Macroeconomics I

Final Exam

Answer both questions time allowed 2 hours

(1) Cocoanut model with money (base on Kiyotaki and Wright *JET* 91

Time: Discrete, infinite horizon.

Demography: A continuum, mass 1 of individuals

Geography: Two islands, trading and production

Preferences: Consumption of own output yields 0 utils, the utility derived from consumption of someone else's output yields depned on whether you happen to like it or not. You get u utils if you like their good and 0 utils if you don't. You can tell whether you will like the good or not by simply looking at it. The ex ante probability that you like their good is x. If you don't like their good you will not trade with them as you cannot store their good at all. There is a common discount rate r. Output is non-divisible and non-storable except by the original producer. There is a maximum inventory of 1.

Endowments: A fraction $1 - m$ of the population are initially endowed with their own output. The remainder are endowed with an intrinsically valueless commodity called money. (So everyone starts out on the trading island.) Anyone can hold money but it is indivisible and it is subject to the maximum inventory rule too. That means someone can hold their own output or a unit of money but not both. (They cannot hold anyone else's output.) We *will* allow for free disposal of money.

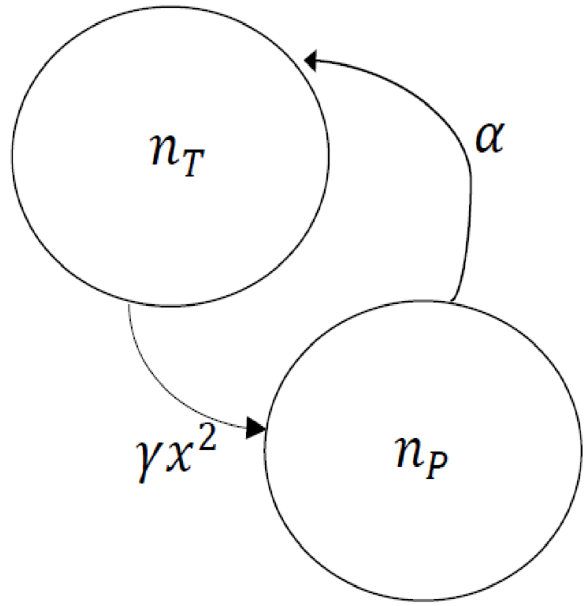
Productive Technology: Individuals on the production island get a production opportunity each period with probability α. Production is costless – nuts lying on the ground. (Once they produce they travel to the trading island)

Matching technology: On the trading island an individual meets another with probability γ each period.

Notes: Use the following notation: V_P, V_T, V_M represent the values to being respectively on the production island, on the trading island holding one's own output, and on the trading island holding money. Use n_i, $i = P, T, M$ to similarly represent the number of individuals in each state. Notice that, as long as money is valued (i.e. it curculates) n_M is equal to m which is exogenous. Let μ be the proportion of people on the trading island who hold money. So, as long as money is valued,

$$\mu = \frac{m}{m + n_T}$$

We will focus entirely on steady-states.



To start with we will assume that money does not circulate. In that case the people who are initially endowed with money will simply throw it way and head for the production island.

(a) As the probability that any individual likes anyone else's good is x. What is the probability that in any meeting trade takes place (i.e. that there is double coincidence of wants)?

The probability is x^2.

(b) Write down the flow asset value or Bellman equations under the supposition that money does **not** circulate (but there is free disposal of money).

$$\begin{aligned} rV_P &= \alpha(V_T - V_P) \\ rV_T &= \gamma x^2(u + V_P - V_T) \end{aligned}$$

These values were defined in the question.

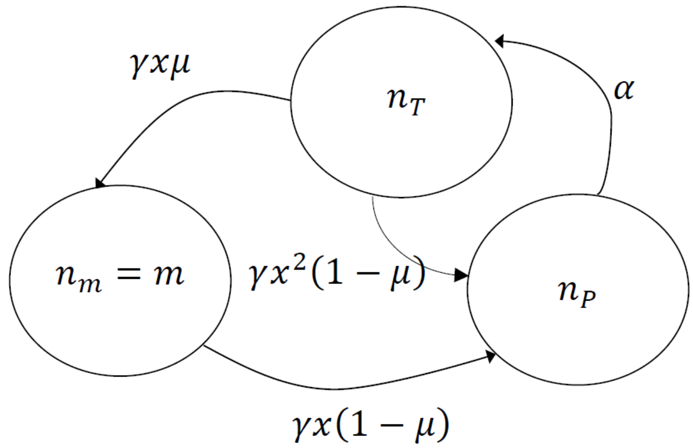
(c) Solve for the values to being on the production and trading islands.

$$\begin{aligned} rV_P &= \frac{\alpha\gamma x^2 u}{r + \alpha + \gamma x^2} \\ rV_T &= \frac{(r+\alpha)\gamma x^2 u}{r + \alpha + \gamma x^2} \end{aligned}$$

(d) Draw a diagram showing the flows between the states

(e) Obtain and solve the set of equations that can be used to determine steady state values of n_T and n_P.

$$\begin{aligned} \gamma x^2 n_T &= \alpha n_P \\ n_T + n_P &= 1 \end{aligned}$$



So

$$n_T = \frac{\alpha}{\alpha+\gamma x^2}, \qquad n_P = \frac{\gamma x^2}{\alpha+\gamma x^2}$$

Now assume instead that money circulates. That is, people will take money in exchange for their good.

(f) Write down the flow asset value or Bellman equations under the supposition that money circulates. (Hint: the probability that a random individual on the trading island holds money is μ. Money holders do not trade with other money holders as their are no gains from trade. When someone with a good meets someone with money trade will occur as long as the person with money likes the good on offer.) **Do not attempt to solve these equations!**

$$\begin{aligned}
rV_P &= \alpha(V_T - V_P) \\
rV_T &= \gamma\left[\mu x(V_M - V_T) + (1-\mu)x^2(u + V_P - V_T)\right] \\
rV_M &= \gamma(1-\mu)x(u + V_P - V_M)
\end{aligned}$$

(g) Draw a diagram showing the flows between the states when money circulates

(f) Write down but do **not** solve a system of equations that can me used to determine the steady state populations in each state in terms of the parameters of the model. (Recall that $n_M = m$ by assumption but that μ is endogenous.)

The set of equations can be any 3 of the following 5 associated with flows in and out of each state being equal and an overall adding up constraint. They determine n_T, n_P and μ.

$$\begin{aligned}
\gamma x\mu n_T &= \gamma x(1-\mu)m \\
\gamma x(1-\mu)(m + x n_T) &= \alpha n_P \\
\alpha n_P &= \gamma x\left[\mu + x(1-\mu)\right] n_T \\
n_T + n_P &= 1 - m \\
\mu &= \frac{m}{m + n_T}
\end{aligned}$$

(2) **Cash-in-Advance for investment goods only**

Time: Discrete; infinite horizon

Demography: A single representative (price taking) consumer/producer household.

Preferences: the instantaneous household utility function over, individual consumption, c, is $u(c)$ where $u(.)$ is twice differentiable, strictly increasing and strictly concave. The discount factor is $\beta \in (0,1)$.

Technology: There is a constant returns to scale technology over capital and labor such that output per unit of labor employed is $f(k)$, where k is capital input per unit of labor; $f(.)$ is twice differentiable, strictly increasing and concave. Capital depreciates at the rate $\delta < 1$.

Endowments: Households' initial capital stock is k_0, each household has 1 unit of labor and access to the technology (i.e. there are no firms - the household does its own production in its backyard).

Initial cash holdings are H_0 for each household.

Institutions: A central bank issues new currency every period so that the total cash in the economy $H_t = (1+\sigma)^t H_0$

Government distributes the new cash in period t as transfers, τ_t. (These can be negative if σ is negative.)

Legal Tender Requirement: investment has to be paid for with cash. (Consumption does not require cash in advance.)

Markets: The market in consumption goods for money is competitive - the price of goods in terms of money is p_t

(a) Write down and solve the households problem in recursive form (i.e. using dynamic programming).

$$\begin{aligned}
V(M_t, k_t) &= \max_{c_t, k_{t+1}, M_t^d} \{u(c_t) + \beta V(M_{t+1}, k_{t+1})\} \\
\text{Subject to} &: f(k_t) + (1-\delta)k_t + \frac{M_t}{P_t} = c_t + k_{t+1} + \frac{M_t^d}{P_t} \\
M_{t+1} &= M_t^d + \tau_{t+1} \\
\frac{M_t}{P_t} &\geq i_t \qquad \text{where } i_t = k_{t+1} - (1-\delta)k_t
\end{aligned}$$

So

$$\begin{aligned}
\mathcal{L} &= u(c_t) + \beta V(M_{t+1}, k_{t+1}) + \lambda_t \left[f(k_t) + (1-\delta)k_t + \frac{M_t}{P_t} - c_t - k_{t+1} - \frac{M_t^d}{P_t} \right] \\
&\quad + \gamma_t \left[\frac{M_t}{P_t} - k_{t+1} + (1-\delta)k_t \right]
\end{aligned}$$

where $M_{t+1} = M_t^d + \tau_{t+1}$.

F.O.C's:

$$\begin{aligned} c_t &: \quad u'(c_t) - \lambda_t = 0 \\ k_{t+1} &: \quad \beta V_2(t+1) - \lambda_t - \gamma_t = 0 \\ M_t^d &: \quad \beta V_1(t+1) - \frac{\lambda_t}{P_t} = 0 \end{aligned}$$

where $V_i(t) = V_i(M_t, k_t)$. Complementary slackness:

$$\gamma_t \left[\frac{M_t}{P_t} - k_{t+1} + (1-\delta)k_t \right] = 0 \qquad \gamma_t \geq 0$$

TVC:

$$\lim_{t\to\infty} \beta^t u'(c_t) k_t = 0$$

Envelope:

$$\begin{aligned} V_1(t) &= \frac{\lambda_t}{P_t} + \frac{\gamma_t}{P_t} \\ V_2(t) &= \lambda_t \left[f'(k_t) + 1 - \delta \right] + \gamma_t (1-\delta) \end{aligned}$$

(b) Solve for the household's Euler equation and interpret it.

Substitute envelope conditions into FOC's to get

$$\begin{aligned} \beta\lambda_{t+1} \left[\lambda_{t+1} f'(k_{t+1}) + (\lambda_{t+1} + \gamma_{t+1})(1-\delta) \right] - \lambda_t - \gamma_t &= 0 \\ \beta \left(\frac{\lambda_{t+1} + \gamma_{t+1}}{P_{t+1}} \right) - \frac{\lambda_t}{P_t} &= 0 \end{aligned}$$

so

$$\lambda_{t+1} + \gamma_{t+1} = \frac{P_{t+1} u'(c_t)}{P_t \beta}.$$

The implied Euler equation is then

$$u'(c_t) = \beta \left(\frac{P_{t+2}}{P_{t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) (1-\delta) u'(c_{t+1}) + \beta^2 \left(\frac{P_t}{P_{t+1}} \right) f'(k_{t+2}) u'(c_{t+2}).$$

To understand why this is optimal consider giving up an infinitesimal unit of consumption. It costs me $u'(c_t)$ right now. I cannot save it as capital because I need cash to buy capital. So, I convert it to cash. This gives real return next period of $\left(\frac{P_t}{P_{t+1}}\right)$ units of purchasable capital. I buy the capital which gives a direct return of $f'(k_{t+2})$ two periods hence. The middle term is a bit trickier. Now we get the undepreciated capital back 2 periods from now so you might expect a term like $\beta^2 \left(\frac{P_t}{P_{t+1}}\right)(1-\delta)u'(c_{t+2})$ on the right hand side. However in anticipation of receiving the extra undepreciated capital output in period $t+2$, the household can consume $\left(\frac{P_{t+2}}{P_{t+1}}\right)$ times that in period $t+1$ because consumption does not require cash in advance.

(c) Write down the government's budget constraint, the market clearing conditions, and define a competitive equilibrium.

Market clearing:

$$\begin{aligned} \text{Money} &: \quad M_t = H_t \\ \text{Goods} &: \quad c_t + k_{t+1} = f(k_t) + (1-\delta)k_t \end{aligned}$$

GBC:

$$\tau_t = \sigma M_{t-1}$$

Definition: A Perfect foresight competitive equilibrium is an allocation, $\{c_t, k_{t+1}, m_t\}$, a price sequence, $\{P_t\}$ and a sequence of transfers $\{\tau_t\}$ such that given prices and transfers, the allocation solves the household's problem, markets clear and the GBC holds.

(d) Solve for a system of equations that characterizes the steady-state competitive equilibrium.

In steady state,

$$\frac{P_t}{P_{t+1}} = \frac{P_{t+1}}{P_{t+2}} = \frac{1}{1+\sigma}$$

this implies that the Euler equation reduces to

$$f'(k^*) = \frac{(1+\sigma)[1-\beta(1-\delta)]}{\beta^2}$$

for all values of σ.

(e) Is money super-neutral? Briefly explain.

This is the same steady state relationship as in the class notes. Money is not super-neutral. The Friedman rule that sets $\beta = 1+\sigma$ is optimal as it equates the rate of return on capital to that of cash.