TMAT/AMAT 118 Limits Quiz

Name: ____________________________

1 [2 points]. For $f : S \to T$ a function and $U \subset S$, give the definition of $f(U)$ (that is, give the definition of the image of U under f). **Answer**:

$$f(U) = \{t \in T \mid t = f(s) \text{ for some } s \in U.\}$$

2 [3 points]. Let $S = \{A, B, C\}$, and let $f : S \to S$ be given by $f(A) = B$, $f(B) = C$, and $f(C) = A$. For each of the following choices of U, what is $f(U)$?

(a) $U = S$, **Answer**: S.[1]
(b) $U = \{A, B\}$, **Answer**: $\{B, C\}$.
(c) $U = \{B\}$, **Answer**: $\{C\}$.

3 [4 points]. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x + 1$. For each of the following choices of U, what is $f(U)$?

(a) $U = \{0\}$, **Answer**: $\{1\}$
(b) U is the interval $(0, 1)$, **Answer**: $(1, 2)$
(c) U is the interval $(0, \infty)$, **Answer**: $(1, \infty)$
(d) $U = \mathbb{R}$ **Answer**: $\mathbb{R}$.

4 [1 point]. True or False: For every even integer z, there exists an odd integer y such that $y = 2z$. HINT: To get an idea of the answer, it may help to look at a few examples of even integers. **Answer**: False. For example take $z = 2$. $2z = 4$ is even, so there is no such y.

5 [1 point]. Define a punctured ball centered at $p \in \mathbb{R}$, as I defined it in the notes and in class. **Answer**: A punctured ball centered at p is a set of the form

$$(p - \epsilon, p) \cup (p, p + \epsilon)$$

for some $\epsilon > 0$.

6 [1 point]. Is the singleton set $\{0\} \subset \mathbb{R}$ a punctured ball centered at 0? **Answer**: No.

7 [3 points]. Complete the following to give the intuitive definition of a limit:

Let f be a function defined near $p \in \mathbb{R}$, but not necessarily at p. We write

$$\lim_{x \to p} f(x) = L$$

if ___.

[1] There was a typo in this question. It originally said $U = \{S\}$ instead of $U = S$. (For $U = \{S\}$, $f(U)$ is not defined, which wasn't what I intended.) Given the typo, I awarded everyone full credit for this subproblem.

8. [3 points] Fill in the three blanks in the following precise definition of a limit. (Write the answers below the question). **Answer**: See the beginning of Section 2.2 in the textbook.

Suppose we are given:

- $S \subset \mathbb{R}$,
- $p, L \in \mathbb{R}$ such that S contains a punctured ball centered at p,
- a function $f : S \to \mathbb{R}$.

We write

$$\lim_{x \to p} f(x) = L$$

if for every ___(a)___, there exists ___(b)___ such that ___(c)___.

Answer: There are two correct answers:

Answer 1:
(a) ball C centered at L
(b) a punctured ball B centered at p
(c) $f(B) \subset C$.

Answer 2:
(a) $\epsilon > 0$
(b) $\delta > 0$
(c) if $0 < |x - p| < \delta$, then $|f(x) - L| < \epsilon$.