

Macroeconomics I
Final Exam

Answer both questions time allowed 2 hours

- (1) Search with on-the-job wage changes.** (Adapted from Rogerson Shimer & Wright)

Time: Discrete, infinite horizon.

Demography: Single worker who lives for ever.

Preferences: The worker is risk-neutral (i.e. $u(x) = x$) and discounts the future at the rate r .

Endowments: *When unemployed:* The worker receives b units of the consumption good per period. Also, with probability α she gets to sample a wage from the continuous distribution F with support $(0, \bar{w}]$ where $\bar{w} > b$.

When employed: The worker receives her current wage but the wage can change. There is no lay-off as such but instead with probability λ a new wage is drawn from F . The worker can quit if she considers the new wage to be too low. Otherwise, she remains employed at the new wage and is again subject to the same probability of a new draw. (Note: the worker does not have the option of remaining in the job at her old wage.)

- (a) Obtain the flow asset value equations for V_u (the value to being unemployed) and $V_e(w)$ (the value to being employed at the wage w).
- (b) Explain how these asset value equations imply that the reservation wage, w^* for unemployed workers is the same as the threshold wage below which employed workers will quit - provide intuition.
- (c) Derive the reservation wage equation. (Hint: Obtain an expression for $V_e(w) - V_u$ then evaluate it at $w = w^*$ and substitute it back in to the expression.)
- (d) If $\lambda = \alpha$ how does the reservation wage compare to b ? What is the intuition for this result?
- (e) If there is a large number of such workers with mass normalized to 1 provide the steady-state flow diagram for movements between employment and unemployment. Use this to obtain an expression for the steady-state unemployment rate, u , in terms of model parameters and w^* .

(2) Cash-in-Advance with Firms

Time: Discrete; infinite horizon

Demography: Continuum of mass 1 of (representative) consumer/worker households, and a large number of profit maximizing firms, owned jointly by the households.

Preferences: the instantaneous household utility function over individual consumption, c , is $u(c)$ where $u(\cdot)$ is strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology over capital and labor such that output per unit of labor employed is $f(k)$, where k is capital input per unit of labor; $f(\cdot)$ is strictly increasing and concave. Capital depreciates at the rate $\delta < 1$.

Endowments: Households' initial capital stock is k_0 , each household has 1 unit of labor.

Initial cash holdings, H_0 for each household.

Institutions: A central bank issues new currency every period so that the total cash in the economy $H_t = (1 + \sigma)^t H_0$

Government distributes the new cash in period t as transfers, τ_t . (These can be negative if σ is negative)

Legal Tender Requirement: Households can consume or save undepreciated capital, $(1 - \delta)k_t$, but (as in class) any additional consumption or investment has to be paid for with cash.

Markets: Every period households rent out their capital and sell their labor in competitive markets. The implied prices are r_t and w_t . (Assume that the households incur the cost of depreciation.) The market in consumption goods for money is also competitive - the price of goods in terms of money is p_t

- (a) Write down and solve the households problem in recursive form (i.e. using dynamic programming).
- (b) Write down the solution to the firm's problem, the government budget constraint, the market clearing conditions and define a competitive equilibrium.
- (c) Solve for a system of equations that characterizes the competitive equilibrium.
- (d) Explain why the Friedman rule, $(1 + \sigma) = \beta$ should be optimal in this environment.