Final Exam: Answer both questions time allowed 3 hours. You may consult your notes or text books but you may not communicate with anyone about the exam. You must email your answers to me as a pdf by 11:30am US Eastern time. Please make sure that the files you upload are legible. There are 2 questions over 2 pages in this exam. Read all of the question before starting to answer it.

1. **Optimal growth with government spending preferences**

   Time: Discrete; infinite horizon

   Demography: A single representative (price taking) consumer/producer household and a single representative (price taking) firm.

   Preferences: the instantaneous household utility function over, individual consumption, c, and government spending, g, is $U(c, g) = u(c) + v(g)$. Both $u(.)$ and $v(.)$ are twice differentiable, strictly increasing and strictly concave and bounded above with marginal utility going to infinity as c or g go to zero. The discount factor is $\beta \in (0, 1)$.

   Productive Technology: There is a constant returns to scale aggregate technology over capital and labor such that output per unit of labor employed is $f(k)$, where k is capital input per unit of labor; $f(.)$ is twice differentiable, strictly increasing and concave with $f(0) = 0$, $\lim_{k\to 0} f'(k) = \infty$ and $\lim_{k\to\infty} f'(k) = 0$. Capital depreciates at the rate $\delta < 1$.

   Endowments: The households' initial capital stock is k_0, it also has 1 unit of labor each period. Firms have access to the technology, $f(.)$.

   (a) The Social Planner wants to choose paths for consumption **and government spending** to maximize welfare. Write down the Planner's problem in recursive form (i.e. using value functions) and solve it.

   Market Economy Institutions: In the market economy there is a government that has to meet an exogenous (for now) stream a of expenditures $\{g_t\}$. To do so it imposes a sequence of proportional tax rates $\{\tau_t\}$. The whole of the household's current period income (from capital and labor) is taxed proportionally each period.

   (b) Write down and solve the household's problem

   (c) Write down and solve the firm's problem

   (d) Write down the market clearing conditions and the government budget constraint. Define a competitive equilibrium.

   (e) Obtain a system of 2 difference equations that characterizes equilibrium in terms of c_t and k_t for given $\{g_t\}$.

   (f) Is the competitive equilibrium efficient for given g_t? Explain your answer

   (g) Write down the system of equations that characterize a **steady state** equilibrium (k^*, c^*)

   (h) Now we consider a situation in which the government can control g but does so using τ a constant tax rate. Obtain a system of equations in k^* and τ that characterizes the maximum level of government spending, g, achievable in steady state.

   (i) Now let $f(k) = k^\alpha$ and obtain the government spending maximizing tax rate, $\hat{\tau}$. Explain why this exists.

2. **Diamond Coconut Economy with random nut quality**

   Time: Discrete, infinite horizon

   Geography: A trading island and a production island.

   Demography: A mass 1 of ex ante identical individuals with infinite lives.

   Preferences: The common discount rate is r. Consumption of one's own produce yields 0 utils. Consumption of someone's output of quality $u \in [0, \bar{u}]$ yields u utils. **Note:** the value of u is not match specific, everyone agrees on the quality of a given nut which is also known to its producer.

   Productive Technology: On the production island individuals come across a tree with a coconut with probability α each period. Once they find a tree obtaining a coconut is costless (i.e. there is no distribution of tree climbing costs and $c = 0$). The quality of the nut they find is continuously distributed $F(u)$ on $[0, \bar{u}]$. The producer observes the quality of the nut and decides whether to pick it up and carry it to the trading island or whether to ignore that nut and search for a better one. (The reason you might not pick up a nut is that if it is low quality, people on the trading island may not give up their nut for yours. Then you would remain on the trading island forever.)

   Matching Technology: On the trading island people with coconuts meet each other with probability γ each period; γ is a constant (i.e. invariant to the number of individuals on the trading island). When you meet someone, you both decide whether to trade based on the quality of the nuts you are both carrying.

   Navigation: Travel between islands is free and instantaneous.

   Endowments: Everyone has a boat and starts off with one of their own coconuts of quality $\bar{u}$. (This is just to set the economy going we will focus on steady state equilibrium.)

   (a) Suppose that everyone else has a reservation nut quality, u^*. This means that on the production island no one will pick up a nut of quality less than u^* and on the trading island, they will not give up their nut unless the one you offer them is at least of quality u^*. On that basis write down the flow value equations for the individual who has to decide for him/herself which nuts to accept on the trading island.

   (b) Given u^*, obtain an implicit equation for $\hat{u}$ the reservation quality accepted by the individual in exchange for the nut s/he is carrying.

   (c) Define a steady state search equilibrium in which everyone on the trading island accepts all coconuts. (**Hint:** in equilibrium $u^* = \hat{u}$.)

   (d) Obtain a characterization of equilibrium.

   (e) Draw a diagram showing the flow rates between and populations on the islands. Use this to obtain an expression in terms of equilibrium u^* for the steady state population, n_T on the trading island.

   (f) Now let $F(u)$ be uniform on $[0, 1]$ so that

   $$F(u) = \begin{cases} 0 \text{ for } u < 0 \\ u \text{ for } u \in [0,1] \\ 1 \text{ for } u > 1 \end{cases} .$$

   Use the characterization of equilibrium to obtain an implicit function for u^*.

   (g) Now simplify the problem further by setting $r = 0$. Obtain the possible values of u^*. Explain the result.