TMAT/AMAT 119   Practice Midterm 2

Name: _____

Information: Solutions to this practice exam will not be collected. The exam
may take you longer than today's class time, and you are welcome to finish at
home. Solutions will be posted online this weekend, and you are welcome to come
discuss the exam at office hours, including those after this class.
For this practice exam, you can use your notes. For the actual exam, you may
bring one page of handwritten notes (front and back). **Note that I will not be
as generous with hints on the actual exam and you should come
prepared with the necessary formulae.** No calculators, phones, etc. will be
allowed. The actual exam may be longer than this one.

1. Evaluate $\int \sin^3 x \, dx$.

2. Evaluate $\int \sin^4 x \, dx$. [HINT: Use the half-angle formula $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.]

3. Evaluate $\int \sin 3x \cos 2x \, dx$. [HINT: $\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$.

4. Compute $\int \frac{x^3}{\sqrt{x^2 + 4}} \, dx$. [HINT: use substitution $x = 2\tan\theta$, $\theta \in (-\pi/2, \pi/2)$.]

5. Compute $\int \frac{x}{\sqrt{x^2 + 4}} \, dx$. [HINT: Trig substitution is not the best approach
here!]

6. Explain what it means for the improper integral $\int_0^\infty f(x) \, dx$ to be convergent.
Use a limit in your answer.

7. Compute $\int_1^\infty \frac{1}{x^3} \, dx$. (do not just use a formula for this, but compute this
directly.)

8. Compute $\int_{-\infty}^0 \frac{1}{1 + x^2} \, dx$. [HINT: $\int \frac{1}{1 + x^2} \, dx = \tan^{-1}(x)$.]

9. Determine whether the improper integral $\int_{-1}^1 \frac{1}{x^{1/3}} \, dx$ is convergent or
divergent. If it is convergent, evaluate it.

10. Find the length of the arc $x = y^{3/2}$ between the points $(1, 1)$ and $(27, 9)$.

11. Consider the surface obtained by rotating the curve of $y = \frac{1}{3}x^3$ from $x = 0$ to
$x = 1$ about the y-axis. Write down an integral representing its surface area (you
don't need to compute it though).

12. Compute the center of mass of the region R in the right half-plane bounded by the curves $y = x^2$ and $y = x^3$.

13. Let X be a random variable with probably density function f.

   (i) What is $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx$?
   (ii) Express the probability $\Pr(0 \le X \le 1)$ in terms of f.
   (iii) Express the probability $\Pr(X \le 1)$ in terms of f.
   (iv) Express the mean of X in terms of f.