AECO601 Fall 2007

Macroeconomics I

Final Exam

Answer both questions time allowed 2 hours

(1) **One sided search with "business cycles"**.

Time: Discrete, infinite horizon.

Demography: Single worker who lives for ever.

Preferences: The worker is risk-neutral (i.e. $u(x) = x$). He discounts the future at the rate r.

Endowments: The worker receives income b per period when unemployed. Every period with probability α he gets an offer of employment.

The "aggregate state" of the economy affects the wages the worker can receive. In a "recession" the wage is w_R. In booms it is $w_B > w_R$.

To clarify, when unemployed the worker gets b which is unaffected by the aggregate state of the economy. If the economy is in recession then the wage offer is w_R. If the economy is in a boom the offer is w_B. At any point in time the offer distribution (that we called F, is degenerate, there is only one wage). Any one hired in a boom suffers a wage decline when a recession hits and vice-versa.

The economy switches aggregate state with probability γ every period. (That is, if this period is a boom, next period will be a recession with probability γ. If this period is a recession, next period will be a boom with probability γ.)

When employed the worker looses his job with probability λ.

Assume for simplicity that events are mutually exclusive. So for example,when the worker is employed he looses his job with probability λ or there is a change in the aggregate state with probability γ or nothing changes with probability $1 - \lambda - \gamma$.

(a) Write down the relevant value functions (or flow value equations). Assume that $w_R > b$ so workers always accept job offers. (Hint: there are 4 states for the individual.)

The 4 value functions are:

$$
\begin{aligned}
rV_e^R &= w_R + \lambda(V_u^R - V_e^R) + \gamma(V_e^B - V_e^R) \\
rV_u^R &= b + \alpha(V_e^R - V_u^R) + \gamma(V_u^B - V_u^R) \\
rV_e^B &= w_B + \lambda(V_u^B - V_e^B) + \gamma(V_e^R - V_e^B) \\
rV_u^B &= b + \alpha(V_e^B - V_u^B) + \gamma(V_u^R - V_u^B)
\end{aligned}
$$

(b) Show that being unemployed in a recession is worse than being unemployed in a boom.

We basically want to show that $V_u^B > V_u^R$. From the value functions we have

$$r(V_u^B - V_u^R) = \alpha(V_e^B - V_e^R + V_u^R - V_u^B) + \gamma(V_u^R - V_u^B + V_u^R - V_u^B)$$

so

$$(r + \alpha + 2\gamma)(V_u^B - V_u^R) = \alpha(V_e^B - V_e^R) \tag{1}$$

Now

$$r(V_e^B - V_e^R) = w_B - w_R + \lambda(V_u^B - V_u^R + V_e^R - V_e^B) + \gamma(V_e^R - V_e^B + V_e^R - V_e^B)$$

So

$$(r + \lambda + 2\gamma)(V_e^B - V_e^R) = w_B - w_R + \lambda(V_u^B - V_u^R)$$

Into equation (1),

$$[(r + \alpha + 2\gamma)(r + \lambda + 2\gamma) - \alpha\lambda]\,(V_u^B - V_u^R) = \alpha(w_B - w_R)$$

As the term in square brackets is positive, and $w_B > w_R$, $V_u^B > V_u^R$.

(c) Is it possible that the worker would take the job in a recession even if $w_R < b$? You do not need to derive the conditions just provide some intuition.

This is true. Basically taking the job during a recession has an option value. If α and λ are small relative to γ, it could be the case that $V_e^R > V_u^R$ even if $w_B < b$. The worker will take the low paid job because as soon as the boom hits she gets the higher paid job. The alternative is turn down the offer during a recession. But if α is low there is no guarantee of getting a high paid job. This is a case of where "a bird in hand is worth 2 in the bush" applies.

To show it algebraically it is necessary to derive an expression for $V_e^R - V_u^R$ as was done under part (b) and show that the sign does not necessarily depend on that of $w_B - b$. The pdf of the Maple file shows that

$$V_e^R - V_u^R = \frac{\gamma(w_R - b) + (r + \alpha + \gamma + \lambda)(w_B - b)}{(r + \alpha + \lambda)(r + \alpha + 2\gamma + \lambda)}$$

This was not required in the exam.

(2) **Optimal Growth with Infrastructure.**

Time: Discrete; infinite horizon

Demography: Continuum of mass 1 of (representative) consumer/worker households, and a large number of profit maximizing firms, owned jointly by the households.

Preferences: the instantaneous household utility function over, individual consumption, c, is $u(c)$ where $u(.)$ is strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.

Technology: There is a constant returns to scale technology over capital and labor such that output per unit of labor employed is $z(g)f(k)$, where k is capital input per unit of labor and g is per household government spending. $f(.)$ and $z(.)$ are both strictly increasing and concave. Capital depreciates at the rate $\delta < 1$.

Endowments: Households' initial capital stock is k_0, each household has 1 unit of labor.

Institutions: A government levies lump sum taxes τ_t on the the population and provides government spending g_t. There is always a balanced budget.

Markets: Every period households rent out their capital and sell their labor in competitive markets. The implied prices are r_t and w_t.

(a) For now, solve the household's problem taking as given the path of taxes.

Household solves,

$$\begin{aligned} V(k_t) &= \max_{k_{t+1}, c_t} \{u(c_t) + \beta V(k_{t+1})\} \\ \text{s.t. } c_t &= r_t k_t + w_t - k_{t+1} - \tau_t \end{aligned}$$

Subbing in for c_t yields the first order condition for k_{t+1},

$$-u'(c_t) + \beta V'(k_{t+1}) = 0$$

The envelope condition yields

$$V'(k_t) = u'(c_t) r_t$$

Which give an Euler equation:

$$u'(c_t) - \beta u'(c_{t+1}) r_{t+1} = 0$$

(b) Write down the solution to the firm's problem, the market clearing conditions and define a competitive equilibrium.

$$
\begin{aligned}
r_t &= z(g_t)f'(k_t^f) + 1 - \delta \\
w_t &= z(g_t)(f(k_t^f) - f'(k_t^f)k_t^f)
\end{aligned}
$$

Market clearing: $k_t^f = k_t$

A *Perfect-Foresight Competitive Equilibrium* is an allocation, $\{k_t, c_t\}$ and a sequence of prices, $\{r_t, w_t\}$ such that given prices the allocation solves the household and firms' problems and markets clear.

(c) Solve for a system of equations that characterizes the competitive equilibrium (still taking taxes as given).

$$
\begin{aligned}
\frac{u'(c_t)}{u'(c_{t+1})} &= \beta\left[z(g_t)f'(k_{t+1}) + 1 - \delta\right] \\
c_t &= z(g_t)f(k_t) + (1-\delta)\,k_t - k_{t+1} - \tau_t
\end{aligned}
$$

There are boundary conditions: k_0 given and a transversality condition.

(d) Now consider the benevolent government's problem. (This is the same as the Planner's problem which takes account of the affect of government spending on output).

$$
V(k_t) = \max_{k_{t+1}, g_t} \left\{u(z(g_t)f(k_t) + (1-\delta)\,k_t - k_{t+1} - g_t) + \beta V(k_{t+1})\right\}
$$

(e) Solve the government problem.

FOC's

k_{t+1} :

$$
-u'(c_t) + \beta V'(k_{t+1}) = 0
$$

g_t :

$$
u'(c_t)\left[z'(g_t)f(k_t) - 1\right] = 0
$$

Envelope:

$$
V'(k_t) = u'(c_t)\left[z(g_t)f'(k_t) + 1 - \delta\right]
$$

(f) What is the implied relationship between optimal government spending, g_t and k_t (i.e. is g_t increasing or decreasing in k_t)?

Let $\Omega(g_t, k_t) \equiv z'(g_t)f(k_t) - 1$. Then

$$
\frac{dg_t}{dk_t} = -\frac{f'(k_t)z'(g_t)}{f(k_t)z''(g_t)} > 0 \tag{2}
$$

from concavity of $z(.)$.

(g) Write down a system of equations that characterizes the steady state values of c_t, k_t and g_t (i.e. c^*, k^*, and g^*).

$$\begin{aligned} c^* &= z(g^*)f(k^*) + \delta k^* - g^* \\ 1 &= \beta\left[z(g^*)f'(k^*) + 1 - \delta\right] \\ 1 &= z'(g^*)f(k^*) \end{aligned}$$

(h) What condition must $z(.)$ and $f(.)$ satisfy to guarantee a unique value for k^*?

We need $z(g^*)f'(k^*)$ to be monotone decreasing in k (I guess monotone increasing should be OK too but in the standard model concavity of f implies zf' is decreasing.) That is we need

$$\frac{d}{dk}\left[z(g)f'(k)\right] = z'(g)f'(k)\frac{dg}{dk} + z(g)f''(k) < 0$$

Which requires from (2),

$$\left[z'(g)f'(k)\right]^2 < z(g)f''(k)f(k_t)z''(g_t)$$