TMAT/AMAT 118 Practice Midterm 1

Name: ______________________________

1. [3 points] Let $S = \{1, 2, 3\}$ and $T = \{2, 3, 4, 5\}$.

   (a) What is $S \cap T$? **Answer**: $\{2, 3\}$
   (b) What is $S \cup T$? $\{1, 2, 3, 4, 5\}$
   (c) What are all the subsets of S? HINT: There are eight of them.
   Answer: $\{1, 2, 3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{\}$.

2. [6 points] Let S and T be as in the previous question, and let $f : S \to T$ be given by

$$
\begin{aligned}
f(1) &= 3,\\
f(2) &= 4,\\
f(3) &= 5.
\end{aligned}
$$

   (a) What is the domain of f? **Answer**: S.
   (b) What is range f? **Answer**: $\{3, 4, 5\}$
   (c) Is f 1-1? **Answer**: yes. f sends no two distinct elements in S to the same element of T.
   (d) Is f onto? **Answer**: no: $2 \in T$, but $2 \notin$ range T.
   (e) Does f have an inverse? If so, what is it? **Answer**: no it doesn't. A function has an inverse if and only if it is a bijection, and f is not a bijection.
   (f) Fill in the blank so that the function $g : S \to T$ defined by the following is *not* 1-1:

$$
\begin{aligned}
g(1) &= 3,\\
g(2) &= 4,\\
g(3) &= \_\_\_.
\end{aligned}
$$

Answer: There two possible answers: 3 and 4. Either one is correct.

3. [5 points] Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2$.

(a) Draw the graph of f.
(b) What is range f? **Answer**: $[0, \infty)$
(c) Is f onto? **Answer**: No, because $[0, \infty) \neq \mathbb{R}$.
(d) Is f 1-1? **Answer**: No, because for $x > 0$, $f(x) = f(-x)$.
(e) Does f have an inverse? **Answer**: No. It is not a bijection.

4. [2 points] Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = -2x$. Find the inverse of f. [Hint: Write $y = -2x$, and solve for x in terms of y. You do not have to explain why the function you get is indeed the inverse.]. **Answer**: we solve $y = -2x$ for x to get $x = -y/2$. Thus, the inverse is given by $g(y) = -y/2$.

5. [2 points] Is the function

$$f(x) = 1 + \sin x$$

1-1? Explain. **Answer**: No. For example $f(0) = 0 = f(\pi)$. (To understand this in terms of the graph of f, note that we can draw a horizontal line that intersects the graph of f in more than one place. This means f is not 1-1.)

6. [11 points] Give each of the following limits. Note that some limits may exist but be ∞ or $-\infty$. If the limit does not exist (even as ∞ or $-\infty$), write DNE.

$$\begin{aligned}
(i)\ & \lim_{x\to 2} 3 = 3 \\
(ii)\ & \lim_{x\to 2} x^2 + 3x + 1 = 11 \\
(iii)\ & \lim_{x\to 1} \frac{1}{x^3} + 4x = 5 \\
(iv)\ & \lim_{x\to 0} \frac{3}{x^2} = \infty. \\
(v)\ & \lim_{x\to 0} \frac{2}{x} = \text{DNE} \\
(vi)\ & \lim_{x\to 1} \frac{x^2+1}{x+2} = 2/3 \\
(vii)\ & \lim_{x\to 1} \frac{1}{x^{1/3}} = 1 \\
(viii)\ & \lim_{x\to 1} \frac{1}{x^{1/3}} - \frac{1}{x^3} = 0 \\
(ix)\ & \lim_{x\to 1} \frac{x-2x+1}{x-1} = \lim_{x\to 1} x - 1 = 0 \\
(x)\ & \lim_{x\to 4^-} 3x + 1 = 13
\end{aligned}$$

7. [3 points] Let

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ x^2 + 1 & \text{if } x \geq 1. \end{cases}$$

Give each of the following limits. If the limit does not exist, write DNE.

$$(i) \lim_{x \to 1^-} f(x) = 1$$

$$(ii) \lim_{x \to 1^+} f(x) = 2$$

$$(iii) \lim_{x \to 1} f(x) = \text{DNE}$$

8. [3 points] Let

$$g(x) = \begin{cases} -x & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0. \end{cases}$$

Give each of the following limits. If the limit does not exist, write DNE.

$$(i) \lim_{x \to 0^-} f(x) = 0$$

$$(ii) \lim_{x \to 0^+} f(x) = 0$$

$$(iii) \lim_{x \to 0} f(x) = 0$$

9. [1 point] Suppose

$$\lim_{x \to 5} f(x) = 3, \qquad \lim_{x \to 5} g(x) = 7.$$

What is $\lim_{x \to 5^-} \frac{f(x)}{g(x)}$? **Answer**: $\frac{3}{7}$

10. [1 point] What is the domain of the function $f(x) = \sqrt{1 - x^2}$? Express the answer using interval notation. **Answer**: [-1,1]

11. [1 point] Let $f(x) = 3x + 1$ and $g(x) = 3x - 1$. What is $g \circ f(1)$? **Answer**:

$$g \circ f = 3(3x + 1) - 1 = 9x + 2.$$

So $g \circ f(1) = 11$.