AECO601 Fall 2005

Macroeconomics I

Solution to Final Exam

(1) The first 3 parts were straight from class notes.
The equations for the system are:

$$\begin{aligned} c_t &= f(k_t) + (1-\delta)k_t - k_{t+1} \\ \frac{u'(c_t)}{\beta u'(c_{t+1})} &= f'(k_t) + (1-\delta) \end{aligned}$$

In the 4th part of the question there is an unforeseen decline in the discount rate. Recall that β is the discount *factor* and that if ρ is used for the discount rate then $\beta = \frac{1}{1+\rho}$ and in steady-state the second equation becomes

$$f'(k^*) = \rho + \delta$$

A decline in ρ therefore leads to an increase in k^* because $f(.)$ is concave. (I did not penalize any one for mistaking the discount rate for the discount factor. If that mistake was made then k^* would fall.) The main diagram for part 4 is Figure 1.

For simplicity we start from an initial steady-state, point A. We know that eventually the economy will reach point C. Other than that we must always be in equilibrium and there are no anticipated jumps in c. When the unanticipated fall in ρ happens the economy jumps from A to B immediately (i.e. on Jan 1). After that the economy converges over time (asymptotically) to point C.

Figure 2 shows at the case when the change in ρ is anticipated (on Nov 1).

In this case we know that we have to be on the saddle-path through point C on Jan 1 but only the news in Nov 1 can cause a jump in c. On Nov 1 the economy jumps to point B1 between Nov 1 and Jan 1 it is on the path from B1 to B2. The economy reaches B2 exactly on Jan 1. Thereafter it tracks toward point C.

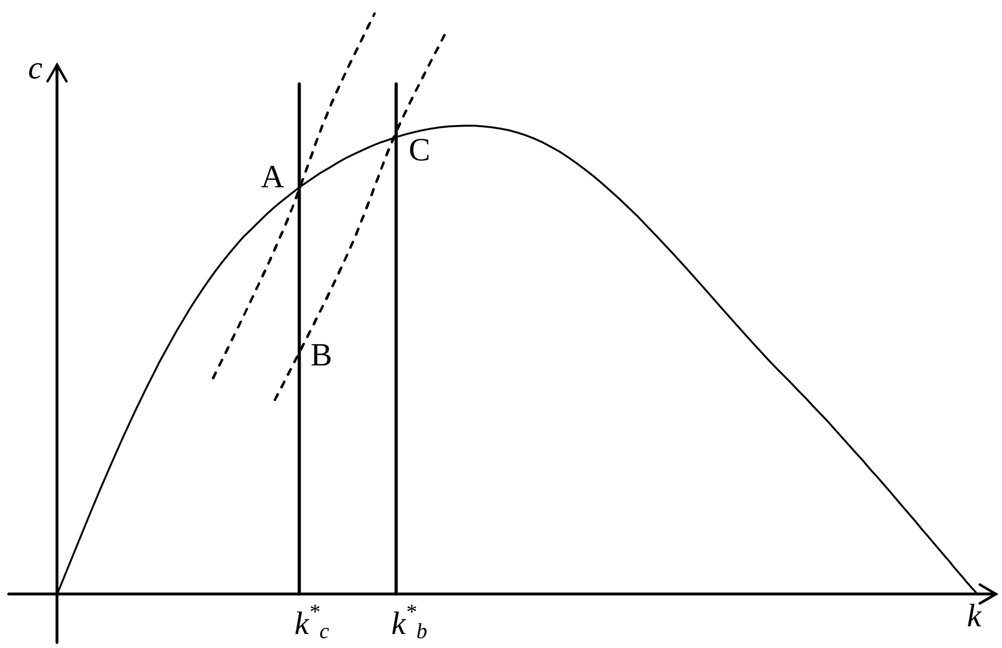

Figure 1: Unanticipated decrease in ρ.

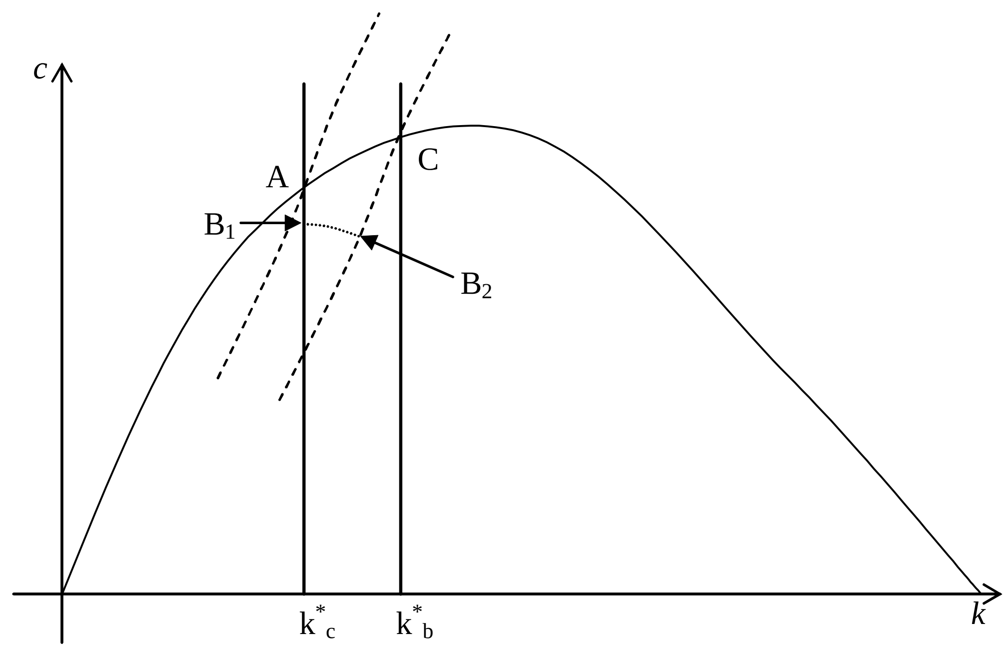

Figure 2: Anticipated decrease in ρ.

(2.1)There are 3 employment states: unemployed, value V_u, employed without promotion, V_e and promoted, V_p. (The wage is another dimension to the state.)

Asset value equations. (Ignoring the possibility of both events happening in any period means that an employed worker gets promoted with probability γ, laid-off with probability λ and remains employed with probability $1-\gamma-\lambda$.)

$$\begin{aligned} rV_u &= b + \alpha \mathbf{E}_w \left[\max\left\{V_e(w) - V_u, 0\right\}\right] \\ rV_e(w) &= w + \lambda(V_u - V_e(w)) + \gamma(V_p(w) - V_e(w)) \\ rV_p(w) &= \phi w + \lambda(V_u - V_p(w)) \end{aligned}$$

To show that promotion is always a good thing we evaluate $V_p(w) - V_e(w)$.

$$r\left(V_p(w) - V_e(w)\right) = (\phi - 1)w - \lambda\left(V_p(w) - V_e(w)\right) - \gamma(V_p(w) - V_e(w))$$

So

$$V_p(w) - V_e(w) = \frac{(\phi-1)w}{r+\lambda+\gamma} > 0$$

(2.2) The 3 unknowns in population are n_u, n_e and n_p. Figure 3 shows the flow diagram.

Equations:

$$\begin{aligned} n_u + n_e + n_p &= 1 \\ \lambda n_e + \gamma n_e &= \alpha(1 - F(w^*))n_u \\ \gamma n_e &= \lambda n_p \\ \lambda n_e + \lambda n_p &= \alpha(1 - F(w^*))n_u \end{aligned}$$

One of these linear equations is superfluous.

(2.3) This is just a bunch of algebra which I put at the end to keep people who finished the more important first 2 parts entertained. You have to substitute V_p out and get equations in V_e and V_u. Then follow the class derivations. For the reservation wage I got

$$\frac{(r+\lambda+\gamma\phi)w^*}{(r+\lambda+\gamma)} = b + \frac{\alpha(r+\lambda+\gamma\phi)}{(r+\lambda)(r+\lambda+\gamma)}\int_{w^*}^{\bar{w}} w - w^* dF(w)$$

so that putting $w^* = b$ leads to an implicit equation in γ_c:

$$(r+\lambda)b\gamma_c(\phi-1) = \alpha(r+\lambda+\gamma_c\phi)\int_b^{\bar{w}} w - b dF(w)$$

Because $\phi > 1$, if γ is big enough, unemployed workers will take jobs at less than they are currently because of the chance of future promotion.

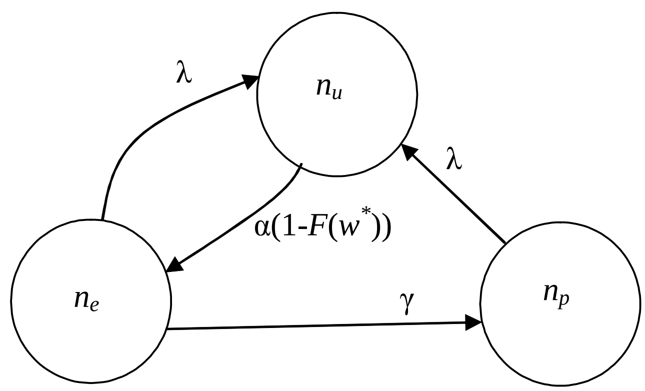

Figure 3: Flow diagram.