AECO601 Fall 2005

Macroeconomics I

Assignment 5

(1) Consider the following infinite-horizon economy consisting of a representative consumer and a representative firm. The consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(l_t)], \ 0 < \beta < 1,$$

where c_t is consumption and l_t is leisure. The consumer is endowed with one unit of time in each period which can be allocated between work and leisure. The representative firm has access to a production technology

$$y_t = z_t n_t$$

where y_t is output, n_t is labor, and z_t determines aggregate labor productivity. There is a government that purchases g_t units of output in period t and throws them into the ocean. Purchases are financed through lump-sum taxation and by issuing one-period bonds. The period-t government budget constraint is

$$g_t + (1 + r_t) b_t^g = \tau_t + b_{t+1}^g$$

where b_{t+1}^g is the quantity of bonds issued by the government in period t, τ_t is period-t taxes, and r_t is the real interest rate on a bond purchased in period $t-1$. In general we can allow the government debt to go negative. In that case the representative household is issuing debt (inside money) which the government buys. Letting b_{t+1}^p denote the quantity of bonds purchased by the consumer in period t, the period-t budget constraint of the consumer is written

$$c_t + b_{t+1}^p = w_t(1 - l_t) - \tau_t + (1 + r_t) b_t^p$$

where w_t is the real wage. We rule out Ponzi games by assuming that the discounted quantity of debt is zero in the limit:

$$\lim_{T\to\infty} \frac{b_t^p}{\Pi_{i=1}^{T-1}(1 + r_i)} = 0.$$

Also, assume that private and public debt is initially zero: $b_0^p = b_0^g = 0$.

(a) Derive a condition that implicitly determines the Pareto-optimal quantity of leisure in period t as a function of period-t productivity and period-t government purchases.

(b) Suppose that $g_t = g > 0$ and $z_t = z > 0$ for $t = 0, 1, 2, ...$ Determine the competitive equilibrium interest rate in each period.

(c) Suppose that z_t increases temporarily with $g_t = g$ as in part (b). That is, suppose that $z_t = z^* > z$ for $t = T$ and $z_t = z$ for $t = 0, 1, 2, ..., T-1, T+1, T+2, ...$ How does the time path of interest rates $\{r_t\}_{t=0}^{\infty}$ differ from that of part (b)? Be as specific as you can. (Hint: It is useful to draw a diagram that illustrates the qualitative features of the time path.)

(d) Now, suppose that z_t increases permanently with $g_t = g$ as in parts (b) and (c). That is, suppose that $z_t = z^* > z$ for $t = T, T+1, T+2, ...$ and $z_t = z$ for $t = 0, 1, 2, ..., T-1$. How does the time path of interest rates $\{r_t\}_{t=0}^{\infty}$ differ from that of part (b)? Be as specific as you can. (Again, it is useful to draw a diagram that illustrates the qualitative features of the time path.)

(e) What is the economic intuition behind the differences in parts (c) and (d)?

(2) Consider a representative agent model where the representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t \log c_t,$$

where $0 < \beta < 1$ and c_t is consumption in period t. The production technology is given by

$$y_t = \alpha_t k_t$$

where $\alpha_t = \alpha^*$ for $t = 0, 2, 4, ...,$ and $\alpha_t = \alpha^{**}$ for $t = 1, 3, 5, ...$. Assume that $\alpha^* \beta > 1$ and $\alpha^{**} \beta < 1$. Also assume 100% depreciation.

(a) Solve for the planners value function and allocation by using guess-and-verify methods (note that in general the value function will be different in even and odd periods). Show that the capital stock, output, and consumption increase in even periods and decrease in odd periods.

(b) Show that trend consumption increases (that is, consumption increases from period t to period $t+2$ for all t) if $\alpha^* \alpha^{**} \beta^2 > 1$ and decreases if $\alpha^* \alpha^{**} \beta^2 < 1$. Explain your results.

3. A consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$, c_t is consumption, and $u(.)$ is a strictly increasing, strictly concave function with $u'(0) = \infty$. The consumer has A_0 assets at the beginning of period 0, and faces a market real interest rate of r in each period $t = 0, 1, 2, ...$ The consumer earns no income during his or her lifetime, so his or her period t budget constraint is given by

$$A_{t+1} = (1 + r)(A_t - c_t)$$

where A_t is the quantity of assets at the beginning of period t.

Show that, if $\beta(1 + r) > 1$, then consumption will increase over time for the consumer, if $\beta(1 + r) < 1$, then consumption decreases over time, and if $\beta(1 + r) = 1$ then consumption will be constant over time. Explain these results.