TMAT/AMAT 118 Partial Practice Final

Name: ____________________________

Note: As a reminder, the actual final is cumulative, and will also cover material from the first two midterms. This partial practice final only covers material from after the cutoff for the second midterm.

1. The area of a circular disc is expanding at a rate of $10m^2/s$. What is the rate of change of the radius of the disc when the area of the disc is 400π?

2. Let $y(t)$ denote the population of squirrels in Albany, NY t years after the year 2000. Suppose that were 1000 squirrels in Albany in 2000, and that $y(t)$ satisfies $y' = 1.01y$. How many squirrels will there be in Albany in 2019?

3. Give the definition of a local minimum of a function f.

4. If the domain of $f(x)$ is $[0, \infty)$, is it possible for f to have a global maximum at $x = 0$? What about a local maximum?

5. If the domain of f is $\mathbb{R}$ and the values of the local maxima of f are $0, 1$, and 2, what can be said about a global maximum of f?

6. True or false: Every function $f : [-1, 1] \to \mathbb{R}$ has an absolute maximum.

7. Find all critical numbers of each of the following functions.

- $f(x) = |x|$,
- $f(x) = \sqrt[3]{x+1}$, (Hint: Use the chain rule to compute the derivative of f),
- $f(x) = \frac{1}{3}x^3 - 4x + 7$.

8. Let $f : [-1, 5] \to \mathbb{R}$ be given by $f(x) = \frac{1}{3}x^3 - 4x + 7$.

- On what interval(s) is f increasing? On what interval(s) is f decreasing?
- On what interval(s) is f concave up? On what interval(s) is f concave down?
- At which x does f have a point of inflection?
- What are the absolute max and absolute min of f? At what values of x are these attained?
- What are local maxes and mins of f? At what values of x are these attained?

9. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable, $f(0) = 0$, and $f(1) = 1$. Use the mean value theorem to complete the following: There exists $c \in (0, 1)$ such that _______.

10. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable, $f(0) = 0$, and $f'(x) \leq -1$ for all $x \in (0, 1)$. What is the maximum value that $f(1)$ can possibly take? Hint: apply the mean value theorem to f on the interval $[0, 1]$.

11. Apply l'Hospital's rule to compute $\lim_{x \to 0} \frac{x^2}{\ln(x+1)}$

12. Find the most general antiderivative of $\frac{1}{\sqrt{x}} + 2\cos(x) + 5 + 4^x$.

13. Solve the differential equation $f''(x) = 2x - \sin(x) + 1$, $f'(0) = 1$, $f(0) = 1$.