

Macroeconomics I

Final Exam**Answer both questions time allowed 2 hours****(1) Sidrauski model with special people.****Time:** Discrete; infinite horizon**Demography:** Continuum of mass 1 of infinite lived consumer/worker households. A proportion λ of the households are special in a way made clear below. There is also a large number of profit maximizing firms, owned jointly by the households.**Preferences:** the instantaneous household utility function over, individual consumption, c , and real money holding, m is $u(c, m)$ where $u(., .)$ is increasing in both arguments and concave. We have $\lim_{c \rightarrow 0} u_1(c, m) = \infty$ and $\lim_{m \rightarrow 0} u_2(c, m) = \infty$ where $u_i(c, m)$ is the derivative of u with respect to its i th argument. The discount factor is $\beta \in (0, 1)$.**Technology:** Each firm has access to a technology which has constant returns to scale with respect to the capital and labor it hires. Thus output per unit of labor employed is $f(k^f)$ where k^f is capital hired per unit of labor. Capital depreciates at the rate $\delta < 1$.**Endowments:** Households' initial capital stock is k_0 . Each household has 1 unit of labor each period. Each household has an initial endowment of money, H_0 . The stock of money grows at the rate σ so that the nominal amount of money in circulation in period t is $H_t = (1 + \sigma)H_0$. The additional money is distributed by helicopter drop but only to the special people. If you are one of the λ you get cash transfers, τ_t (if $\sigma < 0$, $\tau_t < 0$) if you are one of the $1 - \lambda$ nothing happens to your cash balances over night.**Institutions:** Every period there are markets for capital, labor and money. (Firms behave competitively).

- (a) Write down and characterize the solution to both the special and non-special households' problems in terms of wages, w_t , the rental rate of capital, r_t , the price of goods, p_t and transfers, τ_t . You can write out one of the problems if you like and simply point out how they will differ from each other.
- (b) Write down and solve the firm's problem given prices.
- (c) Write down the market clearing conditions for money and capital, and the government budget constraint. Define a competitive equilibrium. Hint you will need to think about each group holding different amounts of money and having different allocations.
- (d) Assuming that $\frac{p_{t+1}}{p_t} = 1 + \sigma$, use the household budget constraints to derive expressions for the steady state consumption of the special and non-special households in terms of the other steady state variables.

(2) Mortensen-Pissarides with closed hut bargaining

Time: Discrete, infinite horizon

Demography: A mass of 1 of ex ante identical workers with infinite lives and a large mass of firms who create individual vacancies.

Preferences: Workers and firms are risk neutral (i.e. $u(x) = x$). The common discount rate is r . The value of leisure for workers is b utils per period. The cost of holding a vacancy for firms is a utils per period.

Productive Technology: Matched firm/worker pairs produce p units of the consumption good per period. With probability λ each period, jobs experience a catastrophic productivity shock and the job is destroyed. Assume $p > 2b$

Matching Technology: Unemployed workers encounter vacancies at the rate $m(\theta)$ where $\theta = v/u$, v is the mass of vacancies and u is the mass of unemployed workers. The function $m(\cdot)$ is increasing concave and $m(\theta) < 1$ for all θ . Also $\lim_{\theta \rightarrow 0} m'(\theta) = 1$, $\lim_{\theta \rightarrow \infty} m'(\theta) = 0$, and $m(\theta) > \theta m'(\theta)$. The rate at which vacancies encounter unemployed workers is then $m(\theta)/\theta$.

Institutions: The terms of trade are determined by symmetric Rubinstein type bargaining which means that the wage $w = p/2$ (as long as the outside option of the worker does not bind which will not happen while $p > 2b$).

- (a) Write down the set of flow value equations or Bellman equations for workers and firms.
- (b) Define a steady state free-entry equilibrium and solve for a single equation in θ .
- (c) Obtain an expression for the unemployment rate, u in terms of θ .
- (d) How do changes in b and a affect the level of unemployment? Provide intuition for your results.