exogenous stream of expenditures, $\{g_t\}$. Government spending is thrown into the ocean. The government can levy taxes and issue bonds in order to meet its expenditure requirement. Taxes are restricted to being proportional to labor income so that in period t, the tax revenue from a household which provides labor services h_t is then $\tau_t w_t h_t$ where τ_t is the period t tax rate and w_t is the wage rate. Every period there are markets for labor, government bonds and consumption goods.

(a) Write down and solve the problems faced by the representative household and the representative firm.

Households solve: (Students might spot immediately that households do not value leisure and so $h_t = 1$ for all t in which case as long as they mention it, they can drop h_t from the household's problem.)

$$\max_{\{c_t, h_t, s_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } c_t = (1 - \tau_t) w_t h_t + (1 + r_t) s_{t-1} - s_t$$

where s_t is the saving (in bonds) by a household in period t and r_t is the interest on bonds

Clearly,

$$h_t = 1 \text{ for all } t$$

and there is an intertemporal optimality condition

$$u'(c_t) = \beta\,(1 + r_{t+1})\,u'(c_{t+1})$$

Firms solve

$$\max_{h_t^f} \left\{ z h_t^f - w_t h_t^f \right\}$$

So for any firm that hires a strictly positive (but finite) amount of labor,

$$w_t = z \text{ for all } t$$

(b) Write down the government's (period by period) budget constraint.

$$g_t + (1 + r_t) b_{t-1} = b_t + \tau_t w_t h_t$$

(c) Define and a characterize a competitive equilibrium.

A *competitive equilibrium* is a sequence of prices, $\{w_t, r_t\}$ a sequence of tax rates $\{\tau_t\}$ and an allocation $\{c_t, h_{t,} h_t^f, b_t, s_t\}$ such that, given prices and tax rates, the allocation solves the households' problem and the firms' problem; the government budget constraint holds and the markets for labor, consumption goods and bonds all clear.

The market clearing conditions are:

$$\begin{aligned} h_t^f &= h_t = 1 \\ b_t &= s_t \\ z &= g_t + c_t \end{aligned}$$

Using these we get

$$c_t = (1 - \tau_t) z + (1 + r_t)\,b_{t-1} - b_t$$

and the government budget constraint becomes

$$g_t + (1 + r_t) b_{t-1} = b_t + \tau_t z$$

Characterization of equilibrium:

$$1 + r_{t+1} = \frac{u'(z - g_t)}{\beta u'(z - g_{t+1})}$$

(d) Does Ricardian equivalence hold? Explain

Yes, the timing τ_t does not affect the equilibrium allocation which is summarized by $c_t = z - g_t$ for all t.

(e) How would your answer to part (d) change if the utility function was replaced by $u(c_t, 1 - h_t)$ and $u(.,.)$ is strictly increasing in both arguments? Explain your answer.

If households care about leisure then the proportional tax will distort the labor-leisure time allocation. Goods market clearing will be $zh_t = g_t + c_t$ and h_t will depend on the timing of taxes. The equilibrium wage will still be z for all t but the first-order condition for the labor leisure choice will imply

$$u_1(zh_t - g_t, 1 - h_t)z(1 - \tau_t) = u_2(zh_t - g_t, 1 - h_t) \tag{1}$$

So $c_t = zh_t - g_t$ for all t but h_t is a function of τ_t.

(f) Solve for the equilibrium values of c_t and h_t in terms of the exogenous variables and model parameters if $u(c_t, 1 - h_t) = A \log c_t + \log(1 - h_t)$ where A is a preference parameter.

Into equation (1):

$$\frac{Az(1 - \tau_t)}{zh_t - g_t} = \frac{1}{1 - h_t}$$

So

$$\begin{aligned} h_t &= \frac{Az(1 - \tau_t) + g_t}{z\left[A(1 - \tau_t) + 1\right]} \\ c_t &= \frac{A(1 - \tau_t)(z - g_t)}{z\left[A(1 - \tau_t) + 1\right]} \end{aligned}$$

2. **Simple Dynamic system**

Let

$$x_{t+1} = \Omega \ln(x_t)$$

represent a dynamic system in x, where Ω is a positive constant.

(a) Given $\Omega > e$ (the exponential constant) draw a graph of x_{t+1} against x_t with a $45°$ line.

(b) How many steady states are there?

[Picture on next page]

(c) Characterize the dynamic properties of each steady state.

x_L^* is unstable, x_H^* is stable neither oscillate.

(d) Provide a mathematical argument to support your assertion as to the dynamic properties of each steady state.

Slope of law of motion is $\frac{\Omega}{x}$. Value of x where slope is 1 is Ω. That is where the cuves are futhest appart. So $x_L^* < \Omega$ and slope at that point is $\frac{\Omega}{x_L^*} > 1$. Slope at x_H^* is $\frac{\Omega}{x_H^*} < 1$.