AECO601 Masters

Macroeconomics I

Assignment 3

(1) Consider the following model:

Time: discrete, infinite horizon

Demography: $N_t = (1+n)^t N_0$ newborns in period t. Everyone lives for 2 periods

Preferences: $U_t(c_{1t}, c_{2t+1}) = (c_{1t})^{\frac{1}{2}} + \beta(c_{2t+1})^{\frac{1}{2}}$,

Endowments: $e_1 = e$, $e_2 = 0$, of the perishable consumption good

H units per original old person of storable "money".

Information: complete, perfect foresight

Suppose the economy is in a stationary monetary equilibrium.

(a) How do changes in H affect the equilibrium allocation? How do changes in growth rate of H change the allocation if (i) the additional money is issued to those holding money in proportion to the amount they hold, (ii) the additional money is distributed equally to every old person without regard to how much they currently hold.

(b) Suppose $N_0 = 1$ and $n = 0$. Also the government now creates new money and uses it to fund fixed spending $G < e$ per period time (i.e. rather then handing out money it buys goods at the current market clearing price level with the new money). How does the equilibrium inflation rate,

$$\pi_{t+1} \equiv (p_{t+1} - p_t)/p_{t+1}$$

depend on G? (Defined in this way $\pi_t = -g_t$, the net rate of return on money used in class and in Blanchard and Fischer) Assume we are looking at steady-states so that the growth rate of the money supply, σ, is constant. Is there a maximum sustainable level of G consistent with a monetary equilibrium?

(2) Consider the following model:

Time: discrete, infinite horizon

Demography: N newborns in every period. Everyone lives for 2 periods except for the first generation of old people.

Preferences: for the generations born in and after period 0; $U_t(c_{1t}, c_{2t+1}) = u(c_{1t}) + \beta u(c_{2t+1})$ where c_{it} is consumption in period t and stage i of life, $u(.)$ is increasing strictly concave and twice differentiable. For the initial old generation $\tilde{U}(c_{2\,0}) = u(c_{2\,0})$.

Endowments: of the single perishable consumption good are $\{e_t, e_t\}$ where $e_t = (1+\gamma)^t e$, $\gamma > 0$. That is everyone gets the same endowment in youth and old age but each subsequent generation gets a larger endowment than the last generation. Endowments grow at the rate γ.

Consider the following institutional arrangements:

(a) Inside money. Define, characterize and solve for the competitive equilibrium

(b) Now suppose the government is considering a pay-as-you-go social security scheme. Derive the competitive equilibrium allocation for a given sequence $\{d_t\}$, the per capita transfer from young to old (which could be negative).

(c) Ignore for now, the first generation of old and solve the government's problem for the optimal transfers of the form $d_t = \bar{d}(1+\gamma)^t$. (That is, solve for optimal $\bar{d}$)

(d) Now taking account of the first generation of old, $u(x) = x^{\frac{1}{2}}$, under what conditions on parameters can the scheme bring about a Pareto improvement?