

Macroeconomics I

Final Exam

Answer both questions time allowed 2 hours

1. Diamond Coconut Economy with idiosyncratic preferences

Time: Discrete, infinite horizon

Geography: A trading island and a production island.

Demography: A mass of 1 of ex ante identical individuals with infinite lives.

Preferences: The common discount rate is r , consumption of my own produce or the produce of someone whose coconut I do not like yields 0 utils. Consumption of produce from someone whose good I like yields u utils. The share of individuals whose goods I like is ϕ . Whether an individual likes my good or not is independent of whether I like hers. (So she likes my good with probability ϕ too.)

Productive Technology: On the production island individuals come across a tree with a coconut with probability α each period. The cost of obtaining the coconut is $c \sim F$. The distribution function $F(\cdot)$ is continuous over its support, $(0, \bar{c})$ where $\bar{c} > u$ (so the coconuts from some trees will get rejected).

Matching Technology: On the trading island people with coconuts meet each with probability γ , a constant.

Navigation: Travel between islands is instantaneous.

Endowments: Everyone has a boat and starts off with one of their own coconuts

- (a) Write down the asset value (Bellman) equations for this economy. Define the terms you introduce.

The regular asset value equations are

$$V_T = \frac{1}{1+r} \{ \gamma \phi^2 (u + V_P) + (1 - \gamma \phi^2) V_T \}$$

$$V_P = \frac{1}{1+r} \{ \alpha E [\max \{ V_T - c, V_P \}] + (1 - \alpha) V_P \}$$

where V_T is the value to being on the trading island and V_P is the value to being on the production island

The flow value equations are

$$rV_T = \gamma \phi^2 (u + V_P - V_T)$$

$$rV_P = \alpha E [\max \{ V_T - c - V_P, 0 \}]$$

(b) Define a search equilibrium

A search equilibrium is a pattern of trade such that given everyone else conforms to that patterns no individual will wish to deviate from it.

(c) Solve for an implicit equation that specifies the reservation tree “height”, c^* in terms of the parameters of the model.

Let $c^* = V_T - V_P$. Then rewrite V_P as

$$rV_P = \alpha \int_0^{c^*} [V_T - c - V_P] dF(c)$$

now subtract rV_P from rV_T to get

$$r(V_T - V_P) = \gamma\phi^2 u - \gamma\phi^2 (V_T - V_P) - \alpha \int_0^{c^*} [V_T - c - V_P] dF(c)$$

or by definition of c^* ,

$$(r + \gamma\phi^2)c^* = \gamma\phi^2 u - \alpha \int_0^{c^*} [c^* - c] dF(c) \quad (1)$$

(d) How does c^* change with respect to ϕ (i.e. obtain the sign of the comparative static)?

Define

$$\Psi(c, \phi) \equiv (r + \gamma\phi^2)c - \gamma\phi^2 u + \alpha \int_0^c [c - \eta] dF(\eta)$$

so that $\Psi(c^*, \phi) = 0$. Then

$$\frac{dc^*}{d\phi} = \frac{-\frac{\partial \Psi}{\partial \phi}}{\frac{\partial \Psi}{\partial c}} \Big|_{c=c^*}$$

Then

$$\frac{\partial \Psi}{\partial \phi} \Big|_{c=c^*} = 2\phi\gamma(c^* - u) < 0$$

(sign comes from equation (1)). And

$$\frac{\partial \Psi}{\partial c} \Big|_{c=c^*} = (r + \gamma\phi^2) + \alpha \int_0^{c^*} dF(\eta) = (r + \gamma\phi^2) + \alpha F(c^*) > 0.$$

So

$$\frac{dc^*}{d\phi} > 0$$

Increasing ϕ rapidly improves my chances of getting to trade my good and return to the production island. Investment (in inventory) has a higher return and I am therefore more ready to invest.

- (e) Draw a diagram showing the population flows between the islands. Write down the steady-state equations and solve for the population on the trading island as a function of c^* .

The diagram is the same as in the class notes for constant γ except that γ should be replaced with $\gamma\phi^2$. Steady-state equations are:

$$\begin{aligned}\gamma\phi^2 n_T &= \alpha F(c^*) n_P \\ n_T + n_P &= 1\end{aligned}$$

where n_T is proportion of the population on the trading island and n_P is the proportion of the population on the production island. Thus

$$n_T = \frac{\alpha F(c^*)}{\gamma\phi^2 + \alpha F(c^*)}$$

2. Optimal growth with proportional labor taxes

Time: Discrete; infinite horizon

Demography: A single representative consumer/worker household, and a single representative profit maximizing firm, owned by the household. (Both act competitively in the market economy.). Both household and firm live for ever.

Preferences: The instantaneous household utility function over, individual consumption, $c \in \mathbf{R}_+$, and leisure $l \in [0, 1]$, is $u(c, l)$ where $u(\cdot, \cdot)$ is strictly increasing in both arguments and strictly concave. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology over capital, k , and labor, h , so that output is $f(k, h)$, where $f(\cdot, \cdot)$ is strictly increasing in both arguments and strictly concave. Capital is fully used up in production (i.e. depreciation rate is 1).

Endowments: Households' initial capital stock is k_0 , each household has 1 unit of time which they can allocate in any proportion to work or leisure.

Institutions: There is a government that has to meet an exogenous spending stream, $\{g_t\}$. In the market economy the government will levy a **proportional** tax $\{\tau_t\}$ on labor income.

Markets: Every period households rent out their capital, sell their labor and buy/sell goods in competitive markets. The implied prices are r_t and w_t . (The consumption good is the numeraire.)

- (a) Write down and solve the household's problem in recursive form (i.e. using dynamic programming).

$$\begin{aligned}V(k_t) &= \max_{\{c_t, h_t, k_{t+1}\}} \{u(c_t, l_t) + \beta V(k_{t+1})\} \\ \text{subject to} & : l_t = 1 - h_t \\ c_t + k_{t+1} &= (1 - \tau_t)w_t h_t + r_t k_t\end{aligned}$$

or

$$V(k_t) = \max_{\{h_t, k_{t+1}\}} \{u((1 - \tau_t)w_t h_t + r_t k_t - k_{t+1}, 1 - h_t) + \beta V(k_{t+1})\}$$

FOC.:

$$\begin{aligned} k_{t+1} &: & -u_1(c_t, l_t) + \beta V'(k_{t+1}) &= 0 \\ h_t &: & u_1(c_t, l_t)(1 - \tau_t)w_t - u_2(c_t, l_t) &= 0 \end{aligned}$$

envelope:

$$V'(k_t) = u_1(c_t, l_t)r_t$$

Sub out $V'(k_{t+1})$ to get the euler equation:

$$u_1(c_t, l_t) - \beta r_{t+1} u_1(c_{t+1}, l_{t+1}) = 0$$

- (b) Write down the solution to the firm's problem, the government budget constraint, the market clearing conditions and define a competitive equilibrium.

Firm's solution

$$\begin{aligned} f_1(k_t^f, h_t^f) &= r_t \\ f_2(k_t^f, h_t^f) &= w_t \end{aligned}$$

GBC:

$$\tau_t w_t h_t = g_t$$

Market clearing:

$$\begin{aligned} k_t^f &= k_t \\ h_t^f &= h_t \\ f(k_t^f, h_t^f) &= c_t + k_{t+1} + g_t \end{aligned}$$

Definition: A competitive equilibrium is an allocation $\{c_t, k_t, h_t, k_t^f, h_t^f\}$ and prices $\{w_t, r_t\}$ such that given prices, the allocation solves the household's and firm's problems, markets clear and the government budget constraint holds.

- (c) Solve for a system of equations that characterizes the competitive equilibrium.

$$\begin{aligned} u_1(c_t, 1 - h_t) - \beta f_1(k_{t+1}, h_{t+1}) u_1(c_{t+1}, 1 - h_{t+1}) &= 0 \\ u_1(c_t, 1 - h_t) \left(1 - \frac{g_t}{f_2(k_t, h_t) h_t}\right) f_2(k_t, h_t) - u_2(c_t, 1 - h_t) &= 0 \\ c_t + k_{t+1} + g_t &= f(k_t, h_t) \end{aligned}$$

- (d) If $g_t = \bar{g}$ for all t , obtain a system of equations that characterizes the steady-state equilibrium, (c^*, h^*, k^*)

$$\begin{aligned} \beta f_1(k^*, h^*) &= 1 \\ u_1(c^*, 1 - h^*) \left(f_2(k^*, h^*) - \frac{\bar{g}}{h^*} \right) &= u_2(c^*, 1 - h^*) \\ c^* + k^* + \bar{g} &= f(k^*, h^*) \end{aligned}$$

- (e) Now solve the Planner's problem with $g_t = \bar{g}$ and solve for the system that characterizes His steady-state solution, (c_p, h_p, k_p) .

$$V^p(k_t) = \max_{\{h_t, k_{t+1}\}} \{u(f(k_t, h_t) - k_{t+1} - \bar{g}, 1 - h_t) + \beta V^p(k_{t+1})\}$$

FOC:

$$\begin{aligned} k_{t+1} &: -u_1(c_t, 1 - h_t) + \beta V^{p'}(k_{t+1}) = 0 \\ h_t &: u_1(c_t, 1 - h_t) f_2(k_t, h_t) - u_2(c_t, 1 - h_t) = 0 \end{aligned}$$

envelope:

$$V^{p'}(k_t) = u_1(c_t, 1 - h_t) f_1(k_t, h_t)$$

Steady state:

$$\begin{aligned} \beta f_1(k_p, h_p) &= 1 \\ u_1(c_p, 1 - h_p) f_2(k_p, h_p) &= u_2(c_p, 1 - h_p) \\ c_p + k_p + \bar{g} &= f(k_p, h_p) \end{aligned}$$

- (f) Briefly discuss any differences between your answers to parts (d) and (e).

The proportional tax introduces a wedge between the contemporaneous marginal utility of consumption and the marginal utility of leisure which is not present in the planner's solution.