

Macroeconomics I: Mid-Term Exam

Answer both questions. Time allowed: 1hr 30min.

1. Proportional income tax in simple dynamic model with a government.

Consider the following economy:

Time: Discrete; infinite horizon

Demography: Continuum of mass 1 of (representative) consumer/worker households, and a large number of profit maximizing firms, owned jointly by the households.

Preferences: the instantaneous household utility function is $u(c)$ where c is household consumption and $u(\cdot)$ is strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology for which labor is the only input so that a firm that hires h units of labor produces zh units of output.

Endowments: Each household has 1 unit of time per period to allocate how ever they like between work and leisure.

Institutions: There is a government that has to meet an exogenous stream of expenditures, $\{g_t\}$. Government spending is thrown into the ocean. The government can levy taxes and issue bonds in order to meet its expenditure requirement. Taxes are restricted to being proportional to labor income so that in period t , the tax revenue from a household which provides labor services h_t is then $\tau_t w_t h_t$ where τ_t is the period t tax rate and w_t is the wage rate. Every period there are markets for labor, government bonds and consumption goods.

- (a) Write down and solve the problems faced by the representative household and the representative firm.
- (b) Write down the government's (period by period) budget constraint.

- (c) Define and characterize a competitive equilibrium.
- (d) Does Ricardian equivalence hold? Explain
- (e) How would your answer to part (d) change if the utility function was replaced by $u(c_t, 1 - h_t)$ and $u(., .)$ is strictly increasing in both arguments? Explain your answer.
- (f) Solve for the equilibrium values of c_t and h_t in terms of the exogenous variables and model parameters if $u(c_t, 1 - h_t) = A \log c_t + \log(1 - h_t)$ where A is a preference parameter.

2. Simple Dynamic system

Let

$$x_{t+1} = \Omega \ln(x_t)$$

represent a dynamic system in x , where Ω is a positive constant.

- (a) Given $\Omega > e$ (the exponential constant) draw a graph of x_{t+1} against x_t with a 45° line.
- (b) How many steady states are there?
- (c) Characterize the dynamic properties of each steady state.
- (d) Provide a mathematical argument to support your assertion as to the dynamic properties of each steady state.