AECO601 Fall 2012

Macroeconomics I

Final Exam

Answer both questions time allowed 2 hours

1. **One-sided search with benefit replacement rate**

   Time: Discrete, infinite horizon

   Demography: A single worker

   Preferences: The worker discounts the future at the rate r. The worker is risk neutral, i.e. $u(x) = x$. (You, therefore, do not need to use a utility function in the analysis.)

   Endowments: When "employed" at the wage, w, the worker gets w units of the consumption good every period but also with probability λ he loses his job. When "unemployed" the worker receives benefit ϕw where w is the wage received in the last job he had. Also with probability α he gets a wage offer from the distribution, $F(.)$. The support of F is $[0, \bar{w}]$.

   (a) Write down the asset value equation for the worker who is unemployed whose previous wage was w.

   $$rV_u(w) = \phi w + \alpha E_{w'} \max\{V_e(w') - V_u(w), 0\}$$

   (b) Write down the asset value (Bellman) equation for the worker employed at wage w.

   $$rV_e(w) = w + \lambda \left[V_u(w) - V_e(w)\right]$$

   (c) Solve for the reservation wage of an unemployed worker whose previous wage was w.

   By the standard analysis from the notes we get:

   $$w^*(w) = \phi w + \frac{\alpha}{r + \lambda} \int_{w^*(w)}^{\bar{w}} w' - w^*(w) dF(w')$$

   (d) Suppose now there are a continuum of similar workers. Derive an implicit formula for $w_{\min}$, the lowest wage observed in the economy.

   The lowest wage in the economy must have property that it is its own reservation wage so that $w^*(w_{\min}) = w_{\min}$. Substituting in we get

   $$w_{\min} = \frac{\alpha}{(r + \lambda)(1 - \phi)} \int_{w_{\min}}^{\bar{w}} w' - w_{\min} dF(w')$$

   (e) What is the relationship between $w_{\min}$ and ϕ?

   It should be obvious that increasing ϕ should make workers picker about new jobs. The derivative is

   $$\frac{dw_{\min}}{d\phi} = \frac{\alpha \int_{w_{\min}}^{\bar{w}} w' - w_{\min} dF(w')}{(1 - \phi)\left((r + \lambda)(1 - \phi) + \alpha\left[1 - F(w_{\min})\right]\right)} > 0$$

(f) Obtaining the unemployment rate, u, in this economy is hard to do. Without attempting to do the math, explain intuitively why

$$u \geq \frac{\lambda}{\lambda + \alpha[1 - F(w_{\min})]}$$

The right hand side would be the unemployment rate if $w_{\min}$ was the reservation wage used by all unemployed workers. But, those that earned more than $w_{\min}$ in their last job will actually have a reservation wage strictly higher than $w_{\min}$ which means the rate at which people flow into employment is generally slower than $\alpha[1 - F(w_{\min})]$.

2. **Optimal growth with non-linear capital formation**

Time: Discrete; infinite horizon

Demography: A single representative consumer/worker household, and a single representative profit maximizing firm, owned by the household. (Both act competitively in the market economy.). Both household and firm live for ever.

Preferences: The instantaneous household utility function over, individual consumption, $c \in \boldsymbol{R}_+$ is $u(c)$ where $u' > 0$, $u'' < 0$ and $\lim_{c \to 0} u'(c) = \infty$. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology over capital and labor so that output per unit of labor is $f(k)$ where $f(.)$ is increasing, concave and satisfies the Inada conditions. Output is in consumption goods which can be converted into capital only by use of the technology $g(.)$. Thus, if the household wants to save $s_t$units of output for conversion into capital it will receive $g(s_t)$ units of capital within the same period so that period t's investment, $i_t = g(s_t)$. Here $g' > 0$, $g'' < 0$, $g(0) = 0$ and assume that there exists a some s^* such that $g(s^*) = s^*$. Capital depreciates at the rate δ every period it is in use.

Endowments: The household's initial capital stock is k_0. Households have a single unit of labor each period.

Institutions: In the decentralized economy there are markets every period for capital, labor and the consumption good. With the consumption good as the numeraire, the implied prices are r_t and w_t.

(a) Write down and solve the household's problem in recursive form (i.e. using dynamic programming).

Household solves,

$$\begin{aligned} & \max_{k_{t+1}, c_t, s_t} \{u(c_t) + \beta V(k_{t+1}, r_{t+1}, w_{t+1})\} \\ \text{Subject to} \quad : \quad & r_t k_t + w_t = c_t + s_t \\ k_{t+1} \quad = \quad & g(s_t) + (1 - \delta) k_t. \end{aligned}$$

Substituting for c_t yields the Lagrangian

$$\mathcal{L} = u(r_t k_t + w_t - s_t) + \beta V(k_{t+1}, r_{t+1}, w_{t+1}) + \lambda_t \left[k_{t+1} - g(s_t) - (1 - \delta) k_t\right]$$

FOCs are:

$$\begin{aligned} k_{t+1} &: \quad \beta V_1(k_{t+1}, r_{t+1}, w_{t+1}) + \lambda_t = 0 \\ s_t &: \quad -u'(c_t) - \lambda_t g'(s_t) = 0 \end{aligned}$$

Envelope:

$$V_1(k_t, r_t, w_t) = u'(c_t) r_t - \lambda_t (1-\delta)$$

(b) Write down the solution to the firm's problem.

$$\begin{aligned} r_t &= f'(k_t^f) \\ w_t &= f(k_t^f) - f'(k_t^f) k_t^f \end{aligned}$$

(c) Write down the market clearing conditions and define a competitive equilibrium.
Market clearing:

$$\begin{aligned} \text{Capital:} &\quad k_t^f = k_t \\ \text{Labor: implicit} & \\ \text{Goods} &: \quad f(k_t) = c_t + s_t \end{aligned}$$

Definition: A *Competitive Equilibrium* is an allocation, $\{c_t, k_t, k_t^f\}$ and prices, $\{r_t, w_t\}$ such that given prices the allocation solves the household and firm's problems and markets clear.

(d) Solve for a system of equations that characterizes the competitive equilibrium.

$$\begin{aligned} \frac{u'(c_t)}{u'(c_{t+1})} &= \beta g'(s_t) \left[f'(k_t) + \frac{1-\delta}{g'(s_{t+1})} \right] \\ k_{t+1} &= g(s_t) + (1-\delta) k_t \\ f(k_t) &= c_t + s_t \end{aligned}$$

(e) Now write down and solve the Planner's problem for this economy.
Planner solves:

$$\begin{aligned} V(k_t) &= \max_{k_{t+1}, c_t, s_t} \{u(c_t) + \beta V(k_{t+1})\} \\ \text{subject to} &: \quad f(k_t) = c_t + s_t \\ k_{t+1} &= g(s_t) + (1-\delta) k_t \end{aligned}$$

Planner's Euler equation is same as competitive equilibrium characterization.

(f) What can you conclude from your answers to parts (d) and (e).
Equilibrium is efficient.