The effects of uncertainty shocks in a model with firm entry and exit

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Abstract
Uncertainty has been shown to act on economic variables through various types of frictions, including adjustment costs, price stickiness and financial frictions. The sunk costs involved in firm entry and exit in many industries constitute another kind of friction. This paper studies the interactions between uncertainty and economic outcomes in a dynamic stochastic general equilibrium model in which firms enter and exit endogenously. Potential entrants face a fixed entry cost, while incumbents face convex and non-convex capital adjustment costs. I find that the incorporation of entry and exit causes an uncertainty shock to be followed by a long period of lower-than-usual aggregate output. This occurs partly through a decline in the total number of firms, with a disproportionately negative effect on potential entering firms. The effect is similar to the “missing generation effect” of a negative first-moment productivity shock discussed in Clementi and Palazzo (2016) and Clementi et al. (2015), though its cause is different. In addition to a decline in the number and capital of firms, the misallocation of resources becomes worse after an uncertainty shock, both due to a wait-and-see effect on incumbent firms as in Bloom et al. (2014) and due to some potentially productive firms choosing not to enter.

1 Introduction
In the various studies of the impact of uncertainty on the economy, conflicting conclusions have been drawn as to the importance of the interaction between uncertainty and capital adjustment frictions in generating an economic downturn. Bloom et al. (2014) find that the real-options effect created by this interaction – where firms pause investment and disinvestment under high uncertainty because of the costliness of making an error – can generate a drop in GDP growth of 2.5% and conclude that uncertainty can be a major factor in driving the business cycle. On the other hand, Bachmann and Bayer (2013) argue, using a model calibrated to German business data, that firm-level uncertainty shocks do not cause business cycles of the magnitudes observed in the data.
Mecikovsky and Meier (2017) question the presence of observable wait-and-see effects in the data, as they find that any significant effects of high uncertainty on variables such as firms’ investment and output act through financial constraints rather than through interaction with real frictions. Gilchrist, Sim and Zakrajšek (2014) find that financial distortions greatly amplify the response of investment to uncertainty shocks as compared to a model with only capital adjustment constraints, in part by interacting with investment irreversibility. In an empirical analysis comparing the effects of uncertainty about financial variables to those of uncertainty about real activity variables, Ludvigson, Ma and Ng (2016) provide evidence that financial uncertainty has a causal effect on the economy, whereas real activity uncertainty does not. Basu and Bundick (2014) argue that the interaction of uncertainty with price stickiness generates the same comovement among consumption, hours, output and investment as observed in the data, which has been difficult to achieve using other types of frictions.

This paper studies another channel through which uncertainty could affect the economy: The life-cycle dynamics of firms. In chapter 2 of this dissertation I found, using vector autoregressions, that job creation by entering firms is not a very important channel for the influence of financial uncertainty on the overall job creation rate, and that job destruction by exiting firms is not very important for the influence of financial uncertainty on the overall job destruction rate. This chapter complements the previous one by studying the effects of uncertainty on a model economy with endogenous entry and exit of firms. Even though I found little importance of the effect of uncertainty on entry and exit on job creation and destruction rates, it could still be that the incorporation of endogenous entry and exit into a DSGE model with capital adjustment frictions causes the effects of uncertainty on aggregate values to change relative to a model in which entry and exit are exogenous. This would suggest that variation in the entry and exit rates of firms may be more important to growth than indicated by the empirical results of chapter 2.

In this paper I extend the Bachmann and Bayer (2013) and Bloom et al. (2014) models to include endogenous entry and exit of firms. My model includes non-convex capital adjustment costs and partially irreversible investment. The model is most similar to Bloom et al. (2014), however, unlike them, I do not include labor adjustment frictions. This is justified by the suggestion in Bloom (2009) that fixed costs of capital adjustment may be more necessary than labor frictions for replicating moments of the data. The first goal of my study is to find the effects of uncertainty on the choices of entering and incumbent firms, such as entry decisions, capital accumulation and the timing of exit. The second is to study how the responses of aggregate variables and firm distributions to an uncertainty shock change when entry and exit are allowed to vary endogenously. In modeling entry and exit, I follow Clementi et al. (2015) and Clementi and Palazzo (2016). The presence of life-cycle dynamics in this model, as in Clementi et al. (2015), allows uncertainty to affect firms of different ages.
differently. Firms tend to start out smaller than average. Surviving firms tend to grow quickly until reaching a capital stock that is optimal in the long term, given the adjustment costs and irreversibility. At some point, a firm will get a sufficiently bad combination of idiosyncratic and aggregate productivity shocks and fixed continuation cost shocks such that it is optimal to exit. In this model, there can be no direct wait-and-see effect for potential entrants because they only get one chance to enter. But uncertainty shocks can still influence entry decisions through a potential entrant’s expectations. Clementi et al. (2015) discuss how the effects of a low aggregate TFP shock are amplified by endogenous entry and exit. In their model, such a shock heavily affects small firms and potential entrants, leading to a “missing generation effect”. A similar channel turns out to exist here with uncertainty shocks.

The assumptions of convex and non-convex adjustment costs and investment irreversibility are important to the results of this paper. There is a large literature on capital adjustment costs and their relationship with investment. Cooper and Haltiwanger (2006) argue that imposing both convex and non-convex adjustment costs improves the fit of a general equilibrium model of investment, compared to only one or the other type of costs. As they explain, the fixed adjustment costs can represent the disruption of production that can occur when firms expand their capacity, for instance by installing new machines which take time to install and to learn how to operate. The addition of partial investment irreversibility to the model has been justified in Cooper and Haltiwanger (2006) by the fact that investment is positive in approximately 80% of their observations, with only 10% of observations showing negative investment. The observed skewness in the investment distribution and lumpiness of investment episodes, with relatively long periods of inaction followed by intense investment activity, has been taken as evidence of irreversibilities and non-convex adjustment costs in Caballero, Engel and Haltiwanger (1995) and Bachmann, Caballero and Engel (2013). Following the literature, this paper adopts both types of capital adjustment costs and partial irreversibility of investment. A fixed cost of continuation in the market is also imposed so that firms sometimes find it optimal to exit. There is a fixed and a convex cost of entry so that the measure and size of entering firms change over time.

2 Model

The model follows Khan and Thomas (2008, 2013), Clementi et al. (2015), Bloom et al. (2014) and Bachmann and Bayer (2013), with some important differences. The economy consists of a continuum of heterogeneous firms, a continuum of potential firms and a representative, infinitely-lived household. Time is discrete and agents discount future payoffs at the common rate $\beta$. The firms produce a homogeneous good in a competitive market according to a decreasing-returns production function $F$, using capital $k$ and labor $n$ as
inputs. The good can be used as consumption or capital. Capital depreciates at the constant rate $\delta$ per period. Firms face both idiosyncratic and aggregate productivity shocks, denoted $\varepsilon$ and $Z$. The conditional standard deviation of the idiosyncratic productivity shocks $\sigma_{\varepsilon}$ is fixed, but the standard deviation of the aggregate productivity shocks $\sigma$ is time-varying. Continuing firms also face a fixed capital-adjustment cost, $\zeta_1 \sim \text{i.i.d.} \phi_1$ with support $[0, \bar{\zeta}_1]$, which must be paid in order for next-period capital to differ from the depreciated value of this period’s capital. In addition, to adjust their capital from $k$ this period to $k'$ next period, they have to pay a convex cost $\xi(k, k')$. Unlike in Bloom et al. (2014), there are no labor adjustment costs. Capital purchases by the firm are partially irreversible; the resale price of capital is $\chi \in [0, 1]$. To continue in the market this period, firms have to pay a random nonconvex continuation cost, $\zeta_0 \sim \text{i.i.d.} \phi_0$, with support $[0, \bar{\zeta}_0]$. This makes it optimal for some firms to exit. A firm that exits does not produce this period and cannot reenter later.

In each period, there is a measure one of potential firms which decide whether to enter the market. Clementi et al. (2015) also have endogenous entry and exit, but in their model the number of potential firms in each period varies so that the total measure of “blueprints” — that is, the total measure of existing firms plus the measure of potential entrants minus the measure of firms that have exited — remains constant. In their model, once a firm has exited, its blueprint can be reused by a new entrant. Here I choose to have a fixed measure of potential firms in each period so that the number of entrants is not constrained by the number of exitors and existing firms.

To enter with starting capital $\kappa$, a potential firm needs to pay a random fixed entry cost, $\zeta_e \sim \text{i.i.d.} \phi_e$ with support $[\bar{\zeta}_e, \tilde{\zeta}_e]$, and a convex cost $\xi_e(\kappa)$. Before making its entry choice, a potential firm receives a signal $\varepsilon$ about its next-period idiosyncratic productivity. If it decides to enter, it has to wait for one period to start producing. Thus a potential entrant could enter and exit in the next period without ever having produced.

The representative household is endowed with one unit of labor in each period and supplies labor to the firms at the competitively determined wage $w$. It also owns the firms, lends and borrows funds, and buys the good for consumption.

Firms’ investment, entry and exit decisions in each period generate a distribution $\mu$ over capital stocks $k$ and idiosyncratic productivities $\varepsilon$, where $\mu(k, \varepsilon)$ is the measure of firms with capital $k$ and idiosyncratic productivity $\varepsilon$. The state variables on which firms base their decisions consist of the idiosyncratic ones $(k, \varepsilon)$ and the aggregate ones $s = (Z, \sigma, \mu)$. In section 4, I will discuss how $\mu$ is proxied by aggregate capital according to the algorithm of Krusell and Smith (1998). The endogenous variables are all functions of the set of state variables. In the equations of the following sections, I omit the dependence on the state variables when it is clear and write, for example, $\zeta_0^*$ instead of $\zeta_0^*(k, \varepsilon; s)$. 

2.1 Incumbent firm’s problem

An incumbent firm (or incumbent) is defined as a firm that was in the market last period (either it produced last period or it was an entrant last period). An incumbent firm’s output is

\[ y = \varepsilon Z F(k, n), \]  

where

\[ F(k, n) = k^\alpha n^\nu, \]  

\[ \alpha + \nu < 1, \]  

\[ \alpha, \nu > 0, \]  

\( \varepsilon \) is the firm’s idiosyncratic productivity shock and \( Z \) is the aggregate productivity shock. The logs of the productivity shocks evolve as AR(1) processes:

\[ \ln(\varepsilon_t) = -\frac{\sigma^2_\varepsilon}{2(1-\rho^2_\varepsilon)} + \rho_\varepsilon \ln(\varepsilon_{t-1}) + \tau_{\varepsilon,t} \]  

with

\[ \tau_{\varepsilon,t} \sim N(0, \sigma^2_\varepsilon), \]  

and

\[ \ln(Z_t) = -\frac{\sigma^2_Z}{2(1-\rho^2_Z)} + \rho_Z \ln(Z_{t-1}) + \tau_{Z,t} \]  

with

\[ \tau_{Z,t} \sim N(0, \sigma^2_Z). \]

The processes are defined in this way to facilitate experimenting with the variance, so that when \( \sigma_\varepsilon \) is changed, the unconditional mean of the process for \( \varepsilon \) stays the same. Similarly, we want the unconditional mean for \( Z \) to stay the same when \( \sigma_Z \) rises in the uncertainty shock experiments, so that responses to a rise in \( \sigma \) are not caused by an increase in the unconditional mean of \( Z \). \( \tau_{\varepsilon,t} \) and \( \tau_{Z,t} \) are independent across time and independent of each other and of the fixed adjustment costs. \( \ln(\sigma_t) \) evolves as an AR(1) process with mixture of normals error.

The timing of incumbent firms’ decisions is as follows. In each period an incumbent firm first observes its idiosyncratic shock \( \varepsilon \), the aggregate TFP shock \( Z \) and the realization of its fixed continuation cost \( \zeta_0 \). Then it decides whether to continue or to exit. If it continues, it then observes the realization of the adjustment cost \( \zeta_1 \) and decides whether to adjust its capital to the optimal value or to let its capital depreciate. Given current capital stock \( k \), a firm can let its capital depreciate to \((1-\delta)k\) without paying any fixed adjustment cost. If a firm adjusts to capital stock \( k' \), it pays the cost

\[ \xi(k, k') = \tau(k, k') \left(k' - (1-\delta)k\right) + c_q \frac{(k' - (1-\delta)k)^2}{k}, \]  

with

\[ \tau(k, k') = \phi(k') \ln(k') - \phi(k) \ln(k), \]  

where

\[ \phi(k) \]  

is the firm’s idiosyncratic productivity shock and \( Z \) is the aggregate productivity shock. The logs of the productivity shocks evolve as AR(1) processes:

\[ \ln(\varepsilon_t) = -\frac{\sigma^2_\varepsilon}{2(1-\rho^2_\varepsilon)} + \rho_\varepsilon \ln(\varepsilon_{t-1}) + \tau_{\varepsilon,t} \]  

with

\[ \tau_{\varepsilon,t} \sim N(0, \sigma^2_\varepsilon), \]  

and

\[ \ln(Z_t) = -\frac{\sigma^2_Z}{2(1-\rho^2_Z)} + \rho_Z \ln(Z_{t-1}) + \tau_{Z,t} \]  

with

\[ \tau_{Z,t} \sim N(0, \sigma^2_Z). \]
where
\[ \tau(k, k') = 1\{k' > (1 - \delta)k\} + \chi 1\{k' < (1 - \delta)k\} \]
and \( \chi \in [0, 1] \) is the irreversibility parameter. The second term on the right-hand side of equation (5) is the quadratic adjustment cost; this allows firms with the same \( \varepsilon \) but different \( k \) to have different target capital levels. A firm starts the next period with the capital stock chosen in this period, i.e. there is a one-period time-to-build assumption.

The timing assumptions for incumbents are made to generate more heterogeneity in the firms’ actions and outcomes, as well as for simplicity of computation. By assuming that \( \zeta_1 \) is independent of \( \zeta_0 \) and not observed when the continuation decision is made, the exit decision can be characterized by the cutoff value \( \zeta_0^{*} \). If firms were to observe their realizations of \( \zeta_1 \) at the same time as their realizations of all the other shocks, for each set of productivity shocks and capital, there would be a region in the space \( S = [0, \tilde{\zeta}_1] \times [0, \tilde{\zeta}_0] \) in which all firms would both continue and adjust, another region in which all firms would continue but not adjust, and another in which all firms would exit. These regions might not be connected, and would be more complicated to describe and work with than the simple cutoff rules when firms observe \( \zeta_1 \) after making their continuation decision. Furthermore, when firms observe \( \zeta_1 \) after the other shocks, they may make an initial choice to continue, based on the expected value over \( \zeta_1 \), that turns out to have been mistaken (they get a very high adjustment cost shock when they expected to invest a non-zero amount). The fact that such an outcome would be impossible in the case where firms observe all shocks at the same time implies that the latter case generates less heterogeneity in firms’ values.

Since the firm does not have labor adjustment costs, the labor choice solves a static problem. The firm’s labor demand is
\[ n_d(k, \varepsilon; s) = \arg\max_n \left\{ \varepsilon Z k^n \nu - w(s) n \right\} = \left( \frac{\varepsilon Z k^\alpha}{w(s)} \right)^{1/(1-\nu)} \]  \hspace{1cm} (6)

Define the firm’s flow profit as
\[ \Pi(k, \varepsilon; s) = \varepsilon Z F(k, n_d) - w(s) n_d = (1 - \nu) \left( \frac{\nu}{w(s)} \right)^{\frac{\nu}{1-\nu}} (\varepsilon Z)^{\frac{1}{1-\nu}} k^\alpha. \]  \hspace{1cm} (7)

The incumbent firm’s value function \( V(k, \varepsilon; s) \) is defined in three steps. First consider the problem of a firm that has already decided to continue and has observed its realization of \( \zeta_1 \), and is deciding whether to adjust to the optimal next-period capital or not to adjust. If the firm chooses not to adjust, its capital will depreciate to
\[ k^{na} = (1 - \delta)k \]
by the next period. The optimal next period capital \( k^a \) conditional on adjusting is defined by
\[ k^a(k, \varepsilon; s) = \arg\max_{k'} \left\{ -\xi(k, k') + E \left[ m' V(k', \varepsilon'; s') \right] \right\}. \]  \hspace{1cm} (8)
where $m'$ is the stochastic discount factor to be defined below, and the expectation is taken over the joint probability distributions of the state variables.

Let $V^a$ be the continuation value of a firm that has chosen to adjust to $k^a$ and $V^{na}$ the continuation value of a firm that has chosen no adjustment. Then

$$V^a(k,\varepsilon; s) = -\xi(k, k^a) + E\left[m'V(k^a, \varepsilon'; s')\right] \quad (9)$$

and

$$V^{na}(k,\varepsilon; s) = E\left[m'V(k^{na}, \varepsilon'; s')\right]. \quad (10)$$

The value of adjusting is nonincreasing in the realization of $\zeta_1$. Therefore, given $(k,\varepsilon; s)$ there is a cutoff adjustment cost $\zeta_1^*$ such that a firm adjusts to $k^a$ if $\zeta_1 \leq \zeta_1^*$ and does not adjust otherwise. $\zeta_1^*$ is such that the firm is indifferent between choosing $k^a$ and choosing $k^{na}$, so

$$\zeta_1^*(k,\varepsilon; s) = \max\left\{\min\left\{\frac{V^a(k,\varepsilon; s) - V^{na}(k,\varepsilon; s)}{w}, \tilde{\zeta}_1\right\}, 0\right\}. \quad (11)$$

Now consider the firm’s choice whether to continue or exit, before $\zeta_1$ has been realized. The expected value of continuing, after paying the fixed continuation cost, is

$$V_1(k,\varepsilon; s) = \Pi(k,\varepsilon; s) + \int_{\zeta_1^*}^{\tilde{\zeta}_1} \left[-w\zeta_1 + V^a(k,\varepsilon; s)\right] d\phi_1(\zeta_1) + \int_{\tilde{\zeta}_1}^{\zeta_1^*} V^{na}(k,\varepsilon; s) d\phi_1(\zeta_1).$$

The value of exiting is just $\chi k$. So the firm is indifferent between exiting and continuing when

$$V_1(k,\varepsilon; s) - \zeta_0 = \chi k.$$

Therefore the cutoff value of $\zeta_0$ between exiting and continuing is

$$\zeta_0^* = \max\left\{\min\left\{V_1(k,\varepsilon; s) - \chi k, \tilde{\zeta}_0\right\}, 0\right\}, \quad (12)$$

and the firm’s expected value before the realization of $\zeta_0$ is

$$V(k,\varepsilon; s) = \int_{0}^{\zeta_0^*} \left[-\zeta_0 + V_1(k,\varepsilon; s)\right] d\phi_0(\zeta_0) + \int_{\zeta_0^*}^{\tilde{\zeta}_0} \chi k d\phi_0(\zeta_0). \quad (13)$$

### 2.2 Potential entrants’ problem

A potential entrant first observes its productivity signal $\varepsilon$ and its fixed entry cost $\zeta_e \sim \phi_e$, then decides whether to enter. If it does not enter, it cannot enter at a later date, and its reservation payoff is zero. If it enters with capital $\kappa$, it pays the fixed cost $\zeta_e$ and a convex cost,

$$\xi_e(\kappa) = \kappa + \frac{\gamma_1}{2}\kappa^2. \quad (14)$$
It begins producing the following period, and from that point its problem becomes that of an incumbent firm. Thus the potential entrant’s problem is summarized by the equations

$$
\kappa^*(\varepsilon; s) \equiv \arg\max \left\{ -\xi_e(\kappa) + E\left[ m' V(\kappa, \varepsilon'; s') \right] \right\}, \\
(15)
$$
determining its optimal entering capital stock should it decide to enter,

$$
V_e(\varepsilon; s) \equiv -\xi_e(\kappa^*(\varepsilon; s)) + E\left[ m' V(\kappa^*(\varepsilon; s), \varepsilon'; s') \right], \\
(16)
$$
determining the firm’s value if it does enter with the optimal capital stock, and

$$
\zeta^*_e(\varepsilon, s) = \max \left\{ \min \left\{ V_e(\varepsilon; s), \zeta_e \right\}, \zeta_e \right\}, \\
(17)
$$
such that a potential entrant with state variables $$\varepsilon; s$$ enters, with initial capital $$\kappa^*(\varepsilon; s)$$, if and only if $$\zeta_e < \zeta^*_e(\varepsilon; s)$$.

### 2.3 Household’s problem

The household chooses sequences $$\{C, N, a'\}$$ of consumption, labor and assets to maximize its value function

$$
W(C, N) = U(C, N) + \beta E[W(C', N')] \\
(18)
$$
subject to

$$
C + a' \leq wN + (1 + r)a, \\
$$
where the wage $$w$$ and interest rate $$r$$ are taken as given by the household, and $$U(C, N)$$ is the flow utility function. The first-order conditions of this problem lead to the following equalities, assuming interior solutions:

$$
w = -\frac{U_2(C, N)}{U_1(C, N)}, \\
(19)
$$

$$
\frac{1}{1 + r} = \beta \frac{U_1(C', N')}{U_1(C, N)}, \\
(20)
$$
where $$U_j$$ refers to the partial derivative of the function $$U$$ with respect to its $$j$$th argument and primes refer to next-period values of variables. The term on the right-hand side of equation (20) is defined as the stochastic discount factor,

$$
m' = \beta \frac{U_1(C', N')}{U_1(C, N)}. \\
(21)
$$

By giving firms the same stochastic discount factor as households, I incorporate the household’s problem into the firm’s problem, as noted by Khan and Thomas (2008, 2013). Define

$$
p \equiv U_1(C, N). \\
$$
As in Khan and Thomas (2008, 2013) and Bloom et al. (2014), I use the utility function

\[ U(C, N) = \ln(C) - \theta N. \tag{22} \]

This specification is chosen for simplicity, as it makes it unnecessary to forecast both the wage and the marginal utility of consumption. Combining the household’s first-order conditions with equation (22) leads to the expression for wage,

\[
W = -\frac{U_2(C, N)}{U_1(C, N)} = \theta C. \tag{23}
\]

Thus, the labor supply is infinitely elastic.

3 Stationary equilibrium

3.1 Definition

The stationary equilibrium of this economy is defined by the equations in this section. These are similar to the equations determining the solutions to the agents’ problems above, except that here the aggregate state variables do not vary. First, the aggregate TFP is fixed at its unconditional mean.

\[ Z_t \equiv Z = 1. \]

Assume that the distributions \( \phi_0 \) and \( \phi_1 \) of a firm’s continuation costs \( \zeta_0 \) and adjustment costs \( \zeta_1 \), respectively, are both uniform on their supports.

The optimal choice of next-period capital conditional on adjustment and continuation is made as above.

\[
k^a(k, \varepsilon) = \arg\max_{k'} \left\{ -\xi(k, k') + \beta E[V^{ss}(k', \varepsilon')|\varepsilon] \right\}, \tag{24}\]

where \( V^{ss} \) is the steady-state value function of the firm. If a firm chooses not to adjust, it starts next period with the depreciated capital stock,

\[ k^{na}(k, \varepsilon) = (1 - \delta)k. \]

The value functions of the firm conditional on adjustment and no adjustment, respectively, are defined as

\[
V_1^a(k, \varepsilon) = -\xi(k, k^a) + \beta E[V^{ss}(k^a, \varepsilon')|\varepsilon], \tag{25}\]

\[
V_1^{na}(k, \varepsilon) = \beta E[V^{ss}(k^{na}, \varepsilon')|\varepsilon]. \tag{26}\]

The cutoff fixed cost for adjustment is defined as

\[
\zeta_1^*(k, \varepsilon) = \max \left\{ \min \left\{ \frac{V_1^a(k, \varepsilon) - V_1^{na}(k, \varepsilon)}{w^{ss}}, \zeta_1 \right\}, 0 \right\}. \tag{27}\]
and the value function of the firm conditional on continuation is

\[ V_{ss}^0(k, \varepsilon) = \Pi(k, \varepsilon; w) + \left(1 - \frac{\zeta^*}{\zeta_1}\right)V_{1na}^s(k, \varepsilon) - w\frac{(\zeta^*_1)^2}{2\zeta_1} + \frac{\zeta^*}{\zeta_1}V_{1a}^s(k, \varepsilon). \]  

(28)

Equation (28) defines the value of continuing as profits plus the expected value over \( \zeta_1 \) of the firm, given that it chooses the optimal capital level for next period. The cutoff value of \( \zeta_0 \), determining whether a firm continues or exits, is defined as

\[ \zeta_0^*(k, \varepsilon) = \max \left\{ \min \left\{ V_{ss}^0(k, \varepsilon) - \chi k, 0 \right\}, 0 \right\}, \]  

(29)

and the steady-state value of the firm before its fixed costs have been realized is

\[ V^{ss}(k, \varepsilon) = \left(1 - \frac{\zeta^*_0}{\zeta_0}\right)\chi k - \frac{w(\zeta^*_1)^2}{2\zeta_1} + \frac{\zeta^*_1}{\zeta_1}V_{ss}^0(k, \varepsilon). \]  

(30)

Since there is a continuum of firms subject to the same laws of motion for productivity and adjustment costs, the law of large numbers implies that the proportion of firms taking any possible action (continuation or exit, adjustment or no adjustment) is equal to the probability of a given firm taking that action. Therefore the total adjustment cost paid by firms at state \((k, \varepsilon)\), denoted \(AC(k, \varepsilon)\), is determined as the weighted sum of the costs paid when taking either type of action.

\[ AC(k, \varepsilon) = \left(1 - \frac{\zeta^*_0}{\zeta_0}\right)\chi k + \frac{(\zeta^*_0)^2}{2\zeta_0} + \frac{\zeta^*_0}{\zeta_0} \left(1 - \frac{\zeta^*_1}{\zeta_1}\right)\xi(k, k^{na}) + \frac{w(\zeta^*_1)^2}{2\zeta_1} + \frac{\zeta^*_1}{\zeta_1}\xi(k, k^a). \]  

(31)

In the two next equations \(\kappa^*\), a potential firm’s initial capital choice conditional on entering, and the fixed entry cost cutoff \(\zeta^*_e\) are defined as functions of the idiosyncratic productivity signal \(\varepsilon\).

\[ \kappa^*(\varepsilon) = \arg\max_{\kappa > 0} \left\{ -\kappa - \gamma_1 k^2 + \beta E[V(\kappa, \varepsilon')|\varepsilon] \right\}, \]  

(32)

\[ \zeta^*_e(\varepsilon) = \max \left\{ \min \left\{ -\kappa^* - \frac{\gamma_1 (\kappa^*^2)}{2} + \beta E[V(\kappa^*, \varepsilon')|\varepsilon], \zeta_e \right\}, \zeta_e \right\}. \]  

(33)

The distribution of entering firms is then given by

\[ EM(k, \varepsilon) = \begin{cases} \left(\frac{\zeta^*_e(\varepsilon) - \zeta_e}{\zeta^*_e - \zeta_e}\right)\bar{\mu}(\varepsilon) & \text{if } k = \kappa^*(\varepsilon), \\ 0, & \text{otherwise} \end{cases} \]  

(34)

where \(\bar{\mu}\) is the ergodic density of the process for \(\varepsilon\). Individual and aggregate entry costs are given by

\[ EC_0(\varepsilon) = \frac{(\zeta^*_e)^2 - \zeta_e^2}{2(\zeta^*_e - \zeta_e)} + \left(\frac{\zeta^*_e - \zeta_e}{\zeta^*_e - \zeta_e}\right)\left(\kappa^*(\varepsilon) + \frac{\gamma_1 (\kappa^*(\varepsilon))^2}{2}\right). \]  

(35)

\[ EC = \int EC_0(\varepsilon)\bar{\mu}(\varepsilon). \]  

(36)
Finally, the aggregate equilibrium conditions are

\[ Y = \left( \frac{\nu}{w} \right)^{\frac{1}{1-\nu}} \int_k \int_\varepsilon \frac{\zeta_0^*(k, \varepsilon)}{\zeta_0} \varepsilon^{\frac{1}{1-\nu}} k^{\frac{\nu}{1-\nu}} d\mu(k, \varepsilon), \]  

(37)

\[ C = Y - AC - EC, \]

(38)

\[ w = \theta C, \]

(39)

\[ N = \left( \frac{\nu}{w} \right)^{\frac{1}{1-\nu}} \int_k \int_\varepsilon \left( \frac{\zeta_0^*(k, \varepsilon)}{\zeta_0} \right) \varepsilon^{\frac{1}{1-\nu}} k^{\frac{\nu}{1-\nu}} d\mu(k, \varepsilon). \]

(40)

The ergodic distribution of individual states \( k \) and \( \varepsilon \) obeys

\[ \mu(k, \varepsilon) = (\Gamma \mu)(k, \varepsilon) + EM(k, \varepsilon), \]

(41)

where \( \Gamma \) is the transition function over \( \mu \) given by

\[ \Gamma \mu(k', \varepsilon'; s) = \int \int \frac{\zeta_0^*}{\zeta_0} \Pr(\varepsilon'|\varepsilon) \times \left[ \left( 1 - \frac{\zeta_1^*}{\zeta_1} \right) 1 \left\{ k^{na}(k, \varepsilon) = k' \right\} + \frac{\zeta_1^*}{\zeta_1} 1 \left\{ k^n(k_i, \varepsilon_j) = k' \right\} \right] d\mu(k, \varepsilon). \]

(42)

### 3.2 Numerical approximations

Since capital is a state variable, it is simpler to calculate the value and decision functions using a grid for capital. I use a grid that is closed under depreciation (the ratio of the \( j \)th to the \((j-1)\)th element of the grid is \( \frac{1}{1-\delta} \), where \( \delta \) is the capital depreciation rate) with minimum value approximately 0.01 and maximum value approximately 6.5. The grid \( \Xi \) and transition matrix \( P_\varepsilon \) for \( \varepsilon \) are constructed using Tauchen’s (1986) approximation with parameters \( \rho_\varepsilon \) and \( \sigma_\varepsilon \) determined from the calibration. The two grids for \( Z \) and its transition matrices are constructed using an extension of Tauchen’s method to a variable with two possible standard deviation values. Denote the grid for \( k \) by \( \Lambda = \{k_1, ..., k_{n_k}\} \), the grid for \( \varepsilon \) by \( \Xi = \{\varepsilon_1, ..., \varepsilon_{n_\varepsilon}\} \) and the grids for \( Z \) by \( \Upsilon_L = \{Z_{1,L}, ..., Z_{n_{\varepsilon}, L}\} \) and \( \Upsilon_H = \{Z_{1,H}, ..., Z_{n_{\varepsilon}, H}\} \), respectively. Then all the equations above have approximate counterparts with discrete state variables. For instance, equation (42) is approximated by

\[ \Gamma \mu(k_i', \varepsilon_j) = \sum_{i=1}^{n_k} \sum_{j=1}^{n_\varepsilon} \frac{\zeta_0^*(k_i, \varepsilon_j)}{\zeta_0} P_\varepsilon(\varepsilon_j, \varepsilon_j') \]

\[ \times \left( \frac{\zeta_1^*(k_i, \varepsilon_j)}{\zeta_1} \right) 1 \left\{ k^n(k_i, \varepsilon_j) = k_{i'} \right\} \left( 1 - \frac{\zeta_1^*(k_i, \varepsilon_j)}{\zeta_1} \right) 1 \left\{ k^{na}(k_i, \varepsilon_j) = k_{i'} \right\} \mu(k_i, \varepsilon_j). \]

(43)
3.3 Calibration

I calibrate the model in stationary equilibrium to match certain moments of the data. A period equals one year. Table 1 shows the calibration targets and model values, and Table 2 shows the corresponding parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital-output ratio</td>
<td>2.1</td>
<td>2-3</td>
<td>standard</td>
</tr>
<tr>
<td>Hours</td>
<td>0.35</td>
<td>0.25-0.33</td>
<td>standard</td>
</tr>
<tr>
<td>Entry/exit rate</td>
<td>7.2%</td>
<td>8.7%</td>
<td>Clementi et al. 2015</td>
</tr>
<tr>
<td>Exit rate age 1</td>
<td>17%</td>
<td>17%</td>
<td>Clementi et al. 2015</td>
</tr>
<tr>
<td>Exit rate age 2</td>
<td>11%</td>
<td>11%</td>
<td>Clementi et al. 2015</td>
</tr>
<tr>
<td>Relative entry size</td>
<td>0.59</td>
<td>0.55</td>
<td>Lee and Mukoyama 2015</td>
</tr>
<tr>
<td>Average investment rate</td>
<td>10.1%</td>
<td>12.2%</td>
<td>Cooper and Haltiwanger 2006, LRD</td>
</tr>
<tr>
<td>Prop. investment spikes</td>
<td>18.9%</td>
<td>18.6%</td>
<td>Cooper and Haltiwanger 2006, LRD</td>
</tr>
<tr>
<td>Survival rate after 5 yrs</td>
<td>50%</td>
<td>45%</td>
<td>Clementi et al. 2015</td>
</tr>
</tbody>
</table>

Note: LRD is the Longitudinal Research Database, using large manufacturing firms from 1972 to 1988. Investment spikes are defined as instances where a firm’s investment is more than 20% of its existing capital.

Table 2. Calibrated parameter values

<table>
<thead>
<tr>
<th>δ</th>
<th>β</th>
<th>α</th>
<th>ν</th>
<th>c_q</th>
<th>ζ_0</th>
<th>ζ_1</th>
<th>ζ_e</th>
<th>ζ_e'</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.91</td>
<td>0.98</td>
<td>0.285</td>
<td>0.46</td>
<td>0.08</td>
<td>0.26</td>
<td>0.007</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>γ_1</td>
<td>θ</td>
<td>ρ_ε</td>
<td>σ_ε</td>
<td>χ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>2.8</td>
<td>0.91</td>
<td>0.075</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: δ is the capital depreciation rate, β the discount factor, α capital output elasticity, ν labor output elasticity, c_q quadratic cost factor, ζ_0 the upper bound for fixed continuation cost, ζ_1 the upper bound for fixed adjustment cost, ζ_e and ζ_e' the bounds for fixed entry cost, γ_1 the quadratic entry cost factor, θ the disutility of labor, and ρ_ε and σ_ε the persistence and standard deviation of ln(ε).

3.4 Steady state experiments

I run three experiments on the steady state economy. In the first, I start with the parameters at the calibrated values and gradually increase σ_ε, the variance of the process for idiosyncratic productivity, common to all firms. Since the process for ε is

\[ \ln(\varepsilon') = \rho_\varepsilon \ln(\varepsilon) - \frac{\sigma_\varepsilon^2}{2(1-\rho_\varepsilon^2)} + \sigma_\varepsilon \omega, \]

with \( \omega \sim \mathcal{N}(0,1) \), an increase in σ_ε preserves the unconditional mean of \varepsilon'.
In the second experiment, I start with the calibrated parameter values and gradually increase the perceived uncertainty $\hat{\sigma}_\varepsilon$ from its initial value, but not the actual uncertainty $\sigma_\varepsilon$. That is, agents choose their decision rules taking $\hat{\sigma}_\varepsilon$ as the conditional standard deviation of $\varepsilon$, but the actual process followed by $\varepsilon$ has standard deviation $\sigma_\varepsilon$. This experiment is meant to isolate the effects of agents’ perception of uncertainty from the effects of the actual variance of the productivity shocks. As can be seen from the measurements of professional forecasters probability scores in chapter 1 of this thesis, forecasters’ perceived uncertainty about a variable is sometimes very different from an ex-post estimate of the variance of that variable. It is useful to know how this difference affects agents’ actions, because this could tell us whether it would be worth making an effort to find more “precise” measures of uncertainty.

The first two experiments involve general equilibrium. In the third experiment I find the stationary partial equilibrium with the wage fixed as $\sigma_\varepsilon$ increases. The purpose is to isolate the impact the consumer side of the market has on changes in steady-state values as the variance of the productivity shock increases (the difference between the general equilibrium values and the partial equilibrium values gives the effect of general equilibrium). This can help to understand the role of the household’s preferences and choices in recursive general equilibrium.

Each of the graphs in Figures 1-4 shows the results from all three experiments for a single variable. For instance, the left hand side of Figure 1 plots the changes in the aggregate steady-state capital stock as $\sigma_\varepsilon$ increases (solid line), as the perceived standard deviation $\hat{\sigma}_\varepsilon$ increases (line with circles), and as $\sigma_\varepsilon$ increases with wage fixed (dashed line).
Figure 1: Aggregate capital (top) and aggregate output (bottom)
Figure 2: aggregate employment and consumption
Figure 3: Aggregate investment and wage
In the stationary equilibrium experiments there is no wait-and-see effect because the change in the productivity variance is recognized to be permanent. The effect of an increase in uncertainty here is to increase activity: Capital, output, employment, consumption and wage all rise. This operates through the convexity of profits in productivity, which causes firms’ value functions to increase at a higher rate in capital (Oi 1961). Firms produce more now and invest more to be able to produce more in the future. Consumption increases because output increases at a higher rate than the total capital adjustment costs that must be paid, and because the demand for labor (and labor income) increases. The only variable in the graphs that decreases is the rate of entry and exit. This can be explained by the following argument: As the variance of the idiosyncratic productivity shock increases, the proportion of firms facing very high and very low productivity shocks increases. The expected return to staying in business rises, especially for larger firms. This effect dominates the effect of the very low shocks.

The partial equilibrium responses to an increase in uncertainty are much stronger than those in general equilibrium. For instance, an actual increase in the standard deviation of productivity from 0.075 to 0.085 causes a more than 100% increase in output in partial equilibrium, whereas in general equilibrium the increase is only about 12%. Capital and employment both increase by about 250% in partial equilibrium, compared to only about 5% in general equilibrium. General equilibrium effects mitigate the expansionary effect of an increase in uncertainty through the rise in wage. When uncertainty rises with the wage fixed, firms earn more profits by producing more and investing more, without having to pay more to their factors of production (because the marginal utility of consumption is constant in steady state, the interest rate, defined by $\frac{1}{1+r} = \beta$, is
automatically fixed here). This leads to a large increase in output, which allows the household increase consumption by about 225%. However, consumption increases at a lower rate than output because the return to savings is higher, so that investment increases at a higher rate than consumption. In addition, the capital adjustment costs that must be paid are greater with larger variance of the idiosyncratic shock, since the actual adjustment that takes place is more. This further mitigates the increase in consumption.

In Figures 1-4, the effects of a change in only perceived uncertainty are close to those when both perceived and actual uncertainty change, at least compared to the changes in partial equilibrium. This suggests that it is mainly the perception of a larger spread in productivity, rather than an actual increased spread in productivity, that causes the variables’ responses. The main difference between the perceived uncertainty and true uncertainty experiment outcomes is the proportion of firms with very high or very low idiosyncratic productivity shocks. When firms get very low productivity shocks they are likely to exit (recall that firms observe their actual productivity before choosing this period’s actions); when they get very high productivity shocks they are likely to make a large positive investment, in accordance with the lumpy investment idea. Therefore the exit rate is higher for the true uncertainty experiment than for the perceived uncertainty experiment, at any value of $\sigma_\varepsilon$ or $\hat{\sigma}_\varepsilon$.

Capital and output increase at a higher rate in the perceived-uncertainty experiment than in the true-uncertainty experiment. This comes from the fact that a smaller proportion of firms is exiting in the perceived uncertainty experiment. Consumption increases by less in the perceived uncertainty experiment because more resources are being used for capital adjustment.

4 Recursive general equilibrium

Given the functional forms of the firms’ production function and household’s utility function, and given the random processes for the idiosyncratic productivity shocks, aggregate productivity shock and standard deviation of the aggregate productivity shock, a recursive general equilibrium in this model is defined by the following conditions.

1. Incumbent firms choose policy functions $(k^a, k^{na}, \zeta_0^*, \zeta_1^*)$ to maximize their value function $V$ at every value of the state variables $(k, \varepsilon; s)$, according to equations (9) to (13).

2. Potential entrants choose entering capital $k^*$ and the fixed-cost entry cut-off $\zeta_\varepsilon^*$ to maximize their value function, given the expected value function for incumbents, according to equations (15) to (17).

3. The representative household chooses sequences $\{C, N, a^t\}$ of consumption, labor supply and asset holdings to maximize its value function $W$ in equation (18).
4. Markets clear in each period: The wage $w$ is such that aggregate labor demand equals labor supply, and the resource constraint

$$Y - AC - EC - C = 0$$

holds, where $Y$ is aggregate output defined in equation (37) (where the variables here are functions of $(k, \varepsilon; s)$), $AC$ equals the aggregate adjustment costs of incumbents defined in equation (31), $EC$ equals the aggregate entry costs defined in equation (36), and $C$ is the household’s consumption. Imposing the stochastic discount factor

$$m' = \frac{\beta U_1(C', N')}{U_1(C, N)}$$
on firms’ decisions ensures that the loans market clears.

5. The transition over $\mu$ is given by

$$\mu' = \Gamma(\mu) + EM$$

(44)
given $s = (Z, \sigma, \mu)$, where

$$\Gamma(\mu)(k', \varepsilon') = \int \int \left( \frac{\zeta_0^*(k, \varepsilon; s)}{\zeta_0} \right) \left[ \frac{\zeta_1^*(k, \varepsilon; s)}{\zeta_1} \right] \mathbf{1}\{k' = k^a(k, \varepsilon; s)\} +$$

$$\left(1 - \frac{\zeta_1^*(k, \varepsilon; s)}{\zeta_1}\right) \mathbf{1}\{k' = k^{aa}(k, \varepsilon; s)\} \right] P_\varepsilon(\varepsilon' | \varepsilon) d\mu(k, \varepsilon)$$

where

$$EM(k', \varepsilon' ; s) = \int \mathbf{1}\{k' = \kappa^*(\varepsilon; s)\} P_\varepsilon(\varepsilon' | \varepsilon) d\bar{\mu}(\varepsilon)$$

(45)
is the distribution of entrants over capital and idiosyncratic productivity. $\mu(k, \varepsilon)$ is the measure of firms with capital $k$ and idiosyncratic productivity $\varepsilon$ that have entered last period or previously and that do not exit this period - that is, those firms that produce this period.

6. Rational expectations hold: The agents know the transition function for $\mu$ and incorporate this expectation of next-period’s $\mu$ into their decision rules.

To be able to solve the model, some approximations need to be made. I discretize the state variables $k, \varepsilon, Z, \sigma$, so that the value and policy functions become arrays. $\sigma$ can assume just two values, $\sigma_L$ and $\sigma_H$, and follows a Markov chain with transition matrix $P_\sigma$, defined by

$$\Pr[\sigma' = \sigma_L | \sigma = \sigma_L] = 0.95, \ Pr[\sigma' = \sigma_L | \sigma = \sigma_H] = 0.3.$$

(46)
The productivity shocks $\varepsilon$ and $Z$ are discretized according to Tauchen (1986). There are two grids, $Z_L$ and $Z_H$, for $Z$, one corresponding to $\sigma_L$ and one to $\sigma_H$. The transition matrix from values of $Z$ on $Z_L$ to $Z_H$ approximates the probabilities of moving from each point on the $Z_L$ grid to each point on the $Z_H$ grid in a way consistent with the Tauchen (1986) method.
In a full rational expectations equilibrium, agents can predict future prices conditional on the realizations of the shocks. In this model, since prices depend on the whole distribution $\mu$ of firms over $(k, \varepsilon)$, it is infeasible for agents to predict prices this way, as they would be integrating over an infinite-dimensional space to find expected values. Therefore, as in Thomas (2003), Khan and Thomas (2008, 2013), Bachmann, Caballero and Engel (2013), and Bloom et al. (2014), I proxy the argument $\mu$ in the equilibrium value, expected value and policy functions by the aggregate capital stock $K = \int_k \int_\varepsilon k d\mu(k, \varepsilon)$, then check whether forecasts based on this proxy generate sufficiently small forecast errors. Young (2007a) demonstrates that in a similar model, the additional knowledge contained in higher moments of the firm distribution would not help agents make significantly better forecasts. It is not certain that this holds for the model in this paper, but the forecast rules generate 100-period ahead forecasts for $K$ that are within 15% of the true values, and for $p = MU_C$ that are within 5%. Log-linear forecasting rules are assumed for both $K$ and $p$, the marginal utility of consumption, i.e. I assume that agents predict using

$$\ln(p) = \alpha_p(Z, \sigma, \sigma_{-1}) + \beta_p(Z, \sigma, \sigma_{-1}) \ln(K),$$

$$\ln(K') = \alpha_K(Z, \sigma, \sigma_{-1}) + \beta_K(Z, \sigma, \sigma_{-1}) \ln(K).$$

The linear forecast coefficients $(\alpha_p, \beta_p)$ for $\ln(p)$ and $(\alpha_K, \beta_K)$ for $\ln(K)$ are allowed to depend on $(Z, \sigma, \sigma_{-1})$ as in Bloom et al. (2014), who argue that inclusion of the lagged aggregate productivity variance in the forecasting rule improves the forecast.

As in Khan and Thomas (2008, 2013), the solution method alternates between an inner and an outer loop. In the inner loop, the model is simulated for $T = 4000$ periods, and the OLS coefficients on $\ln(K)$ and $\ln(p)$ in the forecasting rule are estimated. In the simulation of the model neither $p$ nor $K$ are restricted to follow the forecasting rules. $p$ is calculated in each period from the aggregate resource constraint, and $K$ is computed from firms’ previous-period decision rules, which determine the distribution $\mu$. In the outer loop the firms’ value and policy functions are calculated using the latest $(\alpha_p, \beta_p)$ and $(\alpha_K, \beta_K)$ in the firms’ forecasts (dampening is used as I found this produces faster convergence). The iteration is continued until the 10-year-ahead forecasts for $p$ and $K$ lie within 5% of the actual values found in the simulation. The $R$-squared statistics for the OLS regressions for equations (47) and (48) are all above 0.99.

4.1 Background results

The correlation coefficient between consumption and output in the converged model is 0.80, and between output and lagged consumption is 0.89. The correlation between entry and output is 0.37 (compared to 0.40 in Campbell 1998), the correlation between output and lagged entry is 0.24, and between output and lead entry is 0.29. The correlation between output and the exit rate is -0.63.
(compared to -0.78 in Campbell 1998), between output and lagged exit it is -0.16 and between output and lead exit it is -0.51.

Figures 5-7 are shown to help us understand the variables’ dynamics as they appear in the impulse response graphs. Figure 5 shows the probabilities of continuation by incumbent firms as a function of their capital stock $k$ when $(\varepsilon; Z, K)$ is set to $\varepsilon = 1$, $Z = 1$, and $K = 4$, for all four possible configurations of the current and lagged standard deviation of aggregate productivity $(\sigma, \sigma_{-1})$.

For these values of the state variables $(\varepsilon; Z, K)$, the probability of continuation increases until capital is around 2, then decreases for larger capital. The continuation probability decreases for very large firms because the optimal capital stock at these values of the state variables is lower than 2. Shedding excess capital becomes more and more expensive as a firm gets larger because of the convex adjustment costs. This makes it optimal for very large firms to exit rather than paying these costs if they get a large draw of the fixed continuation cost.

For other values of the state variables, with low to medium idiosyncratic and aggregate productivity, the shape of the graph is similar, but shifted out for higher values of idiosyncratic and aggregate productivity and lower values of aggregate capital. If idiosyncratic and aggregate productivity are high enough, there is no decreasing part of the graph over the entire range of capital, as in Figure 4.1 below. The change in the shape of the graph for different values of the current and lagged standard deviation of productivity is more complicated:
For low levels of aggregate capital, the decreasing part of the graph is shifted inwards for higher $\sigma$, whereas for high levels of aggregate capital it is shifted out when $\sigma$ is higher. Since most firms have less than 2 units of capital in steady state (see Figure 6 below, which plots the steady-state measure of firms at each possible value of capital and idiosyncratic productivity), the more relevant range of capital values is the one over which the continuation probability is increasing. This observation is important in interpreting the reactions of potential entrants to an uncertainty shock.

Figure 6: Stationary measure of firms

Figure 4.1 shows the continuation probability over values of capital for $\varepsilon = 1.15$, $Z = 1.02$ if $\sigma$ is low or $Z = 1.03$ if $\sigma$ is high, $K = 5.4$, and all four possible configurations of the current and lagged standard deviations $(\sigma, \sigma_{-1})$. 


4.2 Impulse responses

Each of Figures 8 to 19 below plots the impulse response function of a variable to an uncertainty shock. An uncertainty shock is defined as a period of imposed high variance of the aggregate productivity shock. I consider shocks lasting 1 period, 2 periods and 5 periods. I simulate 100,000 economies, each with 95 periods, and impose high variance $\sigma$ from period 70 to period $70 + t_0$, where $t_0$ takes the values 0, 1, and 4. From period $70 + t_0 + 1$ on, the economy is allowed to evolve normally again (that is, the process for the variance of the productivity shock follows its Markov chain as in equation (46)). The value of the variable of interest is averaged across economies. The impulse response at period $t \geq 70$ is measured as the percentage difference between the average value of the variable at period $t$ and its average value at period 69, as in Bloom et al. (2014). Figures 8-15 show the impulse responses of the total measure of firms, the measure of entrants and of exitors, the average capital of exitors, of entrants and of incumbents, the output of entrants and of incumbents, and the employment of entrants and of incumbents, for uncertainty shocks lasting one period, 2 periods and 5 periods.
Figure 8: Total measure of firms

Figure 8 shows that an uncertainty shock leads to an immediate and sustained decrease in the measure of firms. Figure 9 below explains this as being mainly caused by a decreased number of entrants.
The top graph in Figure 9 shows that immediately after the shock there is a very large drop of about 5.5% in the measure of entrants. Since in normal times entrants make up around 7 to 9 percent of all firms by measure, this drop leads to a simultaneous drop by over 0.4% in the total measure of firms. The decrease in entrants is compounded by a small initial increase in exitor
measure of around 0.7%. This increase comes from the increased spread in the aggregate productivity shock (recall that these graphs are based on the average of many economies with different paths of aggregate productivity), so that on average more firms lie in the exit region. After the initial onset of the shock, firms adjust to expecting further high uncertainty for a while, as the probability of high uncertainty next period is 0.7 when uncertainty this period is high. Fewer firms now exit because surviving larger firms have responded to the high uncertainty by investing less (see Figure 19 below) and because the total measure of firms is less; the average size of exitors slightly falls because of this adaptation by firms, and because the overall average capital stock of firms is smaller.

By the next period after the initial shock (period 2 in the graph), the measure of entrants has risen back to only 2 percent less than its pre-shock value in Figure 9. This is the same for the 1-period, 2-period and 5-period shock experiments. This bounce back in entrant measure is most likely caused by the decreased interest rate, which comes from overall decreased demand for investment by incumbent firms. It can also be interpreted in terms of consumers’ actions: The household correctly expects output and consumption to decrease over the period from time 1 to time 5. Therefore the time-$(t+1)$ marginal utility of consumption is higher than its value at time $t$ for a few periods, making the expected interest rate lower than usual. This makes it more worthwhile for potential firms to enter in period 2 compared to period 1, and the measure of entrants rises. The measure of entrants returns to the pre-shock level after around 7 periods and stays there for the rest of the sample period. Mechanically, the measure of entrants falls for a only short time because aggregate capital also falls, which in turn makes entry more profitable (aggregate capital, as a state variable in the computation of firms’ value functions, is inversely related to value at each idiosyncratic capital and productivity level). Intuitively, this corresponds to business conditions becoming better because there are fewer firms in the market, opening up possibilities for new entrants.

The temporary drop in entrants and increase in exitors generate a much longer-term dip in the total measure of firms, as seen in Figure 8. The measure of firms starts to rise after 5 periods, but it does not return to its initial level until around period 20. This corresponds to the “missing generation effect” discussed by Clementi et al. (2015) and Siemer (2014), where the effects of a negative first-moment aggregate TFP shock are amplified and prolonged by a lack of new entrants to replace increasing numbers of exiting firms. Here the missing generation effect is caused by the disproportionate negative impact of high uncertainty on small firms, which tend to be younger. Precisely because firms start out small and small firms need a higher idiosyncratic productivity in order to continue, young firms that have survived after a few years tend to have higher than average productivity. Once a firm has grown to a certain size, it is less likely to exit with a bad productivity shock unless it is at the very high end of the capital range. Thus, older firms (which tend to be larger, as
negative investment is rarely undertaken because of the partial irreversibility of investment) are less productive on average than younger ones.

Figure 10 below shows that the average capital stock of entrants, defined as the total capital of entrants divided by the total measure of entrants, rises by 0.6% just after the shock, then declines quickly. Seeing the higher level of uncertainty, potential entrants with low idiosyncratic productivity signals predict that the likelihood of having to exit in the next period or soon after is greater than in normal times. Such an outcome would lead to a negative lifetime value, worse than the zero reservation value of staying out. The productivity signal of a potential firm now has to be quite high, or the fixed random entry cost low, for it to be worth entering, and with higher signal, a higher initial capital stock is optimal. Lee and Mukoyama (2015) note that in the Annual Survey of Manufactures (ASM) data the average size of entering plants in terms of labor, relative to incumbents, is about 25% smaller in booms than in recessions.

It may also be that potential firms with a given idiosyncratic productivity signal enter with higher capital than in normal times. In a sense, capital is more valuable under high uncertainty due to the higher probabilities of extreme aggregate productivity shocks – if a very low shock hits that leads to exit, a higher exit value is obtained with more capital (since it can be resold at a fraction of its value without paying any adjustment costs), and if a very high shock hits, more profits can be obtained immediately without paying adjustment costs to grow.
Figure 10: Average capital of entrants (top) and of incumbents (bottom)

The graph of entrants’ average capital in Figure 10 helps to explain the post-shock behavior of entrants’ total capital, shown in Figure 11. At period 1 of the shock, the average capital of entrants is 0.6% higher than at period 0, because some would-be small entrants are choosing not to enter. By period 2, potential entrants with small optimal capital stock (i.e. with lower idiosyncratic productivity signals) are entering again due to the lower interest rate, which raises their expected value. Notice that after the one-period shock, the average
capital of entrants falls just back to its pre-shock level. By contrast, the two- and five-period shocks lead to a drop to almost 1% less than the pre-shock level.

Figure 11: Total capital of entrants (top) and incumbents (bottom)
Figure 12: Employment by entrants (top) and incumbents (bottom)
In Figures 11-13, the capital, output and employment of entrants are defined as the optimal values for the next period (one period after entry), since entrants have to wait for one period to produce. This is why the capital of entrants drops immediately in response to the shock (the top graph in Figure 11), whereas overall capital responds with a one-period lag (Figure 14 below).

The immediate responses of entrants to the uncertainty shock are much more pronounced than those of incumbents. For instance, the measure of entrants
drops immediately by around 5.4%, while the total measure of firms drops only by about 0.4%, and the capital of entrants drops by about 5%, compared to a drop of only up to 1% by incumbents. The response of employment by entrants is negative while for incumbents the response is positive. The negative response for entrants is due simply to the reduced number of entrants, whereas incumbents replace some capital with labor in production. The total capital stock in the economy falls, both due to more firms choosing inaction because of the interaction of the capital adjustment costs with temporarily higher uncertainty, and due to the temporarily much lower capital of entrants. Since in normal times both entrants and exitors make up around 7 to 9 percent of firms, a large decrease in average entering capital together with a large decrease in the measure of entrants can have a significant effect on aggregate output. This effect is magnified by the fact that surviving young firms tend to grow quickly (Clementi et al. 2015), making up an increasing proportion of the total output over the first few years after their entry.

The disproportionate effect of the uncertainty shock on potential entrants is likely due in part to the form of the adjustment cost functions for entrants and incumbents. The strictly convex part of the adjustment cost function for incumbents is \(\frac{c_0(k^r - (1-\delta)k)2}{k}\), whereas for entrants it is \(\gamma_1 k^2\). This captures the idea that large existing firms find it easier to make an adjustment of a given absolute value. It also makes it unlikely that an entrant will start out large – it would need a high productivity signal for that to be optimal. For a given productivity level, for most realized values of the state variables a smaller firm is at least as likely to exit than a larger one, as suggested by Figures 5 and 6 above. This causes some potential entrants to foresee that they will exit in the next period without ever having produced, thus making a negative payoff rather than the zero reservation payoff they get from staying out.

Figures 14-18 show the impulse responses of aggregate variables to a two-period uncertainty shock for both the baseline model (the one described in the earlier sections) and a model that is the same in all respects except that there is no entry or exit – firms do not need to pay a continuation cost and remain in the market even if their profits are temporarily negative. The differences in the magnitudes, and in the case of output the qualitative features of the responses, are striking.
Figure 14: Aggregate capital for baseline and no entry/exit economies
Figure 15: Aggregate output for baseline and no entry/exit economies
Figure 16: Employment for baseline and no entry/exit economies
Figure 17: Consumption for baseline and no entry/exit economies
Clearly, it makes a big difference whether entry and exit are allowed in the model. Consider first the impact of the uncertainty shocks on the model without entry and exit. This model is like that studied by Bloom et al. (2014) except that I do not include hiring frictions. The variables’ responses to an uncertainty can be divided into short-term and long-term responses. The initial effect of an unexpected high level of uncertainty is to raise employment. Labor replaces capital in production (see Figure 14 for capital and Figure 16 for labor) as higher uncertainty causes firms to become more reluctant to invest in capital. This is because firms face capital adjustment costs, but can hire and fire workers freely. Output also initially rises after the first period of high uncertainty (see Figure 15) because the convexity of profits in productivity makes firms on average initially want to produce more when uncertainty is high, as in the steady state experiments. Consumption initially rises as well, due to the higher output together with lower demand for savings, which makes the return to savings fall. This is unlike in the model with price frictions of Basu and Bundick (2014), where consumers’ precautionary saving drives the economy’s response to an uncertainty shock. After the uncertainty shock ends and the uncertainty process returns to normal, some incumbent firms have pent-up demand for capital as described by Bloom et al (2014): They received good enough idiosyncratic productivity shocks so that they would have invested more had uncertainty
not been high. Now those firms that continue to have good shocks under low uncertainty make up for past inaction by investing at a higher rate, and also by replacing labor by capital. Consumption then falls again briefly (Figure 17) because of the increase in investment and the added capital adjustment costs that are being paid. The decrease in the aggregate capital stock and in investment, shown in Figures 14 and 18, lead to a gradual decrease of aggregate output back to the pre-shock level, without ever falling much below this level (Figure 15).

By contrast, in the baseline model (with entry and exit), aggregate output drops to about 0.1% below its pre-shock value about 2 years after the initial onset of the shock. Investment and aggregate capital drop by much more, and total employment and consumption increase by less, compared to the no entry/exit economy. In addition to allowing the number of firms to vary, entry and exit alters the pre-shock distribution of firms with respect to capital and productivity. Because they can exit when they get a bad set of productivity shocks, firms are both larger and more productive on average in the pre-shock period of the baseline economy compared to the no entry-exit economy.

Bloom et al. (2014) mention that the misallocation of resources due to a rise in uncertainty is another important reason for the resulting economic downturn. In their paper, this misallocation results from firms “freezing” both hiring and investment after an uncertainty shock due to irreversibility and fixed adjustment costs, so that there are some units of labor and capital being used by less productive firms that would have been used by more productive firms in the absence of the shock. Here the entry and exit of firms allow for an additional type of misallocation: There are potential firms who do not enter, yet have higher productivity than some continuing firms. Figure 19 graphs the impulse-response function for the misallocation of labor, measured as the correlation between firms’ productivity and their share of labor.
5 Conclusions

This paper has studied the effects of increases in uncertainty – permanent and temporary, expected and unexpected – on an economy in which firms face both convex and non-convex costs of capital adjustment and can enter and exit the market endogenously. The incorporation of endogenous entry and exit into the model has significant effects on the responses of aggregate variables to an uncertainty shock. The long-term negative response of investment in the model with entry and exit contrasts with a relatively short negative response followed by an overshoot in the model without entry or exit. The long-term responses of output in the two models are qualitatively different, with a long period of output below the ergodic level in the model with entry and exit, and an initial increase followed by a gradual decrease to the ergodic level in the model without entry or exit. The differences between the models are due to the life-cycle dynamics of firms in the baseline model. In this model, small firms and potential entrants are the most negatively affected by an uncertainty shock. This creates a missing generation effect, where firms that would have grown quickly without the shock are absent from the market.