Fall 2006 Mid-term Solution

(a)

$$\max_{s_t} \{u(e_1 - s_t - \tau_{1t}) + \beta u(e_2 + R_{t+1}s_t - \tau_{2t+1})\}$$
$$\implies u'(c_{1t}) = \beta R_{t+1} u'(c_{2t+1})$$

(b)

$$(1+n)^t \tau_{1t} + (1+n)^{t-1}\tau_{2t} + (1+n)^{t+1} b_{t+1} = (1+n)^t R_t b_t + (1+n)^t g_t$$

or some people had

$$(1+n)^t \tau_{1t} + (1+n)^{t-1}\tau_{2t} + (1+n)^{t} b_{t+1} = (1+n)^{t-1} R_t b_t + (1+n)^t g_t$$

(either were OK. The first comes out if $b_t$ is bonds per young person in period they mature the second is if $b_t$ is bonds per young person in period they were issued). Sticking with the first formulation we get

$$\tau_{1t} + \frac{\tau_{2t}}{1+n} + (1+n)b_{t+1} = R_t b_t + g_t$$

(c) A competitive equilibrium is an allocation $\{c_{1t}, c_{2t}, s_t\}$ and prices $\{R_t\}$ such that given prices and government policy, $\{\tau_{1t}, \tau_{2t}, b_t\}$, the allocation solves the individuals' problem at every $t$, the government policy satisfies the government budget constraint and markets clear.

Market clearing requires:

Bonds: $(1+n)b_{t+1} = s_t$

Goods: $c_{1t} + \dfrac{c_{2t}}{1+n} + g_t = e_1 + \dfrac{e_2}{1+n}$

Equilibrium $R^*_{t+1}$ is characterized by

$$u'(e_1 - (1+n)b_{t+1} - \tau_{1t}) = \beta R_{t+1} u'(e_2 + R_{t+1}(1+n)b_{t+1} - \tau_{2t+1})$$

(d) For Ricardian equivalence to hold we need that the allocation depends on the path of $g_t$ only. This is not the case here. It is possible to make some changes to the paths of $\{\tau_{1t}, \tau_{2t}, b_t\}$ such that the allocation is unchanged by containing adjustments within people's lifetimes. But if $\tau_{2t}$ is reduced without the commensurate increase in the tax paid in the first period of

life, people cannot adjust their consumption because old people will not hold bonds.

(e) Lagrangian for the problem is

$$\mathcal{L} = u(c_{20}) + \sum_{t=0}^{\infty} \left\{ (u(c_{1t}) + \beta u(c_{2t+1})) + \lambda_t \left( e_1 + \frac{e_2}{1+n} - c_{1t} - \frac{c_{2t}}{1+n} - g_t \right) \right\}$$

characterization:

$$\beta(1+n)u'(c_{2t}) = u'(c_{1t})$$

(f) for policy to have a role we need that bringing about the planner's outcome involves transfers from the young to the old.

(g) Planner's solution now implies

$$(1+n)c_{1t} = c_{2t}$$

resource constraint says

$$\frac{(2+n)e}{1+n} = c_{1t} + \frac{c_{2t}}{1+n} + g$$

These imply that in the planner's allocation:

$$c_{1t} = \frac{(2+n)e}{2\,(1+n)} - \frac{g}{2}, \quad c_{2t} = \frac{(2+n)e}{2} - \frac{(1+n)g}{2}$$

But in decentralized economy, $c_{it} = e - \tau_{it}$, $i = 1, 2$. So we can manipulate the tax code so as to provide optimal consumption pattern. In particular solving for $\tau_{2t}$ reveals

$$e - \tau_{2t} = \frac{(2+n)e}{2} - \frac{(1+n)g}{2}$$

so

$$\tau_{2t} = \frac{(1+n)g - ne}{2}$$

but $\tau_{2t}$ has to be non-negative so we can only use the tax code to bring about efficiency if $g > \dfrac{ne}{1+n}$. That is, higher government spending affords more flexibility in the tax code to acheve distributional outcomes.

(h) Here $\dfrac{(2+n)\tau}{1+n} = g$ so government budget constraint becomes

$$(1+n)b_{t+1} = R_t b_t$$

which with $\beta = 1$ is consistent with the planners solution for constant $b$. To pin down the value of $b$ that is consistent with competitive equilibrium plug $R_t = (1+n)$ into the characterization of equilibrium to get

$$e + (1+n)^2 b^* - \tau = (1+n)[e - (1+n)b^* - \tau]$$

So

$$b^* = \frac{n(e-\tau)}{2(1+n)^2}$$

if you used the other formulation for the government budget constraint you get $b^* = \dfrac{n(e-\tau)}{2(1+n)}$.