

Macroeconomics I

Final Exam**Answer both questions time allowed 2 hours****1. One-sided search with good and bad jobs****Time:** Discrete, infinite horizon.**Demography:** A single infinite lived worker.**Preferences:** The worker discounts the future at the rate r . When employed the worker gets flow utility w and while unemployed the worker gets flow utility $b < w$.**Endowments:**

Each period while unemployed, with probability α , the worker gets a job offer. With probability γ that offer is for a “good” job and with probability $1 - \gamma$ the offer is for a “bad” job. With probability $1 - \alpha$ the worker gets no offer. Good and bad jobs differ in the probability that the worker will be laid-off. In a good job he gets laid off with probability λ_l and in a bad job he gets laid-off with probability $\lambda_h > \lambda_l$.

(a) Write down the asset value (Bellman) equations for the worker.

Supposing that all meetings lead to matches

$$rV_u = b + \alpha\gamma(V_l - V_u) + \alpha(1 - \gamma)(V_h - V_u)$$

$$rV_i = w + \lambda_i(V_u - V_i), \quad i = h, l$$

(b) Show that no matter how high is λ_h , the worker will always accept offers for bad jobs. Explain this result.

$$\begin{aligned} r(V_h - V_u) &= w - b - [\lambda_h + \alpha(1 - \gamma)](V_h - V_u) - \alpha\gamma(V_l - V_u) \\ [r + \lambda_h + \alpha(1 - \gamma)](V_h - V_u) &= w - b - \alpha\gamma(V_l - V_u) \end{aligned}$$

by symmetry

$$[r + \lambda_l + \alpha\gamma](V_l - V_u) = w - b - \alpha(1 - \gamma)(V_h - V_u).$$

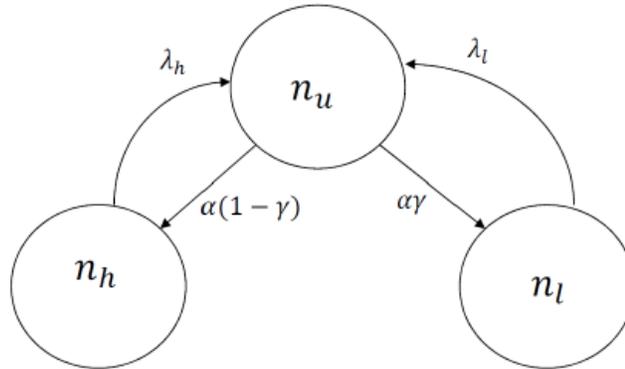
So,

$$[r + \lambda_h + \alpha(1 - \gamma)][r + \lambda_l + \alpha\gamma](V_h - V_u) = (r + \lambda_l)(w - b) + \alpha^2\gamma(1 - \gamma)(V_h - V_u).$$

As $[r + \lambda_h + \alpha(1 - \gamma)][r + \lambda_l + \alpha\gamma] > \alpha^2\gamma(1 - \gamma)$ and $w > b$, $V_h - V_u > 0$ for all λ_h .

Unlike with wages, the expected duration of a job can never be too low. Recall that the worker compares the value to employment in any job with that of unemployment. We know that the $w > w^*$, the reservation wage, and that $w^* = rV_u$. Getting the wage w no matter how briefly is better than remaining unemployed which is equivalent to getting w^* .

- (c) Draw a flow diagram showing the population movements between states when there is a continuum (mass 1) of similar workers.



- (d) Write down a system of equations that can be solved for the steady state populations (you don't need to solve them).

$$\begin{aligned}
 n_u + n_l + n_h &= 1 \\
 \alpha(1 - \gamma)n_u &= \lambda_h n_h \\
 \alpha\gamma n_u &= \lambda_l n_l
 \end{aligned}$$

- (e) Under what condition on γ is the number of workers in good jobs equal to the number in bad jobs?

$$\frac{\gamma}{(1 - \gamma)} = \frac{\lambda_l}{\lambda_h} \implies \gamma = \frac{\lambda_l}{\lambda_l + \lambda_h}$$

2. Optimal growth with costly investment

Time: Discrete; infinite horizon

Demography: A single representative consumer/worker household, and a single representative profit maximizing firm. Both household and firm live for ever.

Preferences: The instantaneous household utility function over, individual consumption, $c \in \mathbf{R}_+$ is $u(c)$ where $u' > 0$, $u'' < 0$ and $\lim_{c \rightarrow 0} u'(c) = \infty$. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology over capital and labor so that output per unit of labor is $f(k)$ where $f(\cdot)$ is increasing, concave and satisfies the Inada conditions. Output is in consumption goods which can be converted into capital at the fixed rate $1/\gamma$, where $\gamma \geq 1$. Thus if the household saves s_t the amount of capital goods it converts into is s_t/γ . Thus for values of $\gamma > 1$ this acts like a tax on savings (at the rate $1/\gamma$) the proceeds of which are thrown into the ocean. The depreciation rate of capital is $\delta < 1$.

Endowments: The household's initial capital stock is k_0 . Households have a single unit of labor each period.

(a) Write down and solve the Planner's problem for this economy.

$$\begin{aligned} & \max_{c_t, s_t, k_{t+1}} \sum_{t=0}^{\infty} u(c_t) \\ \text{subject to: } & f(k_t) = c_t + s_t \\ & s_t = \gamma [k_{t+1} - (1 - \delta)k_t] \end{aligned}$$

Problem becomes

$$V(k_t) = \max_{k_{t+1}} \{u(f(k_t) - \gamma [k_{t+1} - (1 - \delta)k_t]) + \beta V(k_{t+1})\}$$

F.O.C:

$$-\gamma u'(c_t) + \beta V'(k_{t+1})$$

Envelope equation

$$V'(k_t) = u'(c_t) [f'(k_t) + \gamma(1 - \delta)]$$

Eliminating the value function,

$$u'(c_t)\gamma = \beta u'(c_{t+1}) [f'(k_{t+1}) + \gamma(1 - \delta)]$$

- (b) What are the (non-trivial/interior) steady state levels of the capital stock and consumption, (k^*, c^*) ?

$$\begin{aligned} f'(k^*) &= \gamma(\rho + \delta) \\ c^* &= f(k^*) - \gamma\delta k^* \end{aligned}$$

where ρ is the discount rate.

- (c) Derive the loci of the points in (k_t, c_t) space for which $k_{t+1} = k_t$ and $c_{t+1} = c_t$ for all t .

$$k_{t+1} \geq k_t \Rightarrow c_t \leq f(k_t) - \gamma\delta k_t$$

and

$$\begin{aligned} c_{t+1} &\geq c_t \Rightarrow \beta u'(c_{t+1}) \leq \beta u'(c_t) \\ &\Rightarrow \frac{u'(c_t)\gamma}{[f'(k_{t+1}) + \gamma(1 - \delta)]} \leq \beta u'(c_t) \\ &\Rightarrow 1 \leq \beta \frac{[f'(k_{t+1}) + \gamma(1 - \delta)]}{\gamma} \\ &\Rightarrow k_{t+1} \leq k^* \\ &\Rightarrow \frac{f(k_t) - c_t}{\gamma} + (1 - \delta)k_t \leq k^* \\ &\Rightarrow c_t \geq f(k_t) + \gamma[(1 - \delta)k_t - k^*] \end{aligned}$$

- (d) Draw the phase diagram for the system.

Diagram is basically same as in class notes. For $\gamma > 1$ the $k_{t+1} = k_t$ is every where below the one for $\gamma = 1$. The $c_{t+1} = c_t$ can be to the right or the left of the original one depending on the impact on k^* .

- (e) Show by way of a diagram that when an unanticipated increase in γ occurs, consumption can initially jump down (remember that capital cannot jump). Explain how this can happen.

As long as the new saddle path is below the old one this will happen. consumers recognize their reduced ability to create capital and reduce consumption and increase saving to compensate.

- (f) In the case shown in part e., what happens to consumption and the capital stock after the initial jump in consumption?

Both capital and consumption will fall until the new steady state is achieved.