AECO601 Fall 2014

Macroeconomics I

**Final Exam**

**Answer both questions time allowed 2 hours**

**(1) Degenerate Diamond Cocoanut model with money**

**Time:** Discrete, infinite horizon.

**Demography:** A continuum, mass 1 of individuals

**Geography:** Two islands, trading and production

**Preferences:** Consumption of own output yields 0 utils, consumption of someone else's output yields $u$ utils. There is a common discount rate $r$. Output is non-divisible and non-storable except by the original producer. There is a maximum inventory of 1.

**Endowments:** A fraction $1-m$ of the population are initially endowed with their own output. The remainder are endowed with an intrinsically valueless commodity called money. Anyone can hold money but it is indivisible and it is subject to the maximum inventory rule too. That means someone can hold their own output or a unit of money but not both. There is no free disposal of money (you cannot get rid of it).

**Productive Technology:** Individuals on the production island get a production opportunity each period with probability $\alpha$. Production is costless – nuts lying on the ground. (Once they produce they travel to the trading island)

**Matching technology:** On the trading island an individual meets another with probability $\gamma$ each period.

**Notes:** Use the following notation: $V_P$, $V_T$, $V_M$ represent the values to being respectively on the production island, on the trading island holding one's own output, and on the trading island holding money. Use $n_i$, $i = P, T, M$ to similarly represent the number of individuals in each state. You can ignore $n_M$, though, because it is equal to $m$ which is exogenous – whether money changes hands or not the proportion of the population who hold money will not change. Also let $\mu$ be the proportion of people on the trading island who hold money. So

$$\mu = \frac{m}{m + n_T}$$

Of course, $\mu$ is endogenous because $n_T$ is endogenous

**To start with we will assume that money circulates. That is, people will take money in exchange for their good.**

(a) Write down the flow asset value or Bellman equations under the supposition that money circulates. (Hint: the probability that a random individual on the trading island holds money is $\mu$. Also, money holders do not trade with other money holders as there are no gains from trade.)

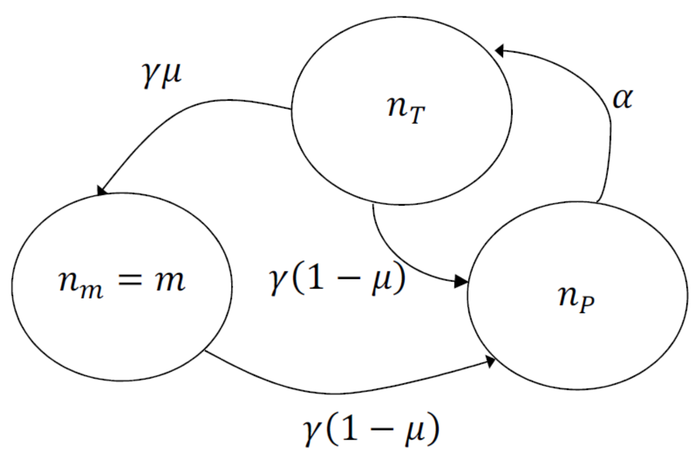
$$\begin{aligned}
rV_P &= \alpha(V_T - V_P) \\
rV_T &= \gamma\left[\mu(V_M - V_T) + (1-\mu)(u + V_P - V_T)\right] \\
rV_M &= \gamma(1-\mu)(u + V_P - V_M)
\end{aligned}$$

(b) Solve for the capital gain associated with accepting money, i.e. $V_M - V_T$. Is this consistent with money circulating? Briefly explain your answer.

$$\begin{aligned}
r(V_T - V_M) &= \gamma\left[\begin{array}{c} \mu(V_M - V_T) + (1-\mu)(u + V_P - V_T) \\ -(1-\mu)(u + V_P - V_M) \end{array}\right] \\
(r + \gamma\mu)(V_T - V_M) &= \gamma(1-\mu)\left[(u + V_P - V_T) - (u + V_P - V_M)\right] \\
&= -\gamma(1-\mu)(V_T - V_M) \\
(V_T - V_M) &= 0
\end{aligned}$$

People are indifferent between holding their own good and money. As each trades equally quickly for the same benefit there is no reason to reject money while others are taking it.

(c) Draw a diagram showing the flows between the states




(d) Write down the system of equations that can be used to obtain the steady state population shares in each state. (Hint: recall that $n_M = m$ is exogenous but $\mu$ is endogenous.)

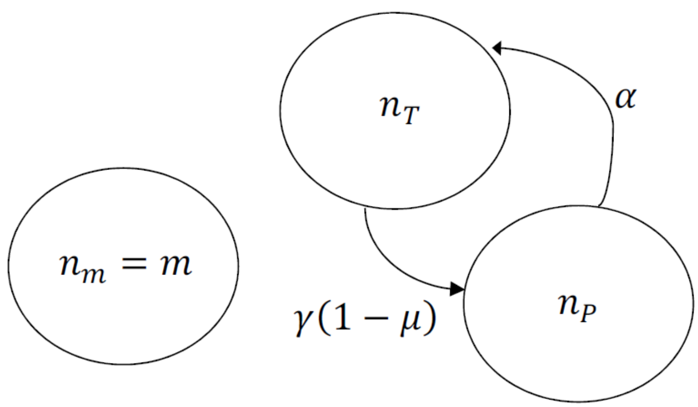
$$\begin{aligned} n_T + n_P &= 1 - m \\ \mu &= \frac{m}{m + n_T} \\ m\gamma(1-\mu) + n_T\gamma(1-\mu) &= \alpha n_P \end{aligned}$$

**Now assume that money does not circulate. Whoever holds money is stuck with it and cannot do anything about it. (This means that** $V_M = 0$.)

(e) Write down the flow asset value or Bellman equations under the supposition that money does **not** circulate. (Again the probability that a random individual on the trading island holds money is $\mu$. People with cocoanuts still meet people with money but they do not trade with them.)

$$\begin{aligned} rV_P &= \alpha(V_T - V_P) \\ rV_T &= \gamma(1-\mu)(u + V_P - V_T) \end{aligned}$$

(f) Draw the flow diagram again and obtain the steady state equations.




$$\begin{aligned} n_T + n_P &= 1 - m \\ \mu &= \frac{m}{m + n_T} \\ n_T\gamma(1-\mu) &= \alpha n_P \end{aligned}$$

(Bonus) Can you tell whether the equilibrium in which money circulates has higher output than the equilibrium in which it does not?

When money circulates we have

$$(m + n_T)\,\gamma(1-\mu) = \alpha n_P$$

and

$$1-\mu = \frac{n_T}{m+n_T}$$

so

$$n_T\gamma = \alpha n_P$$

while when it does not circulate

$$n_T\gamma(1-\mu) = \alpha n_P$$

Which implies that there are fewer people on the production island when money does not circulate. As $\alpha n_P$ is output in the economy, the circulation of money helps to facilitate trade.

(2) **Cash-in-Advance for consumption goods only**

**Time:** Discrete; infinite horizon

**Demography:** A single representative (price taking) consumer/producer household.

**Preferences:** the instantaneous household utility function over, individual consumption, $c$, is $u(c)$ where $u(.)$ is twice differentiable, strictly increasing and strictly concave. The discount factor is $\beta \in (0,1)$.

**Technology:** There is a constant returns to scale technology over capital and labor such that output per unit of labor employed is $f(k)$, where $k$ is capital input per unit of labor; $f(.)$ is twice differentiable, strictly increasing and concave. Capital depreciates at the rate $\delta < 1$.

**Endowments:** Households' initial capital stock is $k_0$, each household has 1 unit of labor and access to the technology (i.e. there are no firms - the household does its own production in its backyard).

Initial cash holdings are $H_0$ for each household.

**Institutions:** A central bank issues new currency every period so that the total cash in the economy $H_t = (1+\sigma)^t H_0$

Government distributes the new cash in period $t$ as transfers, $\tau_t$. (These can be negative if $\sigma$ is negative.)

Legal Tender Requirement: Consumption has to be paid for with cash. (Investment does not require cash in advance.)

**Markets:** The market in consumption goods for money is competitive - the price of goods in terms of money is $p_t$

(a) Write down and solve the households problem in recursive form (i.e. using dynamic programming).

$$
\begin{aligned}
V(M_t, k_t) &= \max_{c_t, k_{t+1}, M_t^d} \{u(c_t) + \beta V(M_{t+1}, k_{t+1})\} \\
\text{Subject to} &: \quad f(k_t) + (1-\delta)k_t + \frac{M_t}{P_t} = c_t + k_{t+1} + \frac{M_t^d}{P_t} \\
M_{t+1} &= M_t^d + \tau_{t+1} \\
\frac{M_t}{P_t} &\geq c_t
\end{aligned}
$$

So

$$
\begin{aligned}
\mathcal{L} &= u(c_t) + \beta V(M_{t+1}, k_{t+1}) + \lambda_t \left[ f(k_t) + (1-\delta)k_t + \frac{M_t}{P_t} - c_t - k_{t+1} - \frac{M_t^d}{P_t} \right] \\
&\quad + \gamma_t \left[ \frac{M_t}{P_t} - c_t \right]
\end{aligned}
$$

where $M_{t+1} = M_t^d + \tau_{t+1}$.

F.O.C's:

$$
\begin{aligned}
c_t &: \quad u'(c_t) - \lambda_t - \gamma_t = 0 \\
k_{t+1} &: \quad \beta V_2(t+1) - \lambda_t = 0 \\
M_t^d &: \quad \beta V_1(t+1) - \frac{\lambda_t}{P_t} = 0
\end{aligned}
$$

where $V_i(t) = V_i(M_t, k_t)$. Complementary slackness:

$$
\gamma_t \left[ \frac{M_t}{P_t} - c_t \right] = 0
$$

TVC:

$$
\lim_{t\to\infty} \beta^t u'(c_t) k_t = 0
$$

Envelope:

$$
\begin{aligned}
V_1(t) &= \frac{\lambda_t}{P_t} + \frac{\gamma_t}{P_t} \\
V_2(t) &= \lambda_t \left[ f'(k_t) + 1 - \delta \right]
\end{aligned}
$$

(b) Solve for the household's Euler equation and interpret it.

Sub envelope conditions into FOC's to get

$$
\begin{aligned}
\beta \lambda_{t+1} \left[ f'(k_{t+1}) + 1 - \delta \right] - \lambda_t &= 0 \\
\beta \left( \frac{\lambda_{t+1} + \gamma_{t+1}}{P_{t+1}} \right) - \frac{\lambda_t}{P_t} &= 0
\end{aligned}
$$

so

$$\lambda_t = \frac{P_t}{P_{t+1}} \beta u'(c_{t+1})$$

and

$$\frac{P_t}{P_{t+1}} \beta u'(c_{t+1}) = \frac{P_{t+1}}{P_{t+2}} \beta^2 u'(c_{t+2}) \left[ f'(k_{t+1}) + 1 - \delta \right]$$

To facilitate an additional amount of consumption next period, I can convert some current savings from investment into money. That does not change current utility but yields $\frac{P_t}{P_{t+1}}$ of the consumption good next period. I value that at $u'(c_{t+1})$ per unit which has to be discounted to convert it into current period utility. The investment I failed to make lowers next period's funds available for conversion into money by $f'(k_{t+1}) + 1 - \delta$ which means a reduction in consumption 2 periods from now by $\frac{P_{t+1}}{P_{t+2}} \left[ f'(k_{t+1}) + 1 - \delta \right]$ which I value at $u'(c_{t+2})$ per unit and I have discount it for 2 periods. At the optimum these two effects have to cancel each other out.

(c) Write down the government's budget constraint, the market clearing conditions, and define a competitive equilibrium.

Market clearing:

$$\begin{aligned} \text{Money} &: \quad M_t = H_t \\ \text{Goods} &: \quad c_t + k_{t+1} = f(k_t) + (1 - \delta) k_t \end{aligned}$$

GBC:

$$\tau_t = \sigma M_{t-1}$$

**Definition:** A Perfect foresight competitive equilibrium is an allocation, $\{c_t, k_{t+1}, m_t\}$, a price sequence, $\{P_t\}$ and a sequence of transfers $\{\tau_t\}$ such that given prices and transfers, the allocation solves the household's problem, markets clear and the GBC holds.

(d) Solve for a system of equations that characterizes the steady-state competitive equilibrium.

In steady state,

$$\frac{P_t}{P_{t+1}} = \frac{P_{t+1}}{P_{t+2}} = \frac{1}{1+\sigma}$$

this implies that the Euler equation reduces to

$$f'(k_{t+1}) + 1 - \delta = \frac{1}{\beta}$$

for all values of $\sigma$.

(e) Is money super-neutral? Briefly explain.

Money is super-neutral. The return on money does not affect the return on capital. Consumers pay a one-time inflation tax to convert money to the consumption good. But this is constant overtime and the tax is fully rebated as transfers.