Final Exam: Answer both questions time allowed 2 hours

1. **One-sided search with declining "benefits"**

   Real world unemployment insurance systems have an initial level of benefits that expire at some point. After expiry the worker moves to some lower level of benefits typically called unemployment (or social) assistance. We will explore, in the simplest possible framework, how such a policy can affect job search behavior.

   Time: Discrete, infinite horizon.

   Demography: A single infinite lived worker.

   Preferences: The worker is risk neutral (i.e. $u(w) = w$) and discounts the future at the rate r.

   Endowments: Throughout any unemployment spell, each period with probability α the worker gets a wage offer, $w \sim F(.)$ on $[0, \bar{w}]$. After any job loss, for each period the worker remains unemployed he initially receives unemployment benefits (UB) which provide $b_H < \bar{w}$ units of the consumption good. But with probability γ every period he switches to unemployment assistance (UA) which only pays $b_L < b_H$ per period. (Assume that getting a job and switching from UB to UA are mutually exclusive events and that $\alpha + \gamma < 1$.) While employed at wage w the worker gets w units of the consumption good each period but is also subject to layoff with probability λ.

   (a) Write down the flow value equations for the worker.

   (b) There will be two reservation wages, w_H^* for those on UB and w_L^* for those on UA. Obtain expressions for V_u^L and V_u^H (the values to being unemployed on UB and UA respectively) in terms of these reservation values and parameters only.

   (c) Use the expressions derived in part b. to obtain a system of two reservation equations that jointly determine the reservation wages in terms of (each other and) model parameters only.

   (d) Now suppose there are a unit mass continuum of such workers. Draw a diagram to show how workers flow between employment states. Without solving them, write down a system of equations that can be used to obtain the steady state proportions of workers in each state.

2. **Endowment model with "cash" and "credit" goods (inspired by Lucas and Stokey [1987])**

   Consider the following economy:

   Time: discrete, infinite horizon, $t = 1, 2, \ldots$

   Demography: A single representative household (takes prices as given)

   Preferences: The household likes to consume two goods. Good 1 is a cash good which requires cash in advance of the period in which the purchase is made. Good 2 is a "credit" good which just means that it does not require cash for its acquisition. The household gets utility $u(c_{1,t}, c_{2,t})$ from consumption of $c_{i,t}$ units of good $i = 1, 2$ at time t. Assume that u is strictly increasing in both arguments and strictly concave with

   $$\lim_{c_{1,t} \to 0} u_1(c_{1,t}, c_{2,t}) = \lim_{c_{2,t} \to 0} u_2(c_{1,t}, c_{2,t}) = \infty$$

   The household's discount factor is $\beta < 1$.

   Endowments: In period t, the household receives y_t units of generic perishable consumption good and can choose freely how much of it to take to the market as a cash good, $y_{1,t}$ and how much to take to the market as a credit good, $y_{2,t}$ (so $y_{1,t} + y_{2,t} = y_t$). The usual assumption that you cannot consume your own output/endowment applies so you have to bring the goods to market. **Note:** This assumption means that both goods will always trade at the same price, p_t. If not, people would only bring the higher priced good to market and our assumptions on preferences rule out such a corner solution.

   Institutions: There is a government that issues currency so that the stock at time t is H_t. the time t growth rate of the money supply is σ_t so that $H_{t+1} = (1 + \sigma_t)H_t$. The money is distributed (or collected if $\sigma_t < 0$) lump sum so that the household's period t transfer (tax) is τ_t. The household can only acquire the cash good using money which has to have been acquired in the previous period. The credit good can be acquired on credit in that the household can use the current period's proceeds from sales to obtain it. This means they can effectively swap their own goods for credit goods and do not need to go through cash.

   (a) Write down the problem faced by the representative household. (**Note:** there is no capital in this model so the household's only state variable will be M_t their nominal money holdings.)

   (b) Obtain the first-order, complementary slackness and envelope conditions.

   (c) Obtain an intertemporal (Euler type) condition that relates the marginal utility of consuming good 2 today and good 1 tomorrow. Explain this expression.

   (d) Write down the market-clearing conditions and the government budget constraint. Define a competitive equilibrium.

   (e) Now suppose $\sigma_t = \sigma$ and $y_t = y$ for all t and that we are in a steady state so that all real variables are constant (e.g. $c_{1,t} = c_1$ and $c_{2,t} = c_2$ for all t). What does this mean for the real return on money, p_t/p_{t+1} and the optimality of the Friedman rule? Explain your answer.