AECO601 Fall 2008

Macroeconomics I

Final Exam

Answer both questions time allowed 2 hours

(1) Search with on-the-job wage changes. (Adapted from Rogerson Shimer & Wright)

Time: Discrete, infinite horizon.

Demography: Single worker who lives for ever.

Preferences: The worker is risk-neutral (i.e. $u(x) = x$) and discounts the future at the rate r.

Endowments: *When unemployed:* The worker receives b units of the consumption good per period. Also, with probability α she gets to sample a wage from the continuous distribution F with support $(0, \bar{w}]$ where $\bar{w} > b$.

When employed: The worker receives her current wage but the wage can change. There is no lay-off as such but instead with probability λ a new wage is drawn from F. The worker can quit if she considers the new wage to be too low. Otherwise, she remains employed at the new wage and is again subject to the same probability of a new draw. (Note: the worker does not have the option of remaining in the job at her old wage.)

(a) Obtain the flow asset value equations for V_u (the value to being unemployed) and $V_e(w)$ (the value to being employed at the wage w).

$$rV_u = b + \alpha \mathbb{E}_w \left[\max\{V_e(w) - V_u, 0\} \right]$$

$$\begin{aligned} rV_e(w) &= w + \lambda \mathbb{E}_{w'} \left[\max\{V_e(w') - V_e(w), V_u - V_e(w)\} \right] \\ &= w + \lambda \mathbb{E}_{w'} \left[\max\{V_e(w') - V_u, 0\} \right] + \lambda(V_u - V_e(w)) \end{aligned}$$

(b) Explain how these asset value equations imply that the reservation wage, w^* for unemployed workers is the same as the threshold wage below which employed workers will quit - provide intuition.

As the worker does not have the option of holding onto his old wage the choice is always between continued employment at the new wage and unemployment. The reservation wage w^* associated with the initial acceptance solves $V_e(w^*) = V_u$ as does that for the choice to quit.

(c) Derive the reservation wage equation. (Hint: Obtain an expression for $V_e(w) - V_u$ then evaluate it at $w = w^*$ and substitute it back in to the expression.)

From above equations and the definition of mathematical expectation we have

$$\begin{aligned} rV_u &= b + \alpha \int_{w^*}^{\bar{w}} (V_e(w) - V_u)\, dF(w) \\ rV_e(w) &= w + \lambda \int_{w^*}^{\bar{w}} (V_e(w') - V_u)\, dF(w') + \lambda(V_u - V_e(w)) \end{aligned}$$

Subtracting yields,

$$r\,(V_e(w) - V_u) = w - b + (\lambda - \alpha) \int_{w^*}^{\bar{w}} (V_e(w') - V_u)\, dF(w') + \lambda(V_u - V_e(w))$$

So

$$(r + \lambda)\,(V_e(w) - V_u) = w - b + (\lambda - \alpha) \int_{w^*}^{\bar{w}} (V_e(w') - V_u)\, dF(w') \tag{1}$$

And, since $V_e(w^*) = V_u$ evaluation at $w = w^*$ implies

$$0 = w^* - b + (\lambda - \alpha) \int_{w^*}^{\bar{w}} (V_e(w') - V_u)\, dF(w') \tag{2}$$

Using (2) into (1) to eliminate the integral term implies

$$(r + \lambda)\,(V_e(w) - V_u) = w - w^*$$

Back into (2) and rearranging we get

$$w^* = b + \frac{\alpha - \lambda}{r + \lambda} \int_{w^*}^{\bar{w}} (w' - w^*)\, dF(w')$$

(d) If $\lambda = \alpha$ how does the reservation wage compare to b? What is the intuition for this result?

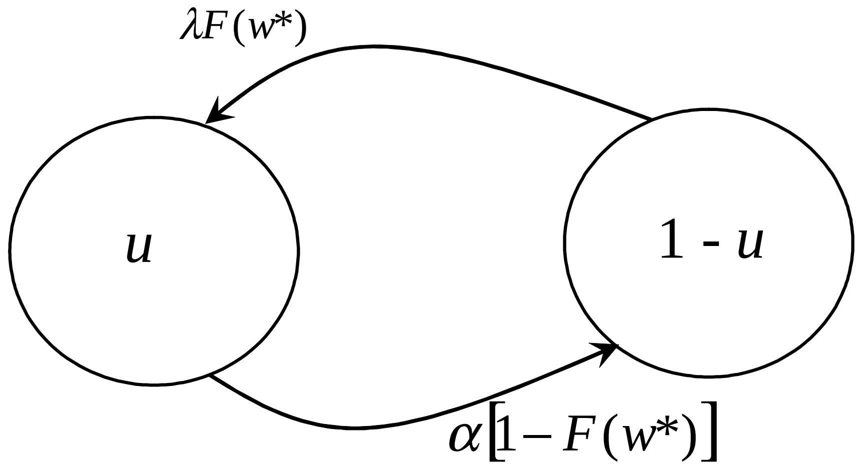
If $\lambda = \alpha$, $w^* = b$. If there is no opportunity cost associated with accepting a job then I accept any offer that is better than my current income.

(e) If there is a large number of such workers with mass normalized to 1 provide the steady-state flow diagram for movements between employment and unemployment. Use this to obtain an expression for the steady-state unemployment rate, u, in terms of model parameters and w^*.

$$\alpha[1 - F(w^*)]u = \lambda F(w^*)(1 - u)$$

So

$$u = \frac{\lambda F(w^*)}{\alpha[1 - F(w^*)] + \lambda F(w^*)}$$



(2) **Cash-in-Advance with Firms**

Time: Discrete; infinite horizon

Demography: Continuum of mass 1 of (representative) consumer/worker households, and a large number of profit maximizing firms, owned jointly by the households.

Preferences: the instantaneous household utility function over, individual consumption, c, is $u(c)$ where $u(.)$ is strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology over capital and labor such that output per unit of labor employed is $f(k)$, where k is capital input per unit of labor; $f(.)$ is strictly increasing and concave. Capital depreciates at the rate $\delta < 1$.

Endowments: Households' initial capital stock is k_0, each household has 1 unit of labor.

Initial cash holdings, H_0 for each household.

Institutions: A central bank issues new currency every period so that the total cash in the economy $H_t = (1 + \sigma)^t H_0$

Government distributes the new cash in period t as transfers, τ_t. (These can be negative if σ if negative)

Legal Tender Requirement: Households can consume or save undepreciated capital, $(1-\delta)k_t$, but (as in class) any additional consumption or investment has to be paid for with cash.

Markets: Every period households rent out their capital and sell their labor in competitive markets. The implied prices are r_t and w_t. (Assume that the households incur the cost of depreciation.) The market in consumption goods for money is also competitive - the price of goods in terms of money is p_t.

(a) Write down and solve the households problem in recursive form (i.e. using dynamic programming).

$$\begin{aligned}
V(M_t, k_t) &= \max_{c_t, k_{t+1}, M_t^d} \{u(c_t) + \beta V(M_{t+1}, k_{t+1})\} \\
\text{subject to} &: \quad w_t + r_t k_t + (1-\delta)k_t + \frac{M_t}{p_t} = c_t + k_{t+1} + \frac{M_t^d}{p_t} \\
\text{and} &: \quad \frac{M_t}{p_t} \geq c_t + k_{t+1} - (1-\delta)k_t \\
\text{where} &: \quad M_t = M_{t-1}^d + \tau_t
\end{aligned}$$

Households take the sequence of prices as given they form the Lagrangian:

$$\begin{aligned}
\boldsymbol{L} &= u(c_t) + \beta V(M_t^d + \tau_{t+1}, k_{t+1}) \\
&\quad + \lambda_t \left[w_t + r_t k_t + (1-\delta)k_t + \frac{M_t}{p_t} - c_t - k_{t+1} - \frac{M_t^d}{p_t} \right] \\
&\quad \gamma_t \left[\frac{M_t}{p_t} - c_t - k_{t+1} + (1-\delta)k_t \right]
\end{aligned}$$

F.O.C's,

$$\begin{aligned}
c_t &: \quad u'(c_t) - \lambda_t - \gamma_t = 0 \\
k_{t+1} &: \quad \beta V_2(M_{t+1}, k_{t+1}) - \lambda_t - \gamma_t = 0 \\
M_t^d &: \quad \beta V_1(M_{t+1}, k_{t+1}) - \frac{\lambda_t}{p_t} = 0
\end{aligned}$$

Complementary slackness conditions:

$$\begin{aligned}
\gamma_t \left[\frac{M_t}{p_t} - c_t - k_{t+1} + (1-\delta)k_t \right] &= 0 \\
\gamma_t &\geq 0
\end{aligned}$$

Transversality condition:

$$\lim_{t\to\infty} \beta^t u'(c_t) k_t = 0$$

Envelope conditions:

$$\begin{aligned} V_2(M_t, k_t) &= \lambda_t \left[r_t + (1-\delta)\right] + \gamma_t(1-\delta) \\ V_1(M_t, k_t) &= \frac{\lambda_t}{p_t} + \frac{\gamma_t}{p_t} \end{aligned}$$

Eliminating the value functions leads to

$$u'(c_t) = (1-\delta)\beta u'(c_{t+1}) + \frac{p_{t+1}}{p_{t+2}} r_{t+1} \beta^2 u'(c_{t+2})$$

(b) Write down the solution to the firm's problem, the market clearing conditions, the government budget constraint, and define a competitive equilibrium.

Firms solve a static profit maximization problem. They obtain

$$\begin{aligned} r_t &= f'(k_t^f) \\ w_t &= f(k_t^f) - f'(k_t^f) k_t^f \end{aligned}$$

Market clearing:

$$\begin{aligned} k_t^f &= k_t \\ M_t &= H_t \\ c_t + k_{t+1} &= f(k_t) + (1-\delta)k_t \end{aligned}$$

Government budget constraint:

$$\tau_{t+1} = \sigma H_t$$

Definition: A C.E. is an allocation, $\{c_t, k_{t+1}, k_t^f, M_t\}$, prices $\{r_t, w_t, p_t\}$ and sequence of transfers $\{\tau_t\}$ such that given prices and transfers, the allocations solves the households' and firms' problems, markets clear and the government budget constraint holds.

(c) Solve for a system of equations that characterizes the competitive equilibrium.

$$\begin{aligned} u'(c_t) &= (1-\delta)\beta u'(c_{t+1}) + \frac{p_{t+1}}{p_{t+2}} f'(k_t) \beta^2 u'(c_{t+2}) \\ c_t + k_{t+1} &= f(k_t) + (1-\delta)k_t \end{aligned}$$

and,

$$\frac{H_t}{p_t} \geq c_t - k_{t+1} + (1-\delta)k_t$$

(d) Explain why the Friedman rule, $(1+\sigma) = \beta$ should be optimal in this environment.

At the Friedman rule CIA just binds. When $(1+\sigma) > \beta$ the return on capital exceeds that of money. People hold money so they can eat next period but doing so is costly. When they are equal there is no opportunity cost in terms of returns on asset holdings from money. It therefore represents the best policy for the households.