TMAT/AMAT 118 Partial Practice Final

Name: ______________________________

Note: As a reminder, the actual final is cumulative, and will also cover material from the first two midterms. This partial practice final only covers material from after the cutoff for the second midterm. That material will account for roughly 40-50% of the exam.

1. Let $\{a_n\}$ and $\{b_n\}$ be sequences. Suppose $\lim_{n\to\infty}\{a_n\} = -1$ and $\lim_{n\to\infty}\{b_n\} = 1$. What can be said about $\lim_{n\to\infty}\{a_n + b_n\}$?

2. Let $\{a_n\} = a_1, a_2, a_3, \dots$ be a sequence with $\lim_{n\to\infty}\{a_n\} = 2$, and let $\{b_n\}$ be the sequence given by

$$b_n = \begin{cases} a_n & \text{for } n > 1, \\ 0 & \text{for } n = 1 \end{cases}.$$

What is $\lim_{n\to\infty} b_n$?

3. For each condition, say whether the series $\sum_{n=1}^{\infty} a_n$ converges, diverges, or you can't tell. Explain your answer.

- $\sum_{n=1}^{\infty} -a_n$ converges.
- $\lim_{n\to\infty} a_n \neq 0$,
- $\lim_{n\to\infty} a_n = 0$,
- $\sum_{n=1}^{\infty} |a_n|$ converges,

- $\sum_{n=1}^{\infty} |a_n|$ diverges.

4. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ converge or diverge? Explain your answer.

5. Does $\sum_{n=1}^{\infty} \frac{1}{n^3+1}$ converge or diverge? Explain your answer.

6. Does $\sum_{n=1}^{\infty} \frac{1}{n^3-1}$ converge or diverge? Explain your answer.

7. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{4^n}$? Explain your answer.

8. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^4}$? Explain your answer.

9. Find an infinite series expression for $\frac{1}{2-x}$.

10. Compute the first two non-zero terms of Taylor series for $\sin x$ centered around $a = \pi/2$. Do this computation directly, don't just quote a formula.

11. Is the series $\sum_{n=1}^{\infty} \frac{1}{2^{n+2}}$ convergent or divergent? If convergent, find the value. If divergent, explain why.