AMAT/TMAT 118

Cheat Sheet for Sets and Functions

As already noted, our midterm will have some material on sets and functions, as in class and my notes notes on the topic. This document concisely reviews the material about sets and functions you will need to know for the midterm. Not everything from my notes on sets and functions is summarized on this cheat sheet, but all that you need for the midterm is summarized here.

- $\{x, y, z\}$ = the set containing the elements x, y, and z.
- $\mathbb{N}$ = natural numbers. That is, $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.
- $\mathbb{Z}$ = integers $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. That is $\mathbb{Z}$ includes both positive and negative numbers, but not fractions.
- $\mathbb{Q}$ = rational numbers. (You will not be asked any tricky theoretical question about rationals or reals on the midterm.)
- $\mathbb{R}$ = real numbers.
- $a \in S$ means "a is an element of the set S."
  - For example, $5 \in \mathbb{Z}$, becuase 5 is an integer.
- $a \notin S$ means "a is not an element of the set S." For example, $\frac{1}{2} \notin \mathbb{Z}$, because $\frac{1}{2}$ is not an integer.
- $\emptyset$ = the empty set, i.e., the set with no elements. This is also can be written as $\{\}$.
- For sets S and T, $S \subset T$ means "S is a subset of the set T." This means that each element of S is also an element of T.
  - For example, $\mathbb{N} \subset \mathbb{Z}$, and $\{x, y\} \subset \{x, y, z\}$.
- Remember that for any set S, $\{\} \subset S$ and $S \subset S$.
- $S \not\subset T$ means "S is not a subset of T".
  - For example, $\{x, y, z\} \not\subset \{x, y\}$.

- $S \cup T$ = the *union* of sets S and T, that is, the set consisting of elements which are either in S or in T.
  - For example,
  $$\{A, B\} \cup \{B, C\} = \{A, B, C\}.$$
- $S \cap T$ = the *intersection* of sets S and T, that is, the set consisting of elements which are both in S and in T.
  - For example,
  $$\{A, B\} \cap \{B, C\} = \{B\}.$$
- Given sets S and T, a *function f from S to T* is a rule which assigns to each element in S a single element of T. We often write this as $f : S \to T$.
  - For example, for $S = \{0, 1, 2\}$ and $T = \{A, B, C\}$, we can define a function $g : S \to T$ by $g(0) = A$, $g(1) = B$, $g(2) = A$.
  - As another example, we can define a function $h : \mathbb{Z} \to \mathbb{Z}$ by the formula $h(z) = 2z$.
- For a function $f : S \to T$, we call S the *domain* of f and T the *codomain* of f.
- The range of a function $f : S \to T$ is the subset of T consisting of elements of the form $f(s)$ for some $s \in S$.
  - For example, for g as in the example above, $\operatorname{range} g = \{A, B\}$.
- We say a function $f : S \to T$ is:
  - *1-1* if $f(a) \neq f(b)$ whenever $a \neq b$.
  - *onto* if $\operatorname{range} f = T$,
  - *a bijection* if f is both 1-1 and onto.
  - For example, the function $g : S \to T$ above is not 1-1, because
  $$g(0) = g(2) = A,$$
  and g is not onto because $C \notin \text{range } g$.
  - As another example, the function $h : \mathbb{Z} \to \mathbb{Z}$ above is 1-1, but h is not onto.

- Functions $f : S \to T$ and $g : T \to S$ are said to be *inverses* if:
  - for every $s \in S$, $g(f(s)) = s$, and
  - for every $t \in T$, $f(g(t)) = t$.
- Fact: A function f has an inverse if and only if f is bijective.