AECO601 Fall 2014

Macroeconomics I

Final Exam

Answer both questions time allowed 2 hours

(1) Degenerate Diamond Cocoanut model with money

Time: Discrete, infinite horizon.

Demography: A continuum, mass 1 of individuals

Geography: Two islands, trading and production

Preferences: Consumption of own output yields 0 utils, consumption of someone else's output yields u utils. There is a common discount rate r. Output is non-divisible and non-storable except by the original producer. There is a maximum inventory of 1.

Endowments: A fraction $1 - m$ of the population are initially endowed with their own output. The remainder are endowed with an intrinsically valueless commodity called money. Anyone can hold money but it is indivisible and it is subject to the maximum inventory rule too. That means someone can hold their own output or a unit of money but not both. There is no free disposal of money (you cannot get rid of it).

Productive Technology: Individuals on the production island get a production opportunity each period with probability α. Production is costless – nuts lying on the ground. (Once they produce they travel to the trading island)

Matching technology: On the trading island an individual meets another with probability γ each period.

Notes: Use the following notation: V_P, V_T, V_M represent the values to being respectively on the production island, on the trading island holding one's own output, and on the trading island holding money. Use n_i, $i = P, T, M$ to similarly represent the number of individuals in each state. You can ignore n_M, though, because it is equal to m which is exogenous – whether money changes hands or not, the proportion of the population who hold money will not change. Also let μ be the proportion of people on the trading island who hold money. So

$$\mu = \frac{m}{m + n_T}$$

Of course, μ is endogenous because n_T is endogenous

To start with we will assume that money circulates. That is, people will take money in exchange for their good.

(a) Write down the flow asset value or Bellman equations under the supposition that money circulates. (Hint: the probability that a random individual on the trading island holds money is μ. Also, money holders do not trade with other money holders as there are no gains from trade.)

(b) Solve for the capital gain associated with accepting money, i.e. $V_M - V_T$. Is this consistent with money circulating? Briefly explain your answer.

(c) Draw a diagram showing the flows between the states

(d) Write down the system of equations that can be used to obtain the steady state population shares in each state. (Hint: recall that $n_M = m$ is exogenous but μ is endogenous.)

Now assume that money does not circulate. Whoever holds money is stuck with it and cannot do anything about it. (This means that $V_M = 0$.)
(e) Write down the flow asset value or Bellman equations under the supposition that money does **not** circulate. (Again the probability that a random individual on the trading island holds money is μ. People with cocoanuts still meet people with money but they do not trade with them.)
(f) Draw the flow diagram again and obtain the steady state equations.
(Bonus) Can you tell whether the equilibrium in which money circulates has higher output than the equilibrium in which it does not?

(2) **Cash-in-Advance for consumption goods only**

Time: Discrete; infinite horizon

Demography: A single representative (price taking) consumer/producer household.

Preferences: the instantaneous household utility function over, individual consumption, c, is $u(c)$ where $u(.)$ is twice differentiable, strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology over capital and labor such that output per unit of labor employed is $f(k)$, where k is capital input per unit of labor; $f(.)$ is twice differentiable, strictly increasing and concave. Capital depreciates at the rate $\delta < 1$.

Endowments: Households' initial capital stock is k_0, each household has 1 unit of labor and access to the technology (i.e. there are no firms - the household does its own production in its backyard).

Initial cash holdings are H_0 for each household.

Institutions: A central bank issues new currency every period so that the total cash in the economy $H_t = (1 + \sigma)^t H_0$.

Government distributes the new cash in period t as transfers, τ_t. (These can be negative if σ is negative.)

Legal Tender Requirement: Consumption has to be paid for with cash. (Investment does not require cash in advance.)

Markets: The market in consumption goods for money is competitive - the price of goods in terms of money is p_t

(a) Write down and solve the households problem in recursive form (i.e. using dynamic programming).
(b) Solve for the household's Euler equation and interpret it.
(c) Write down the government's budget constraint, the market clearing conditions, and define a competitive equilibrium.
(d) Solve for a system of equations that characterizes the steady-state competitive equilibrium.
(e) Is money super-neutral? Briefly explain.