

Macroeconomics I: Mid-Term Exam

Consider the following model:

Time: discrete, infinite horizon

Demography: $N_t = (1 + n)^t N_0$ newborns in period t . Everyone lives for 2 periods

Preferences: $U_t(c_{1t}, c_{2t+1}) = u(c_{1t}) + \beta u(c_{2t+1})$, $u(\cdot)$ strictly increasing and strictly concave

Endowments: every individual receives e_i units of the (perishable) consumption good in period i of his/her life.

Information: complete, perfect foresight

Institutions: A *government* has to meet an *exogenous spending stream*, g_t per young person. It can raise *taxes* τ_{it} per person in life period i and issue *bonds*, b_t per young person to cover expenditures. *Markets* in bonds and consumption goods (ignore inside money).

(a) Write down the problem for an individual born in period t and derive the equation that (implicitly) gives savings as a function of the parameters of the individual's problem.

(b) What is the government budget constraint in period t ? (Assume government Ponzi-games are ruled out)

(c) Define and characterize the competitive equilibrium.

(d) To what extent can you change the paths of b_t , τ_{1t} and τ_{2t} , subject to the government budget constraint, so that the allocation does not change? Comment on the answer.

(e) Write down the Planner's problem and characterize his optimal allocation.

(f) Under what circumstances might government policy be Pareto improving?

Now suppose $u(c) = \ln(c)$, $\beta = 1$, $e_1 = e_2 = e$ and $g_t = g$ for all time.

(g) If the government in the decentralized economy cannot run a deficit (i.e. $b_t = 0$ for all t) and taxes are non-negative, what is the minimum size of g required for the government to achieve the optimal allocation. Explain the answer.

(h) Suppose instead that the government cannot be ageist so that $\tau_{1t} = \tau_{2t} = \tau$. As long as taxes cover the immediate government spending (i.e. $(2 + n)\tau / (1 + n) = g$) solve for the optimal level of (constant) government debt, b^* in terms of n , e and τ .