TMAT/AMAT 118   Practice Midterm 1

Name: _____

1. [3 points] Let $S = \{1, 2, 3\}$ and $T = \{2, 3, 4, 5\}$.

    (a) What is $S \cap T$?

    (b) What is $S \cup T$?

    (c) What are all the subsets of S? HINT: There are eight of them.

2. [6 points] Let S and T be as in the previous question, and let $f : S \to T$ be given by

$$f(1) = 3,$$
$$f(2) = 4,$$
$$f(3) = 5.$$

    (a) What is the domain of f?

    (b) What is range f?

    (c) Is f 1-1?

    (d) Is f onto?

    (e) Does f have an inverse? If so, what is it?

    (f) Fill in the blank so that the function $g : S \to T$ defined by the following is *not* 1-1:

$$g(1) = 3,$$
$$g(2) = 4,$$
$$g(3) = \text{----}.$$

3. [5 points] Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2$.

    (a) Draw the graph of f.

    (b) What is range f?

    (c) Is f onto?

    (d) Is f 1-1?

    (e) Does f have an inverse?

4. [2 points] $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = -2x$. Find the inverse of f. [Hint: Write $y = -2x$, and solve for x in terms of y. You do not have to explain why the function you get is indeed the inverse.]

5. [2 points] Is the function

$$f(x) = 1 + \sin x$$

1-1? Explain.

6. [11 points] Give each of the following limits. Note that some limits may exist but be ∞ or $-\infty$. If the limit does not exist (even as ∞ or $-\infty$), write DNE.

(i) $\lim\limits_{x \to 2} 3 =$

(ii) $\lim\limits_{x \to 2} x^2 + 3x + 1 =$

(iii) $\lim\limits_{x \to 1} \dfrac{1}{x^3} + 4x$

(iv) $\lim\limits_{x \to 0} \dfrac{3}{x^2} =$

(v) $\lim\limits_{x \to 0} \dfrac{2}{x} =$

(vi) $\lim\limits_{x \to 1} \dfrac{x^2 + 1}{x + 2} =$

(vii) $\lim\limits_{x \to 1} \dfrac{1}{x^{1/3}} =$

$(viii)$ $\lim\limits_{x \to 1} \dfrac{1}{x^{1/3}} - \dfrac{1}{x^3} =$

(ix) $\lim\limits_{x \to 1} \dfrac{x - 2x + 1}{x - 1} =$

(x) $\lim\limits_{x \to 4^-} 3x + 1 =$

7. [3 points] Let

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ x^2 + 1 & \text{if } x \geq 1. \end{cases}$$

Give each of the following limits. If the limit does not exist, write DNE.

(i) $\lim\limits_{x \to 1^-} f(x) =$

(ii) $\lim\limits_{x \to 1^+} f(x) =$

(iii) $\lim\limits_{x \to 1} f(x) =$

8. [3 points] Let

$$g(x) = \begin{cases} -x & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0. \end{cases}$$

Give each of the following limits. If the limit does not exist, write DNE.

(i) $\lim\limits_{x \to 0^-} f(x) =$

(ii) $\lim\limits_{x \to 0^+} f(x) =$

(iii) $\lim\limits_{x \to 0} f(x) =$

9. [1 point] Suppose
$$\lim_{x \to 5} f(x) = 3, \qquad \lim_{x \to 5} g(x) = 7.$$

What is $\lim_{x \to 5^-} \frac{f(x)}{g(x)}$?

10. [1 point] What is the domain of the function $f(x) = \sqrt{1 - x^2}$? Express the answer using interval notation.

11. [1 point] Let $f(x) = 3x + 1$ and $g(x) = 3x - 1$. What is $g \circ f(1)$?